Link path and reflection visibility algorithms

Sudebkumar Prasant Pal

Professor

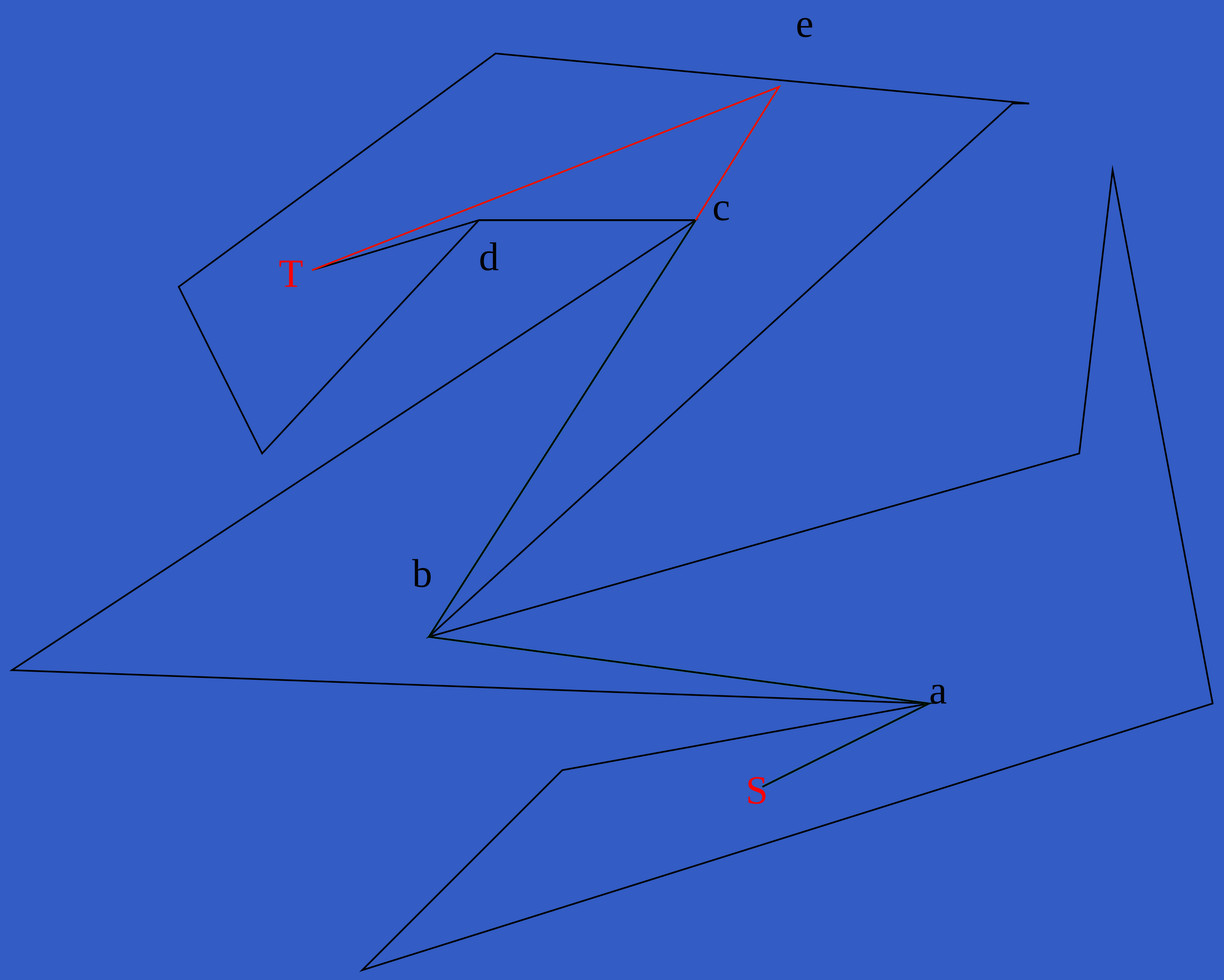
Department of Computer Science and Engineering

