Ramakrishna Mission Vivekananda Educational and Research Institute

Deemed-to-be-University as declared by Government of India under section 3 of UGC Act, 1956

Admission Test for MSc in Big Data Analytics

Model Test Paper

Exam Set: **B**

Instructions:

∗ *Mark your answers on the OMR answer sheet only*

∗ *Correct answer fetches **4** marks, incorrect answer **-1** mark, and unanswered question **0** mark*

Applicant's Name (in block letters):

Application No:

Time: 2 hr

Part A

1. We form a password using two distinct English capital letters followed by two non-zero distinct digits. How many such passwords are possible?

   a. 58500 b. 70 c. 11700 d. 46800 e. 1256640

2. What is the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

   a. $8\sqrt{3}$ b. $12\sqrt{3} - 4$ c. 4 d. $\frac{9}{4}$ e. 3

3. We have to form a committee of four members from a group of nine people. However, if we choose person A, then we cannot choose person B and vice versa. How many different committees can we form?

   a. 126 b. 75 c. 70 d. 80 e. 105

4. In a chess tournament, there are 8 players. Each player plays against every other player exactly once. Additionally, each game can result in a win, a loss, or a draw. A tournament outcome tabulates all the results of the games played. How many distinct tournament outcomes are there?

   a. 48 b. 21952 c. 175616 d. 784 e. None of the above

5. If $y = e^x \sin x$, then what is $\frac{d^2y}{dx^2}$?

   a. $2e^x \sin x$ b. $e^x \cos x$ c. $-2e^x \sin x$ d. $2e^x \cos x$ e. $e^x \sin x$

6. What is $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$?

   a. $\cos 1$ b. ∞ c. $\sin 1$ d. $\ln 2$ e. $\frac{1}{2}\left(e - e^{-1}\right)$

7. Consider a set S of 9 points in a plane, where no three points are collinear. Each point is coloured with either red, green or blue. Consider all triangles whose vertices are from set S. Any of these triangles is called *empty* if its interior does not contain any point from S. Which of the statements below is necessarily true, for any possible colouring, where each colour is used at least once.

   a. There exists an empty triangle where two vertices are blue and one is red.
   b. There exists an empty triangle with vertices of all three colors.
   c. There exists an empty triangle in which two vertices are red and one is green.
   d. There exists an empty triangle all of whose vertices are red.
   e. There exists an empty triangle where two vertices are green and one is red.

8. We draw four lines on a plane with no two of them parallel and no three of them concurrent. Then we additionally draw new lines joining all the points of intersection of the previous four lines. What is the number of such new lines?

   a. 0 b. 2 c. 3 d. 12 e. 5

9. What is $\frac{dy}{dx}$ when $x^2y^2 = (x+y)^5$?

   a. $\frac{5(x+y)^4-2xy^2}{2x^2y-5(x+y)^4}$ b. $\frac{(3x-2y)}{(2x-3y)}$ c. $\frac{5(x+y)^4-2x^2y}{2xy^2-5(x+y)^4}$ d. $\frac{(3x-2y)y}{(2x-3y)x}$ e. $\frac{3(x+y)^4}{2x^2y^2}$

10. The system of equations
    $x + y + z = 1$,
    $2x + 3y - z = 5$, and
    $x + 2y + kz = 4$,
    has an infinite number of solutions for some real number k. What is the value of k?

    a. -2 b. $+2$ c. -1 d. 0 e. $+1$

11. Which number in the finite sequence does not fit in naturally: $28, 84, 112, 196, 308, 504, 872$?

    a. 308 b. 872 c. 112 d. 504 e. 196

12. Consider two series (i) $\sum_{n=1}^{\infty} \sin \frac{x}{n}$ and (ii) $\sum_{n=1}^{\infty} (-1)^n \cos \frac{x}{n}$. Which of the following is true?

    a. (i) converges but (ii) diverges b. both (i) and (ii) converge c. (i) diverges but (ii) converges d. both (i) and (ii) diverge e. both convergences depend on value of x

13. What is the mid-point of the chord formed by the intercept of $x + 3y - 2 = 0$ with the circle $x^2 + y^2 = 4$?

    a. $(-1, 1)$ b. $(1, \frac{1}{3})$ c. $(2, 0)$ d. $(\frac{1}{5}, \frac{3}{5})$ e. $(0, \frac{2}{3})$

14. There are three people P, Q and R, wearing hats that are either black or white, standing in a queue. They know that at least two of them are wearing black hats but they cannot see their own hats. P, who can see Q and R, declares, "I don't know the color of my hat". Q, who can see R, declares, "I know the color of my hat". What colors are Q's and R's hats respectively?

