Using Shape Spaces for Structure from Motion

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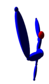
Joint Work: Appu Shaji, Yashoteja Prabhu, Pascal Fua, S. Ladha & other ViGIL students

Note: These slides are best seen with accompanying video

Problem Definition

Can we understand motion using a single camera?



Given 2D point tracks of landmark points from a *single view point*, recover 3D pose and orientation

Assumptions

- 2D tracks of major landmark points are provided
- Scaled-projective/orthographic projection model.

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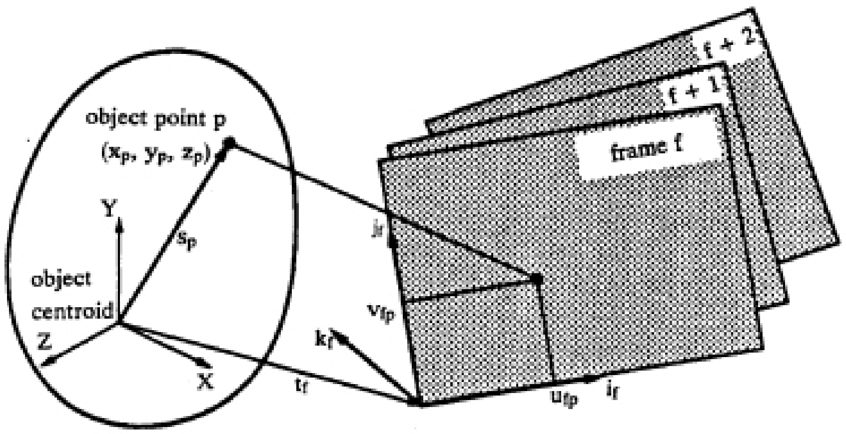
Rigid Body Geometry and Motion



- Object centroid based World Co-ordinate System (WCS)

Rank Theorem

Define $\tilde{x}_{ij} = x_{ij} - \bar{x}_i$ and $\tilde{y}_{ij} = y_{ij} - \bar{y}_i$ where the bar notation refers to the centroid of the points in the ith frame. We have the *measurement matrix*

$$\mathbf{\bar{W}_{2F\times P}} = \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1p} \\ y_{11} & \cdots & y_{1p} \\ \vdots & \vdots & \vdots \\ \tilde{x}_{f1} & \cdots & \tilde{x}_{fp} \\ y_{f1} & \cdots & y_{fp} \end{pmatrix}$$

The matrix $\bar{\mathbf{W}}$ has rank 3

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Rank Theorem Proof

$$x_{ij} = \mathbf{i}_i^T(\mathbf{P_j} - \mathbf{T_i}), \quad \mathbf{y_{ij}} = \mathbf{j_i^T}(\mathbf{P_j} - \mathbf{T_i}), \quad \frac{\mathbf{1}}{\mathbf{n}}\sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{n}} \mathbf{P_j} = \mathbf{0}$$

$$\tilde{x}_{ij} = \mathbf{i}_i^T(\mathbf{P_j} - \mathbf{T_i}) - \frac{\mathbf{1}}{\mathbf{n}}\sum_{\mathbf{m}=\mathbf{1}}^{\mathbf{n}} \mathbf{i_i^T}(\mathbf{P_m} - \mathbf{T_i})$$

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$$\tilde{x}_{ij} = \mathbf{i}_i^T\mathbf{P_j} \qquad \tilde{\mathbf{y}}_{\mathbf{ij}} = \mathbf{j_i^T}\mathbf{P_j}$$

$$\bar{\mathbf{W}} = \mathbf{RS}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{j}_1^T \\ \cdots \\ \mathbf{i}_N^T \\ \mathbf{j}_N^T \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} \mathbf{P_1} & \mathbf{P_2} & \ldots & \mathbf{P_N} \end{bmatrix}$$

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- Without noise $\overline{\mathbf{W}}$ is atmost of rank *three*
- Using SVD, $\overline{\mathbf{W}} = \mathbf{O_1}\Sigma\mathbf{O_2}$ where,
  $\mathbf{O_1}$, $\mathbf{O_2}$ are column orthogonal matrices and Σ is a diagonal matrix with singular values in non-decreasing order
- $\mathbf{O_1}\Sigma\mathbf{O_2} = \mathbf{O}_1^{'}\Sigma^{'}\mathbf{O}_2^{'} + \mathbf{O}_1^{''}\Sigma^{''}\mathbf{O}_2^{''}$ where,
  $\mathbf{O}_1^{'}$ has *first three* columns of $\mathbf{O_1}$, $\mathbf{O}_2^{'}$ has *first three* rows of $\mathbf{O_2}$ and $\Sigma^{'}$ is 3×3 matrix with 3 largest non-singular values.
- The second term is completely due to noise and can be eliminated
- $\hat{\mathbf{R}} = \mathbf{O}_1^{'}\left[\Sigma^{'}\right]^{1/2}$ and $\hat{\mathbf{S}} = \left[\Sigma^{'}\right]^{1/2}\mathbf{O}_2^{'}$