IIT Kharagpur

India

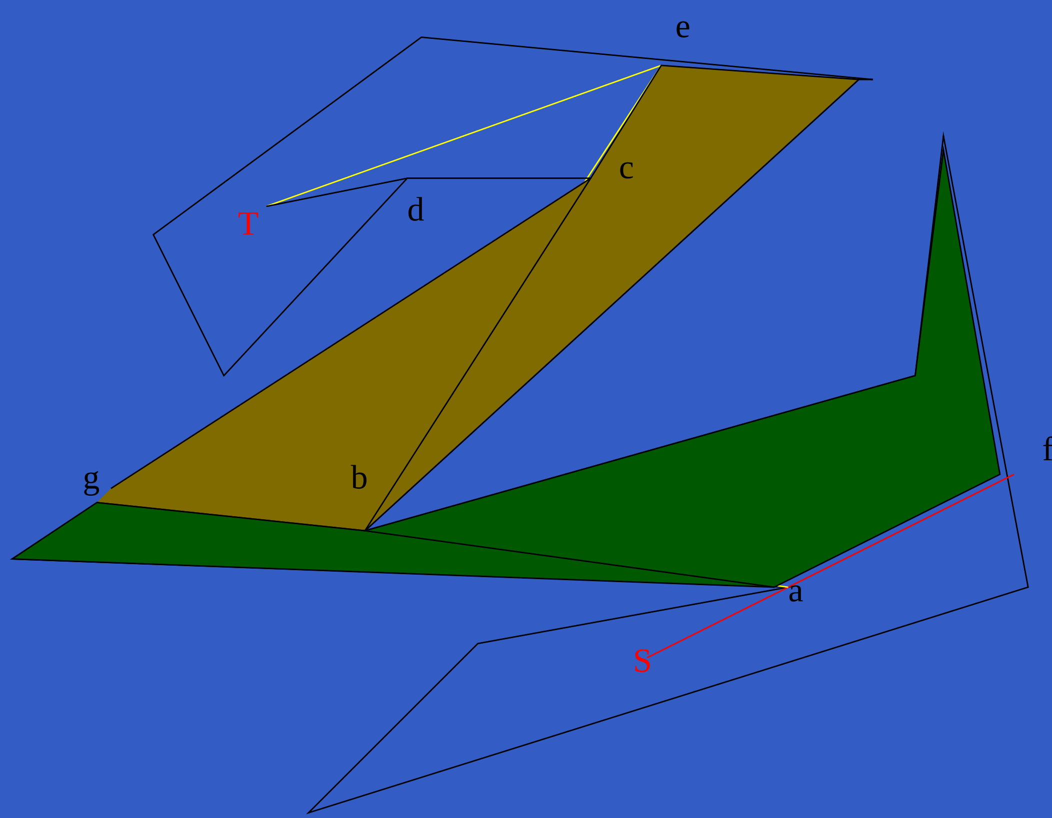
email: spp@cse.iitkgp.ernet.in

Definitions: Link paths



- The link path S, a, b, e, T and shortest path S, a, b, c, d, T, from s to t.