    a. White and white b. White and black c. Black and white d. Black and black e. Cannot be determined from the given conditions

15. Vidyasagar was walking on the street, one boy requested him to donate for social welfare fund. He gave him a rupee more than half the money he had. Then came a girl who requested him to donate for poor people's fund for which he gave two rupees more than half the money he had then. After that, again a boy approached him for an orphanage fund. He gave three rupees more than half of what he had. At last he had just two rupee remaining in his hand. How much amount did Vidyasagar have in his pocket in the beginning?

    a. 72 b. 35 c. 60 d. 50 e. 42

16. There are 8 letters and 8 envelopes. Each letter is destined to a unique distinct address. These addresses are written on the envelope. In how many ways can you misplace exactly 4 letters?
    a. 70 b. 560 c. 720 d. 840 e. 630

Read the text below to answer the next 3 questions:

Seven sports awardees, A, B, C, D, E, F and G were honoured at a function. There were seven seats on the dais in a row. First, A and G came and sat together at one end. Second, B, the Khel Ratna recipient sat at the centre. Next, C and D were bitter rivals and therefore sat as farthest apart as possible. Lastly, others came and sat on remaining seats.

17. Who among C, D, F and G could not sit at any end in any possible arrangements of seating?
    a. F b. G c. D d. C e. Cannot be determined

18. Which of the following pairs cannot occupy the seats on either side of B in any possible arrangements of seating?
    a. F & D b. C & E c. F & C d. A & F e. D & E

19. Which of the following pairs cannot be seated adjacent to each other in any possible arrangements of seating?

 a. C & F b. D & G c. B & F d. A & E e. B & D

20. Two chess tournaments are being held in Chennai and Kolkata at the same time so that the same player cannot attend both the tournaments. A chess club, which has 10 members, decides to send at least one player to each of these tournaments. In how many ways the club can do so?
 a. $3^{10} - 2 \cdot 2^{10}$ b. 3^{10} c. $2(2^{10} - 1)$ d. $3^{10} - 2 \cdot 2^{10} + 1$ e. $3^{10} - 1$

Part B

21. A bag contains 4 red, 3 blue, and 2 green balls. If two balls are drawn randomly, what is the probability that at least one of them is green?

 a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{7}{9}$ d. $\frac{5}{12}$ e. $\frac{7}{12}$

22. The quality assurance team of a chocolate company tests quality in two steps.
 Step 1: 50 out of every 1000 chocolate bars are randomly picked for physical examination, where the shape of the bar is examined.
 Step 2: For every 40 chocolate bars whose shape is examined, 1 chocolate bar is randomly picked for chemical examination, where the composition and taste of the chocolate is examined.
 If one chocolate bar that is randomly picked out of 1000 bars is known to be physically examined, then what is the probability that it was not chemically examined?

 a. 0.40 b. 0.98 c. 0.999 d. 0.50 e. 0.975

23. Let $\mathbf{A}$ be a 4×4 real orthogonal matrix such that $\mathbf{A}^2 = \mathbf{I}$, and $\textsc{Trace}(\mathbf{A}) = 0$. An orthogonal matrix is such that $\mathbf{A}\mathbf{A}^{\mathrm{T}} = \mathbf{I}$. The $\textsc{Trace}$ of a square matrix is the sum of its diagonal elements. What are the eigenvalues of $\mathbf{A}$?

 a. $-1, 0, 0, 1$ b. $-1, -1, 1, 1$ c. $-3, -1, 1, 3$ d. $-2, -1, 1, 2$ e. Depends on $\mathbf{A}$

24. What is the probability that in a 5-card hand dealt from a standard deck of 52 cards, there are exactly 3 cards of one number and 2 cards of another number (a full house)?

 a. $\frac{13^2 \cdot \binom{4}{3}\binom{4}{2}}{\binom{52}{5}}$ b. $\frac{13 \cdot 12 \cdot \binom{4}{3}\binom{4}{2}}{\binom{52}{5}}$ c. $\frac{\binom{13}{3}\binom{13}{2}}{\binom{52}{5}}$ d. $\frac{13^2 \cdot \binom{4}{3}\binom{4}{2}}{52^5}$ e. $\frac{13 \cdot 12 \cdot \binom{4}{3}\binom{4}{2}}{52^5}$

25. Let $\begin{pmatrix} 1 & 5 & 4 \\ 3 & 14 & 10 \\ 6 & 9 & a \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & 16 \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$.
 What is the value of a?