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Rigid Body Geometry and Motion

- Solution is not unique any invertible 3×3, $\mathbf{Q}$ matrix can be written as $\mathbf{R} = (\hat{\mathbf{R}}\mathbf{Q})$ and $\mathbf{S} = (\mathbf{Q}^{-1}\hat{\mathbf{S}})$
- $\hat{\mathbf{R}}$ is a linear transformation of $\mathbf{R}$, similarly $\hat{\mathbf{S}}$ is a linear transformation of $\mathbf{S}$.
- Using the following orthonormality constraints we can find $\mathbf{R}$ and $\mathbf{S}$

$$\hat{\mathbf{i}}_f^T\mathbf{Q}\mathbf{Q}^\mathbf{T}\hat{\mathbf{i}}_f = 1$$
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$$
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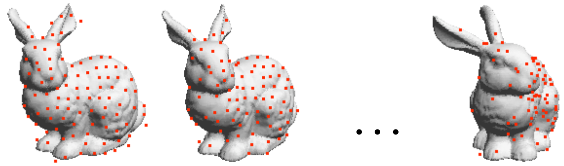
$$

Tomasi Kanade Factorisation (Recap)



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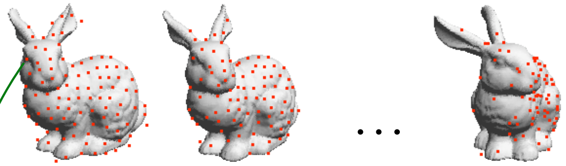

$$\begin{bmatrix} 27 & 61 & \cdots & 96 \\ 97 & 53 & \cdots & 122 \\ 28 & 62 & \cdots & 97 \\ 97 & 53 & \cdots & 122 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 94 & ? & \cdots & 131 \\ 109 & ? & \cdots & 135 \end{bmatrix}$$

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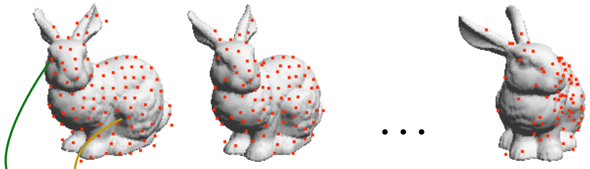

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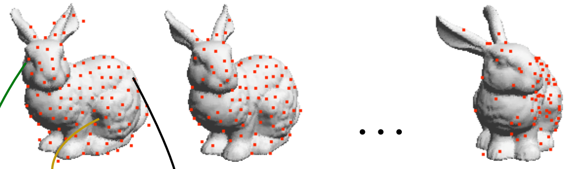

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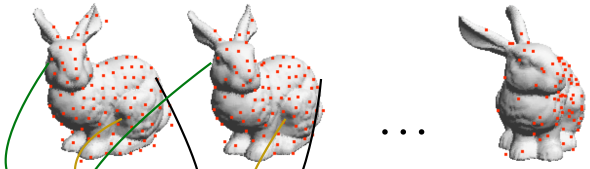

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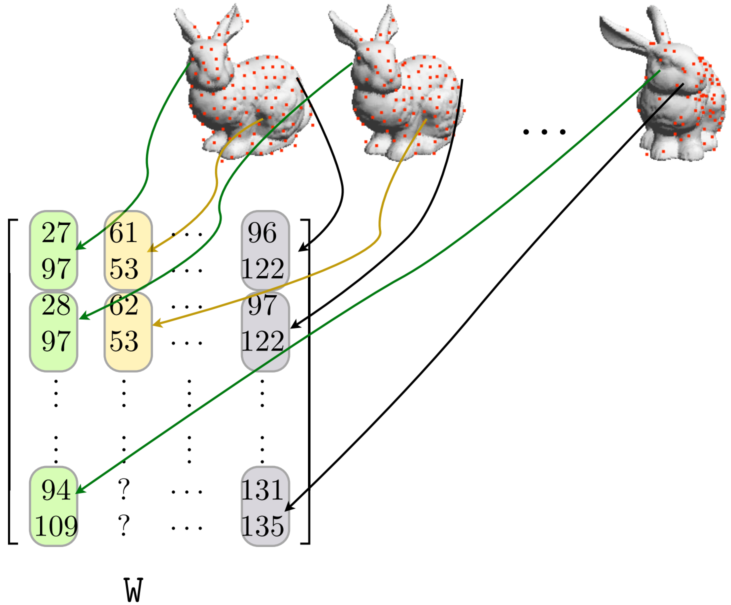