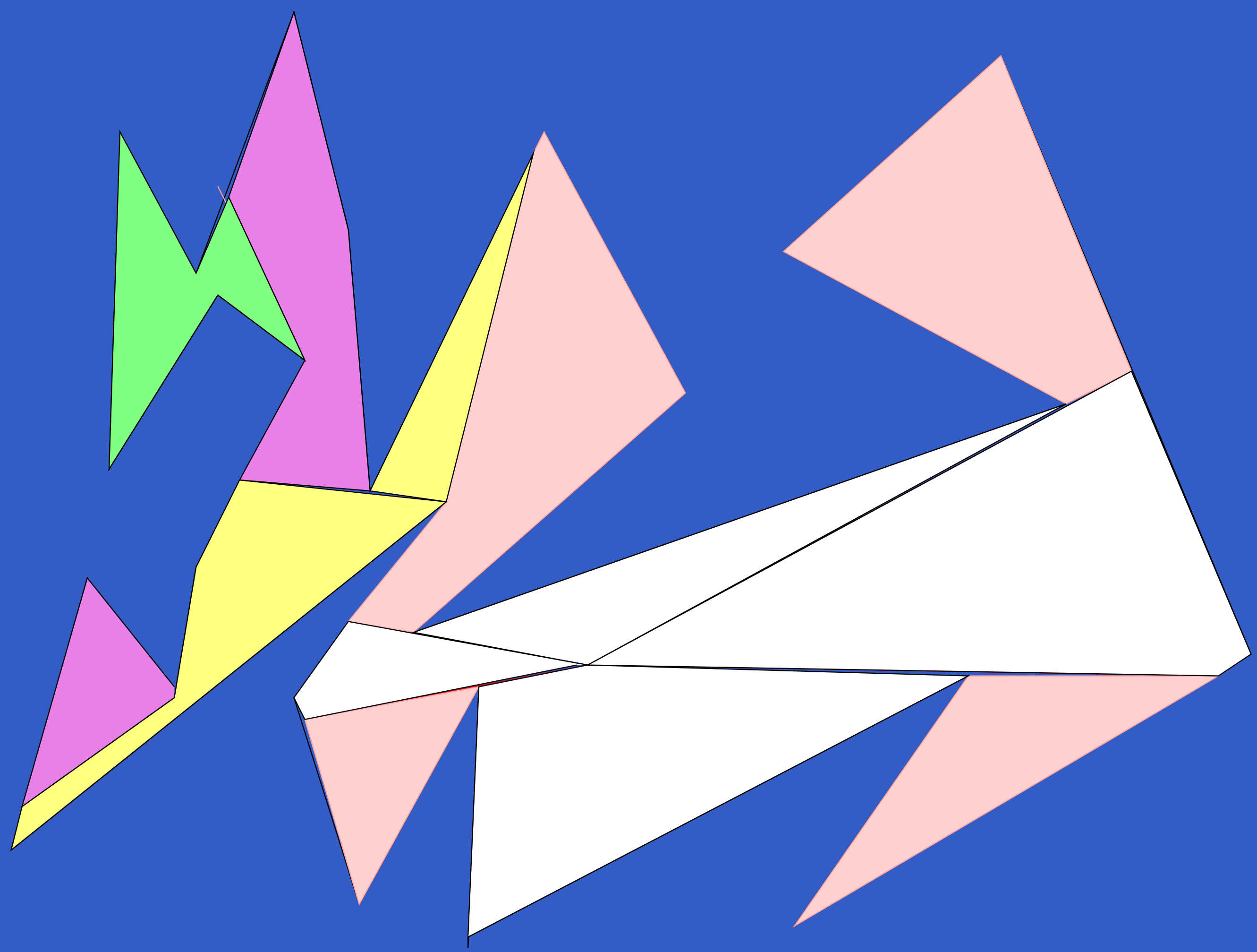
Computing link paths



Compute successive windows, af, bg and ce, of $V(S)$, $V(P, af)$, $V(P, bg)$. The region $V(P, ce)$ has T.

A. Maheshwari, J-R Sack and H. N. Djidjev, Link distance problems, Chapter 12, Handbook of Computational Geometry, Elsevier, 1999.

Computing link paths



Useful in designing communication systems and in robotics where turns are more important than total distance traversed.

Link radius, diameter and center

1. The maximum link distance between a pair of points is the link *diameter d*.
2. The point whose maximum link distance to all points is minimized, defines the link *radius r*. It is known that $\lceil d/2 \rceil \leq r \leq \lceil d/2 \rceil + 1$ [1,2].
3. The set of points with maximum link distance r to points in the polygon, form the link *centre*.
4. The diameter, radius and the centre can all be computed in $O(n^2)$ time [1,2].

[1] *A. Maheshwari, J-R Sack and H. N. Djidjev, Link distance problems, , Handbook of Computational Geometry (Chapter 12), Elsevier, 1999.*

[2] *S. K. Ghosh, Visibility algorithms in the plane (Chapter 7), Cambridge, 2007.*