 a. 0 b. 2 c. 1 d. -1 e. -2

26. The rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ bc & ca & ab \end{pmatrix}$ is less than 3 if and only if

 a. if $a \neq b \neq c$ b. any two of a, b, c be equal but different from third c. $a = b = c$ d. at least two of a, b, c are equal e. None of these

27. Let $\mathbf{S}$ denotes the vector space of real-valued functions on the interval $(0, 4)$ and $\mathbf{A}$ be a set of all real-valued differentiable functions f on the interval $(0, 4)$ such that $f'(2) = b$. Which of the following is a correct statement?

 a. $\mathbf{A}$ is a subspace of $\mathbf{S}$ if and only if $b \neq 0$.
 b. $\mathbf{A}$ is always a subspace of $\mathbf{S}$.
 c. $\mathbf{A}$ is never a subspace of $\mathbf{S}$.
 d. $\mathbf{A}$ is a subspace of $\mathbf{S}$ if and only if $b = 0$.

e. None of the above is correct.

28. Let

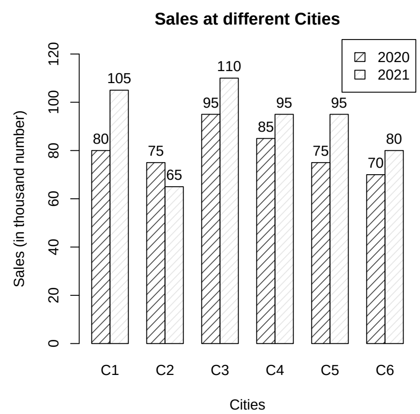
$$f(x,y) = \begin{cases} e^{-1/(x^2+y^2)} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Then which is the following is true?
a. $f(x,y)$ is continuous at $(0,0)$ and has first order partial derivatives, but not differentiable at $(0,0)$.
b. $f(x,y)$ is not continuous at $(0,0)$.
c. $f(x,y)$ is continuous at $(0,0)$ but does not have first order partial derivatives.
d. $f(x,y)$ is differentiable at $(0,0)$.
e. None of the above is correct.

29. Suppose A, B are two events such that $\Pr(A) = \frac{3}{4}$ and $\Pr(B) = \frac{1}{2}$, where $\Pr(A)$, $\Pr(B)$ denote the probability of events A and B respectively. Then which of the following option is true?
a. $\frac{5}{6} \leq \Pr(A \cap B) \leq 1$ b. $\frac{1}{4} \leq \Pr(A \cap B) \leq \frac{1}{2}$ c. $0 \leq \Pr(A \cap B) \leq \frac{1}{8}$ d. $\frac{3}{4} \leq \Pr(A \cap B) \leq \frac{4}{5}$
e. $\frac{3}{4} \leq \Pr(A \cap B) \leq 1$

Read the text below to answer the next 4 questions:

The bar graph below shows the sales of a particular medicine (in thousands) of a pharmaceutical company in six cities, C1, C2, C3, C4, C5 and C6, during two consecutive years 2020 and 2021.



30. What is the ratio of the total sales of city C2 for both years to the total sales of city C4 for both years?

a. 7:9 b. 1:2 c. 3:5 d. 2:3 e. 4:5

31. Total sales of city C6 for both the years is what percent of the total sales of city C3 for both the years?

a. 75.11% b. 75.55% c. 73.17% d. 71.11% e. 68.54%

32. What percent of the average sales of cities C1, C2 and C3 in 2021 is the average sales of cities C1, C3 and C6 in 2020?

a. 87.5% b. 114.2% c. 82.5% d. 75% e. 125%

33. How much more percentage sales were needed in C6 in the year 2020 such that the average sales of all the cities (in thousands) for the year 2020 would have as same as that of for the year 2021?

a. 87.5% b. 73% c. 100% d. 93.3% e. 82.3%

34. Suppose A and B are $n \times n$ positive-definite symmetric matrices and I be the $n \times n$ identity matrix. A symmetric matrix M with real entries is positive-definite if the real number $x^{\mathrm{T}}Mx$ is positive for every nonzero real column vector x . Then which of the following may not be positive-definite?

a. $A + B$ b. AB c. ABA^{T} d. $A^2 + I$ e. None of these.

35. Two real numbers x, y are chosen uniformly at random and independently from the interval $[0, 1)$. Let $\Pr(A)$ and $\Pr(B)$ denote the probabilities of the events $A : x = y$ and $B : x < y$, respectively. Which of the following is true?
a. $\Pr(A) = 0$, $\Pr(B) = \frac{1}{3}$. b. $\Pr(A) = 0$, $\Pr(B) = \frac{1}{2}$. c. $\Pr(A) = 0$, $\Pr(B) = 0$. d. $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{2}$. e. $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$.