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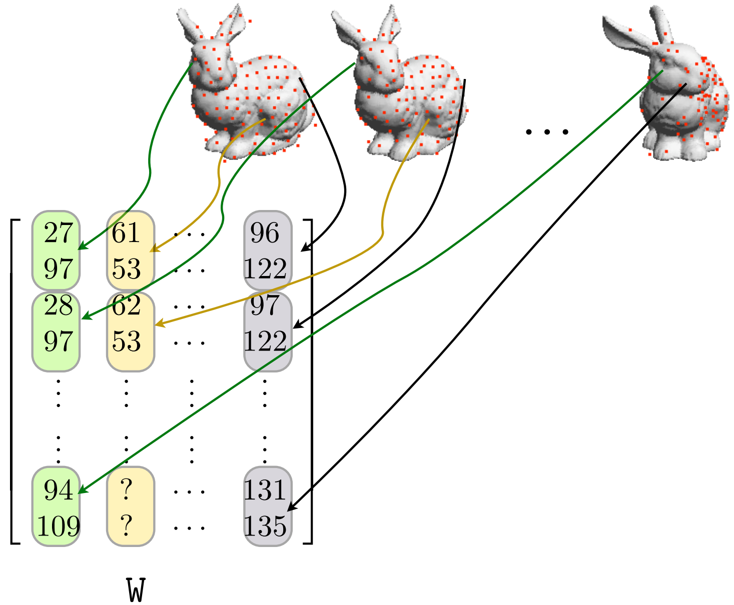

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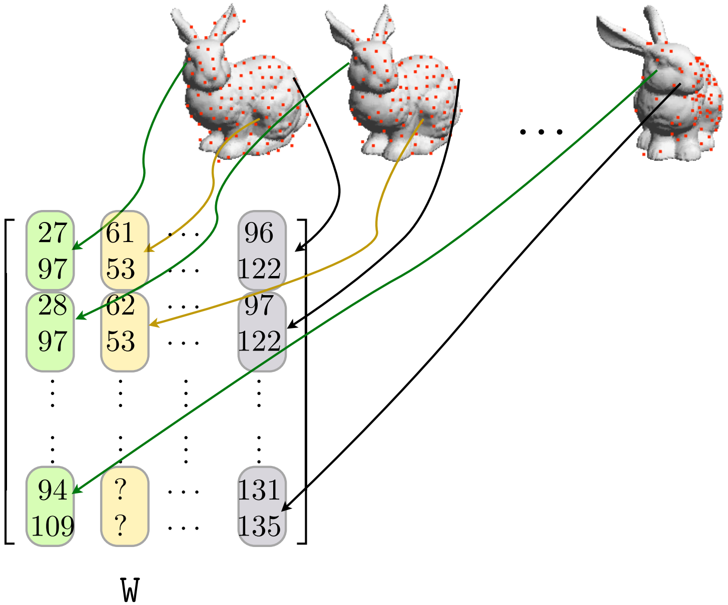

Central Observation: This matrix is rank-limited.
If the object motion is rigid the observation matrix (discounting noise) will have a maximum rank of 4