Open problems for link distances

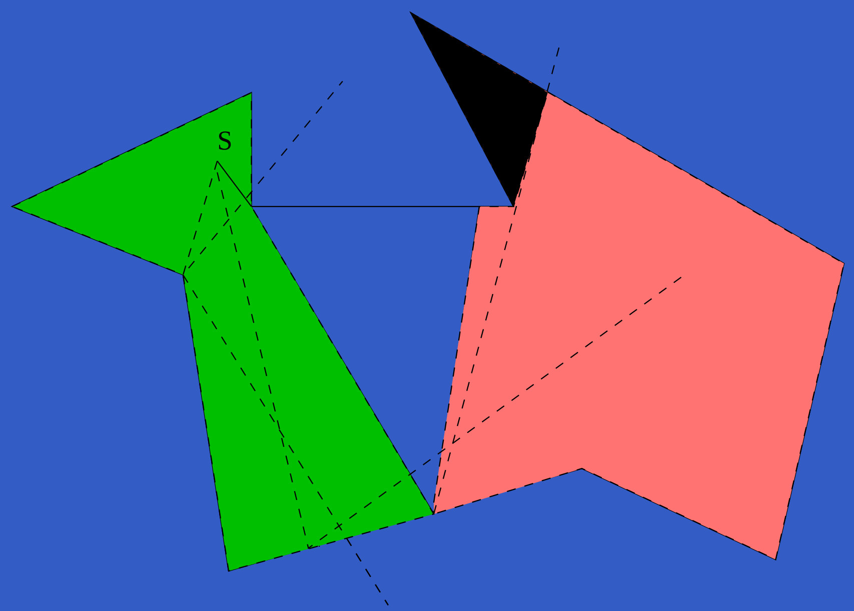
- Computing the link centre in linear time; the best known $O(n \log n)$ algorithm is due to Djidjev et al. [1]
- Computing the minimum link path in subquadratic time for two points inside a polygon with holes remains open. See Mitchell et al. [2] and Maheshwari et al. [3].

[1] *H. N. Djidjev, A. Lingas and J-R. Sack, An $O(n \log n)$ algorithm for computing the link center of a simple polygon, Discrete. Comput. Geom. 8: 131-152, 1992.*

[2] *J. S. B. Mitchell, G. Rote and G. Woginger, Minimum-link paths among obstacles in the plane, Algorithmica 8 (1992), 431-459.*

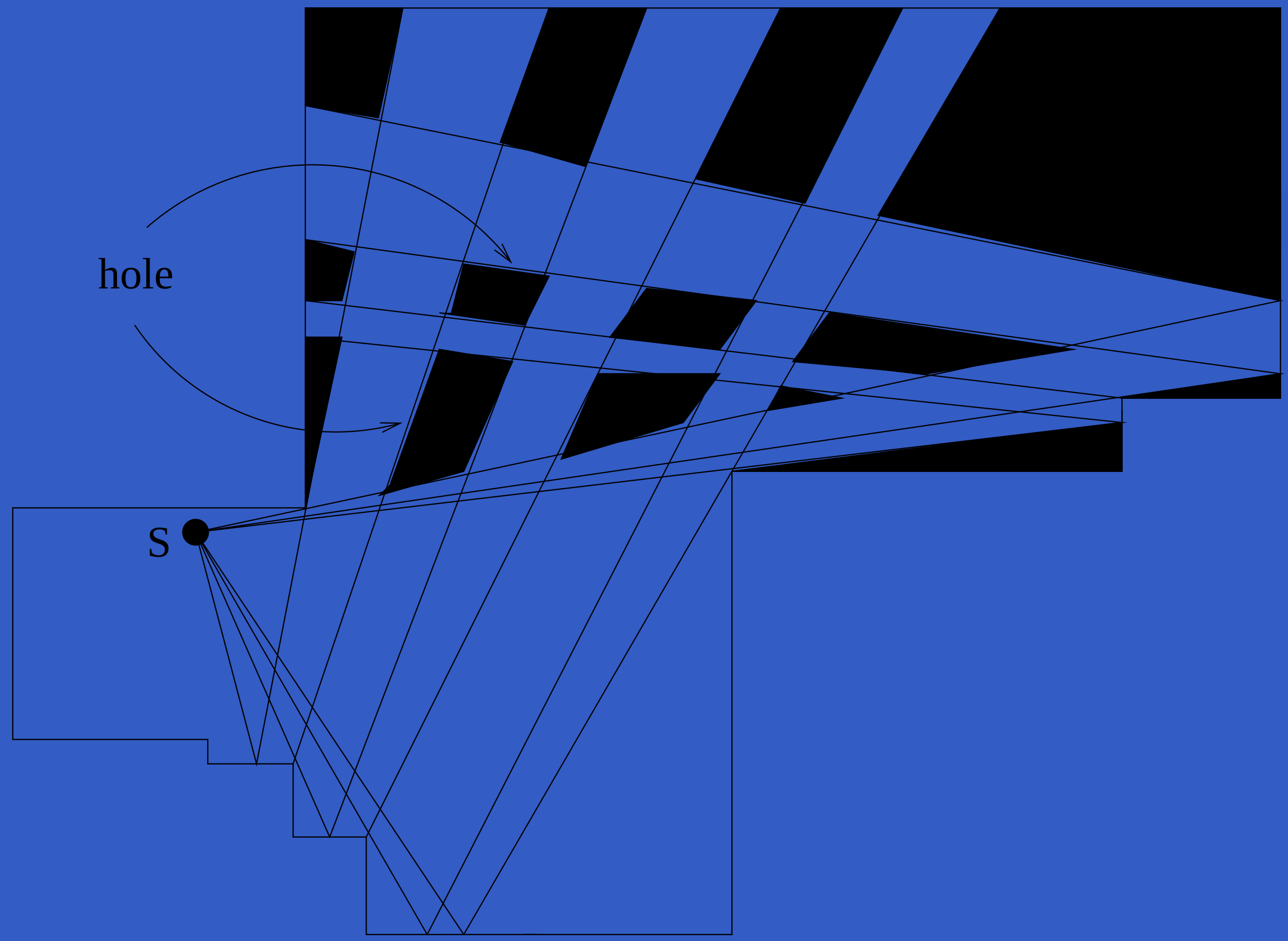
[3] *A. Maheshwari, J-R Sack and H. N. Djidjev, Link distance problems, Chapter 12, Handbook of Computational Geometry, Elsevier, 1999.*