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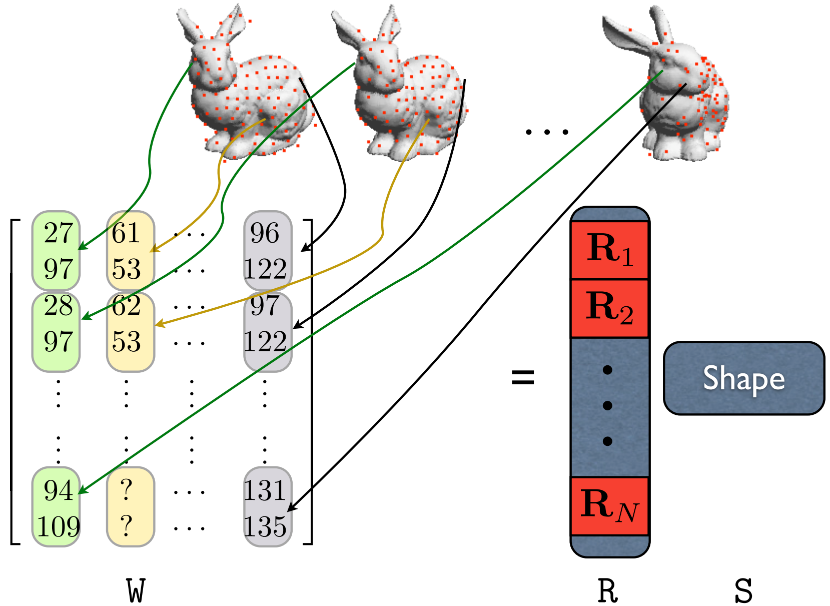

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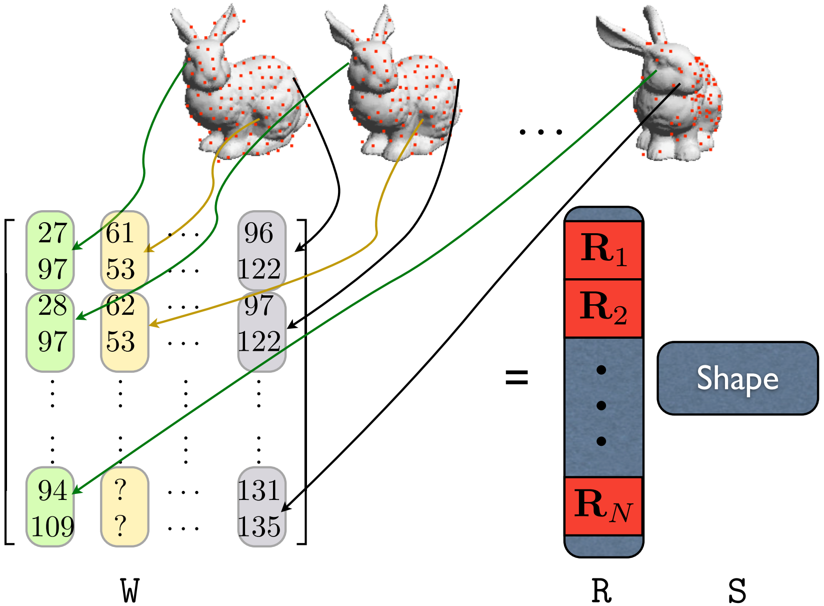

Assumptions

- Orthographic Camera Model
- Single Object in FOV of camera
- Object undergoes rigid motion
- All the points are visible throughout the sequence

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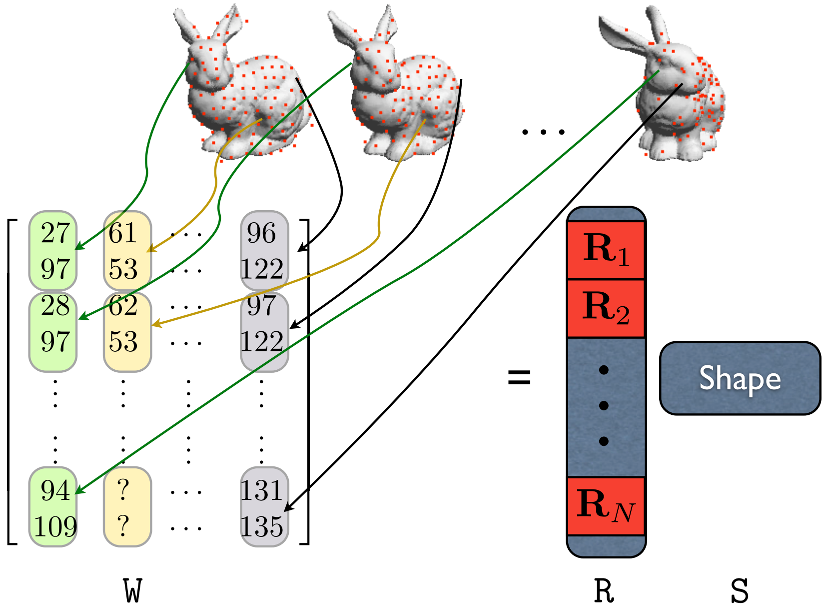

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Non-Rigid Motion

- Many objects are non-rigid
- The parametrisation $\mathbf{S}_{3\times P}$ is no longer valid.
- However, deformable bodies (like human body, face) can be represented using a linear combination of basis shapes

$$\mathbf{S_{morph}} = \sum_{i=1}^{K} c_i \mathbf{S_i} \qquad \mathbf{S_{morph}}, \mathbf{S_i} \in \mathbb{R}^{3\times P}, c_i \in \mathbb{R}$$

where $\mathbf{S_i}$'s are the bases, and c_i are the deformation weights.

Motivation Factorization Non-Rigid Motion Occlusion

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Morphable Models

One popular generalisation (used for human faces): linear combination of shapes

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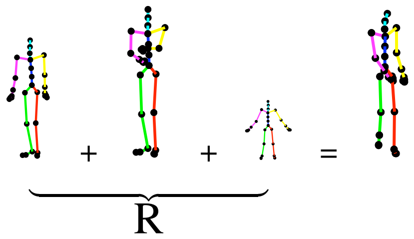


Non-Rigid Framework

- Assume that there are K shape bases $\{\mathbf{B_i} \mid i = 1, ..., K\}$
- The 3D coordinate of point p on frame f is given as,

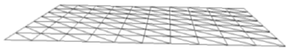
$$\mathbf{X_{fp}} = (x, y, z)^T_{fp} = \sum_{i=1}^{K} c_{fi}\mathbf{b_{ip}} \quad f = 1, ..., F, \; p = 1, ...N \quad (2)$$

- Image coordinate of $\mathbf{X_{fp}}$ under weak perspective projection model is,

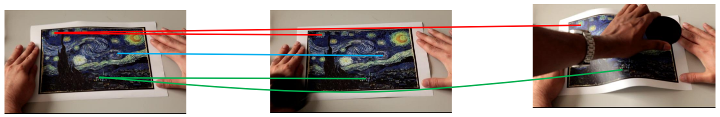
$$\mathbf{x_{fp}} = (u, v)^T_{fp} = s_f(\mathbf{R_f} \cdot \mathbf{X_{fp}} + \mathbf{t_f}) \quad (3)$$

$$\mathbf{x_{fp}} = \begin{pmatrix} c_{f1}\mathbf{R_f} & \dots & c_{fK}\mathbf{R_f} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{b_{1p}} \\ \vdots \\ \mathbf{b_{Kp}} \end{pmatrix} + \mathbf{t_f} \quad (4)$$

The Specific Problem



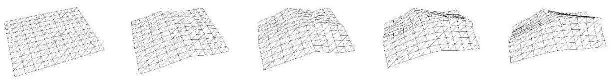
(a) INPUT : 3D mesh



(b) INPUT : Correspondences of Feature Points

Input/Output:

1. A 3D mesh structure of a deformable object;
2. Location of feature points in a video sequence
3. Recover the object 3D shape for all frames.



Motivation Factorization Non-Rigid Motion Occlusion

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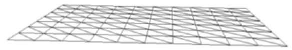

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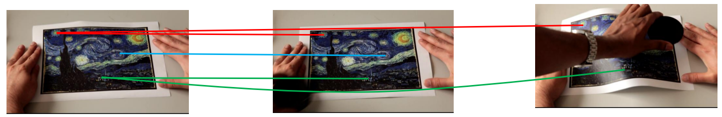

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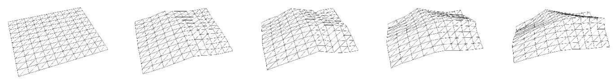

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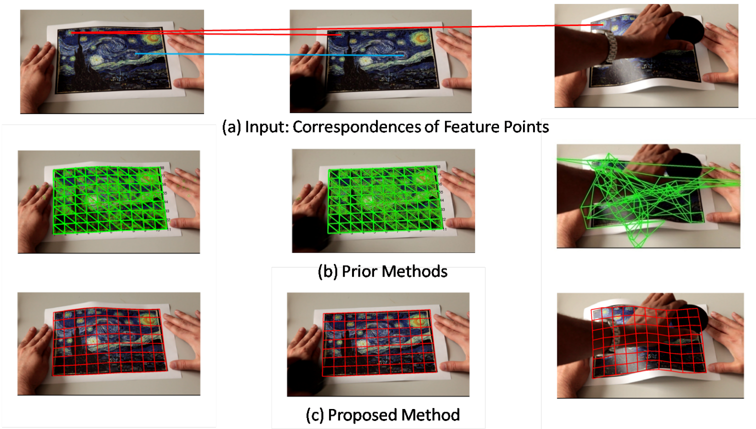

- Feature points may be partially missing due to occlusions, specular effects, etc. ...
- Reconstruction under occlusions is very troublesome[6] and state-of-the-art algorithms are inadequate.