Definitions: Specular and Diffuse reflections



- The green region is directly visible from the point source S.
- The black region is not visible even by diffuse reflection.
- The blue region is visible by specular reflection.
- The red region is visible only by diffuse reflection.

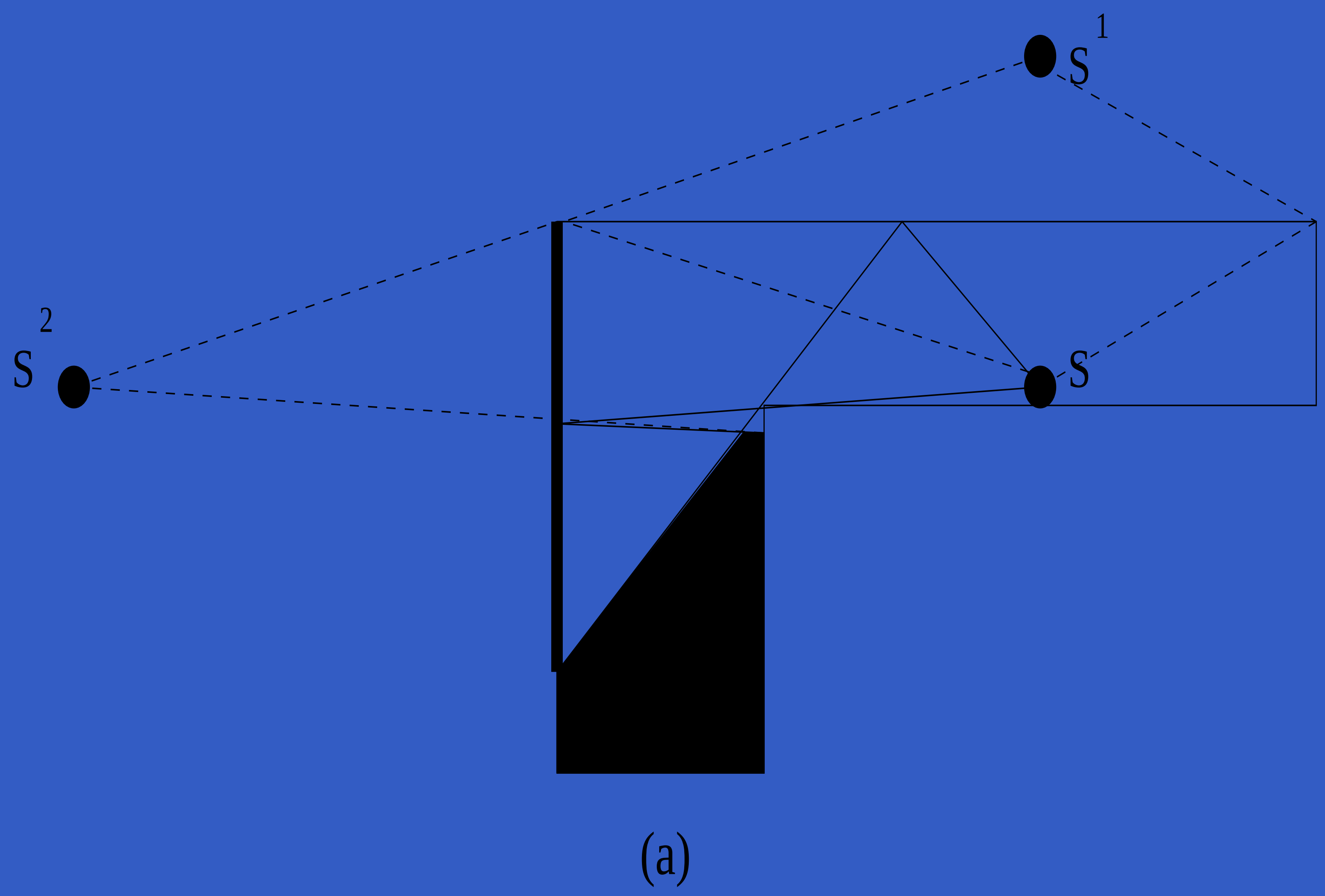
$\Omega(n^2)$ Holes possible with one specular reflection