Motivation Factorization Non-Rigid Motion Occlusion

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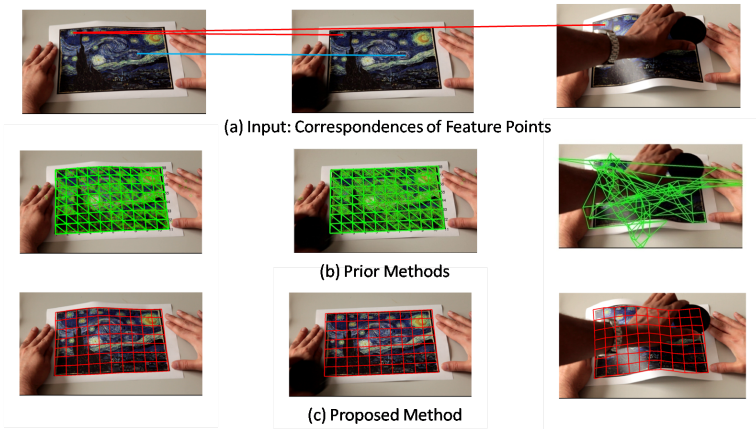

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Intuition and Idea

Intuition

It should be possible to solve for the missing region in a specific frame, based on the data available in the current, previous and subsequent frames.

Idea

- We assume that the surface is inelastic and deformations should preserve the length of every edge in the mesh.
- We want to find a shape that is consistent with temporal constraints, the deformation model, and one that minimizes the reprojection error.
- This is formulated as an optimization problem on the Riemannian Shape Space.

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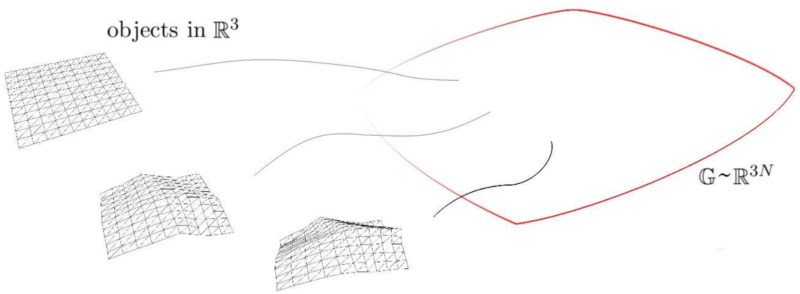
Shape Spaces



- Every point on this space is a 3D mesh.
- A time varying curve in this space corresponds to a deforming shape.
- Technicality: The *local* distance between two neighbouring points is given by the difference in edge lengths of the two meshes.

Motivation Factorization Non-Rigid Motion **Occlusion**

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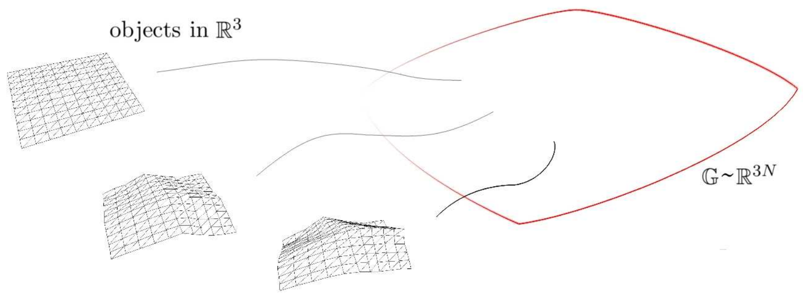

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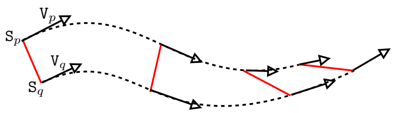
Riemannian Metric



$$\forall \text{ Edge } (p,q) \in \text{Mesh}, \quad \|S_p - S_q\|_2 = \text{const} \tag{5}$$

$$\forall \text{ Edge } (p,q) \in \text{Mesh}, \quad \langle V_p - V_q, S_p - S_q \rangle = 0 \tag{6}$$

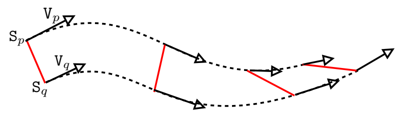
where S_p and S_q are the 3D positions and V_p and V_q are the velocities of vertices p and q respectively.

$$\|V\|_{\text{Iso}} = \sum_{(p,q)\in\text{Mesh}} \langle V_p - V_q, S_p - S_q \rangle$$

A vanishing norm indicates an isometric deformation.