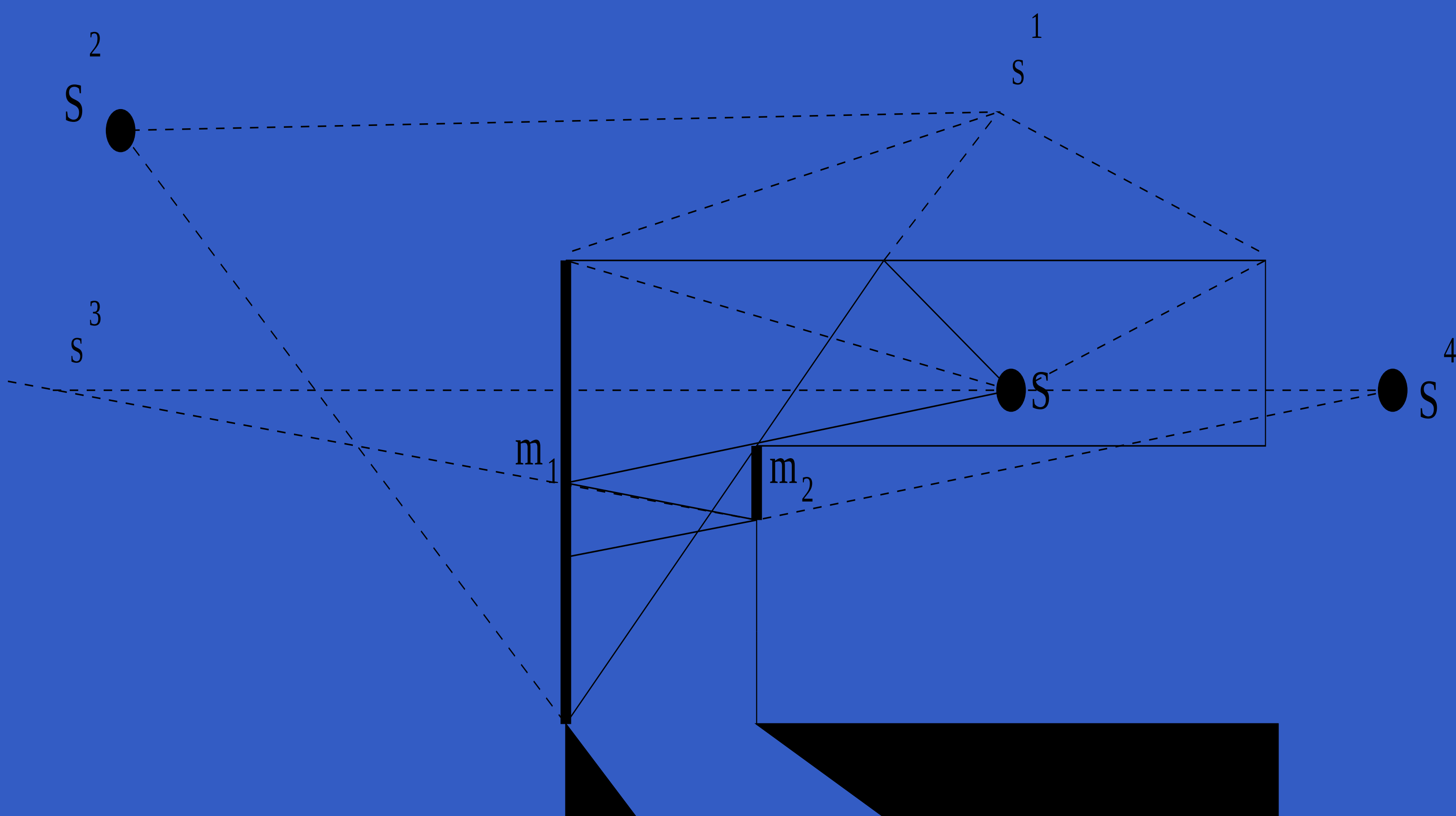
[1] *B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with one reflection, Discrete. Comput. Geom. 19 (1998), 553-574.*

[2] *M. Aanjaneya, S. P. Pal and A. Bishnu, Directly visible pairs and illumination by reflections in orthogonal polygons, 24th European Workshop on Computational Geometry, Nancy, France, 2008.*

Two specular reflections for orthogonal spiral polygons



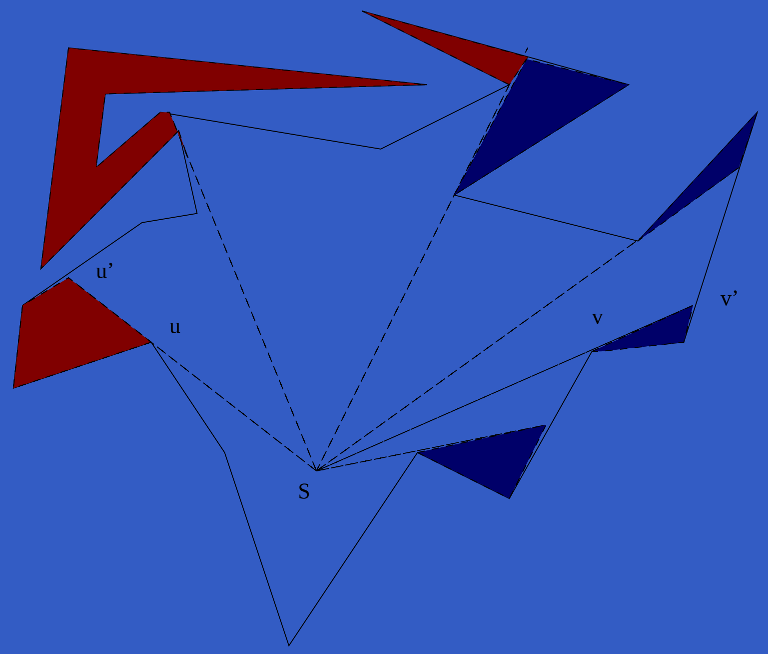
Two specular reflections for orthogonal spiral polygons



(b)