Motivation Factorization Non-Rigid Motion Occlusion

Riemannian Metric



$$\forall \text{ Edge } (p,q) \in \text{Mesh}, \quad \|S_p - S_q\|_2 = \text{const} \tag{5}$$

$$\forall \text{ Edge } (p,q) \in \text{Mesh}, \quad \langle V_p - V_q, S_p - S_q \rangle = 0 \tag{6}$$

where S_p and S_q are the 3D positions and V_p and V_q are the velocities of vertices p and q respectively.

$$\|V\|_{\text{Iso}} = \sum_{(p,q)\in\text{Mesh}} \langle V_p - V_q, S_p - S_q \rangle$$

A vanishing norm indicates an isometric deformation.

Introducing Vision: Reprojection Error

- The 3D coordinates $\mathtt{F}_i^j$ of a feature point j in frame i are given by:

$$\mathtt{F}_i^j = a_j \mathtt{S}_i^{j_1} + b_j \mathtt{S}_i^{j_2} + c_j \mathtt{S}_i^{j_3} \quad (7)$$

where a_j, b_j, c_j are the barycentric coordinates of point j in triangle formed by vertices $\mathtt{S}_i^{j_1}$, $\mathtt{S}_i^{j_2}$, and $\mathtt{S}_i^{j_3}$.

- We have: $\mathtt{f}_i^j = \frac{1}{w_i^j}.\mathbf{C}.\mathtt{F}_i^j$, with $\mathtt{f}_i^j$ the 2D location of feature point j, and $\mathbf{C}$ the perspective projection matrix.
- We can rewrite this equation using Eq. (7) as:

$$\mathbf{m}_i^j.\mathtt{S}_i = 0$$

- By stacking such equation for all feature points, we get the linear system:

$$\mathbf{M}_i.\mathtt{S}_i = 0$$

Therefore, the desired shape $\mathtt{S}_i$ belongs to the null space of $\mathbf{M}_i$.

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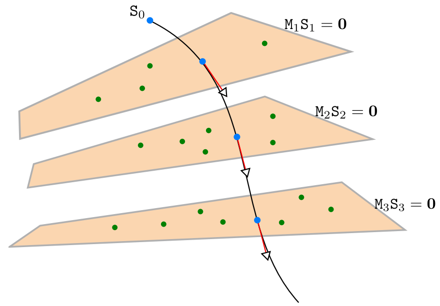
Motivation Factorization Non-Rigid Motion **Occlusion**

Formulation



The goal is to fit a curve $\{\mathtt{S}_i\}$ in the shape space for the input video sequence.

- The curve should be a geodesic curve to respect the edge length constraint;
- The points on curve should belong to the null space of the $\mathbf{M}_i$ matrices.

Computer Vision Approach: Energies

Deformation Error: $\mathrm{E_D}$ measures the non-isometricity in a deformation sequence.

$$\mathrm{E_{Deform}} = \sum_{i=1}^{F} \sum_{(\mathrm{s}_p, \mathrm{s}_q) \in \mathrm{Mesh}} < \dot{\mathrm{s}_p} - \dot{\mathrm{s}_q}, \mathrm{s}_p - \mathrm{s}_q >^2$$

Reprojection Error:

$$\mathrm{E_{Reproj}} = \sum_{j=1}^{F} ||\mathbf{M}_j \mathrm{s}_j||_2^2$$

Optional Temporal Smoothness Error:

$$\mathrm{E_{Temporal}} = \sum_{i=1}^{F-2} \sum_{\mathrm{v}_i^j \in \mathrm{Vertices}} ||\mathrm{v}_i^j + \mathrm{v}_{i+2}^j - 2\mathrm{v}_{i+1}^j||^2$$

Motivation Factorization Non-Rigid Motion Occlusion

Formulation

Cumulative Cost Function

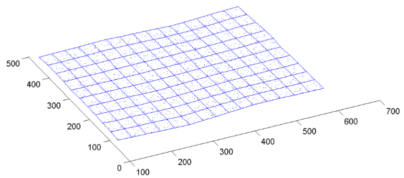
We minimize the following non-convex error term:

$$\min_{s_1 \cdots s_F} \mathrm{E_{Deform}} + \lambda_1 \mathrm{E_{Reproj}} + \lambda_2 \mathrm{E_{Temporal}}$$

- Many commercial softwares can be used, e.g., ‘`fminunc`’ function in matlab.
- However, due to high dimensionality and non-convex nature of problem, we require a reasonable initialization point for the optimization.
- Good initialization leads to faster and better convergence.

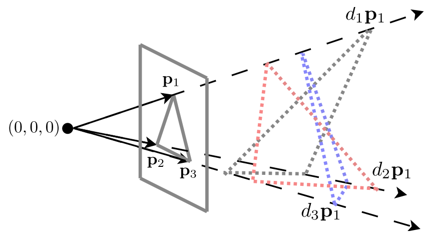
Initialization - Stage 1

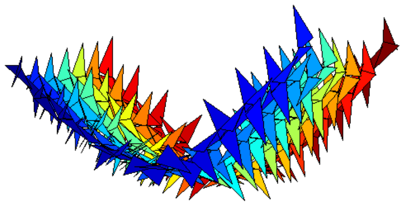
We first recover the *2D projection* of the mesh vertices using weak perspective projection assumption



Initialization - Stage II

- By enforcing mesh length constraints, we recover a maximum of 4 possible shapes for every mesh triangle[1].





1 M. Fischler and R. Bolles. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography. Communications ACM, 24(6):381-395, 1981.

Picking the right triangle

We pick the solution that is the most consistent with its neighbours and minimizes the reprojection error.
This can be expressed by the following quadratic program :

$$\min_{\alpha_i,\beta_i,\gamma_i,\delta_i,s_k,k\in\{1\cdots N_v\}} \quad \lambda_1. \sum_{i\in\mathcal{T}(s_j)} \left(\sum_{j=1}^{N_v} \| T_i^*(s_j) - s_j \|^2 \right) + \lambda_2.\|\mathbf{M}.s\|^2$$

$$\text{subject to} \quad : \quad T_i^* = \alpha_i T_i^{(1)} + \beta_i T_i^{(2)} + \gamma_i T_i^{(3)} + \delta_i T_i^{(4)}$$

$$\alpha_i + \beta_i + \gamma_i + \delta_i = 1, \text{with } \alpha_i, \beta_i, \gamma_i, \delta_i \in [0,1]$$

$$\forall\ i \in \{1, N_{\text{facets}}\}$$

where $\mathcal{T}(s_j)$ is the list of facets to which Vertex s_j can belong.
In practice we relax the integer constraints on α, β, γ and δ to a linear one, and change the equality constraint into an inequality one: $\alpha_i, \beta_i, \gamma_i, \delta_i \leq 1$

Motivation Factorization Non-Rigid Motion **Occlusion**

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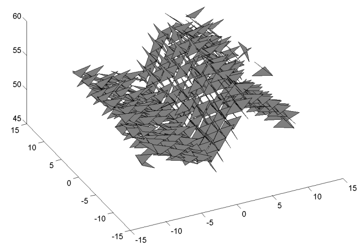
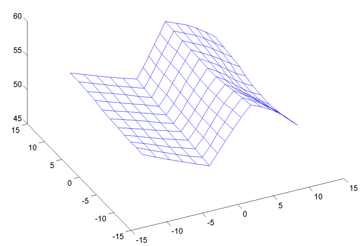
Picking the right triangle: Example

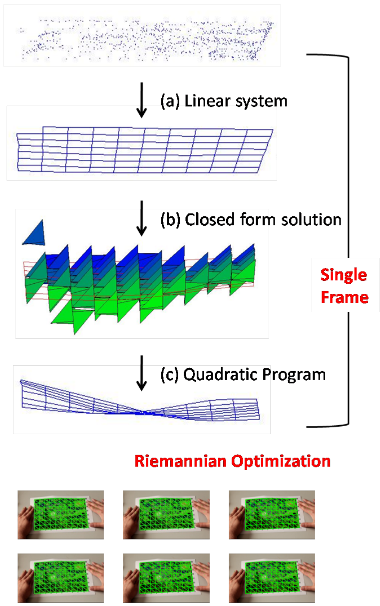


Set of potential triangles



Retrieved (initial) shape

Overall



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For Further Reading II

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