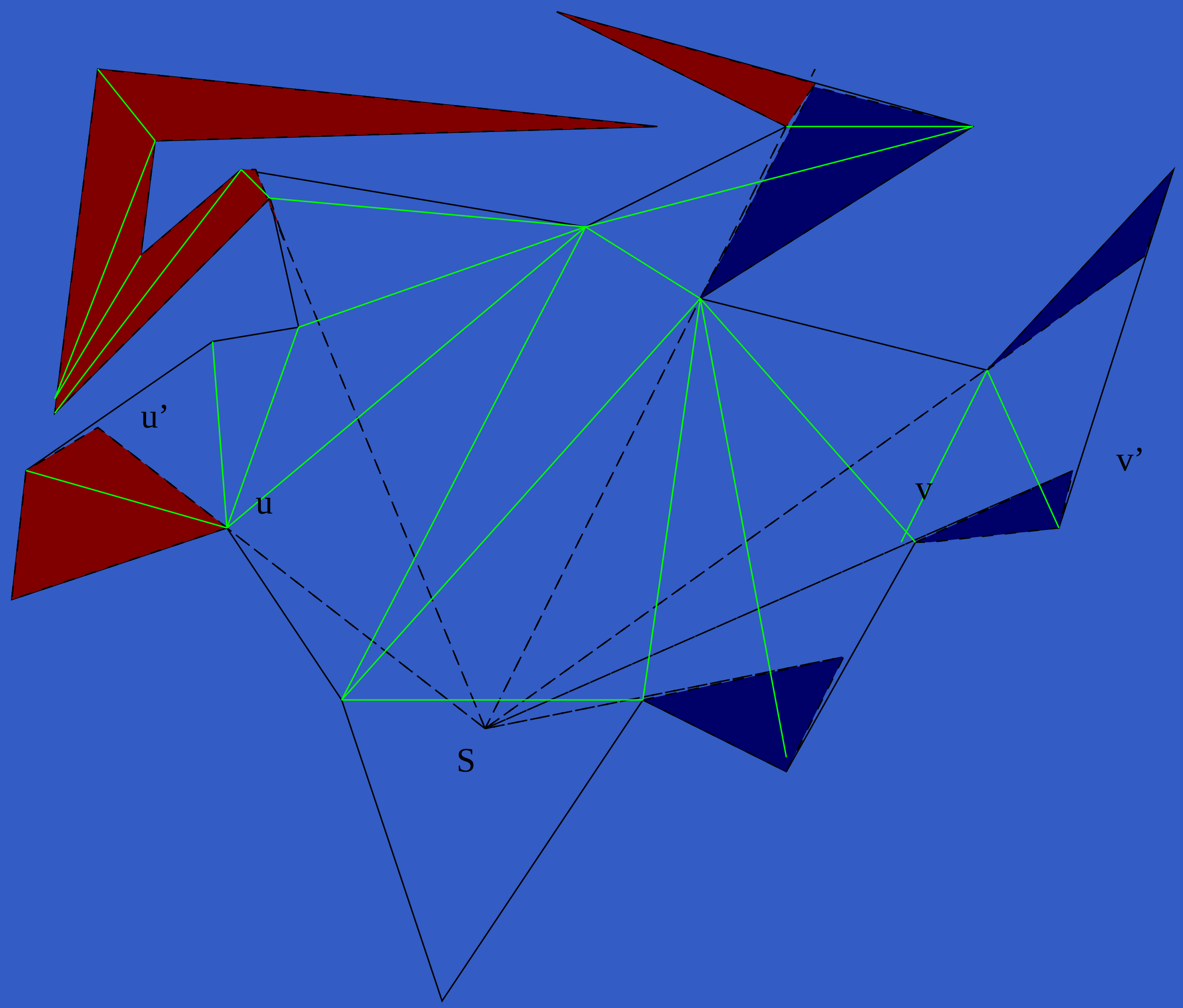
Can one compute the visible region by $k \geq 1$ specular reflections in linear time for orthogonal spiral polygons? [These visible regions do not have holes.]

Computing point-visible regions



- Data structures: Doubly linked list (circular list) of boundary vertices and trees.
- Typical algorithms scan the list clockwise/counterclockwise.
- Given three consecutive vertices u, v, w we need to see if the edges uv and vw form a left/right turn at v.

Computing point-visible regions

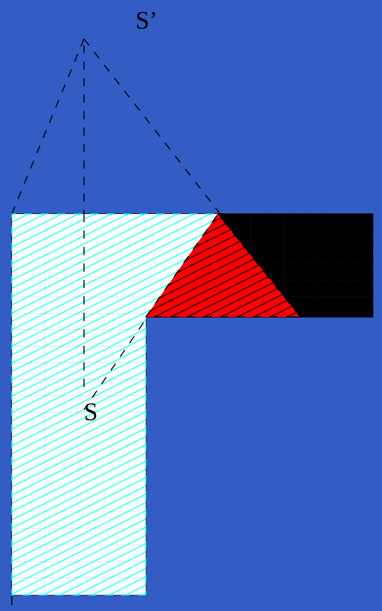


- Traverse the triangles in depth first manner starting from source S.

Computing point-visible regions

- Maintain windows for each child triangle as we go deep cutting triangles.
- The window allows light to enter a triangle from S leaving it from parts of at most two other sides of the triangle.
- Split the beam from S into at most two new windows and continue until you hit the boundary.
- If a triangulation is not available, one has to scan the polygonal boundary and use a stack.

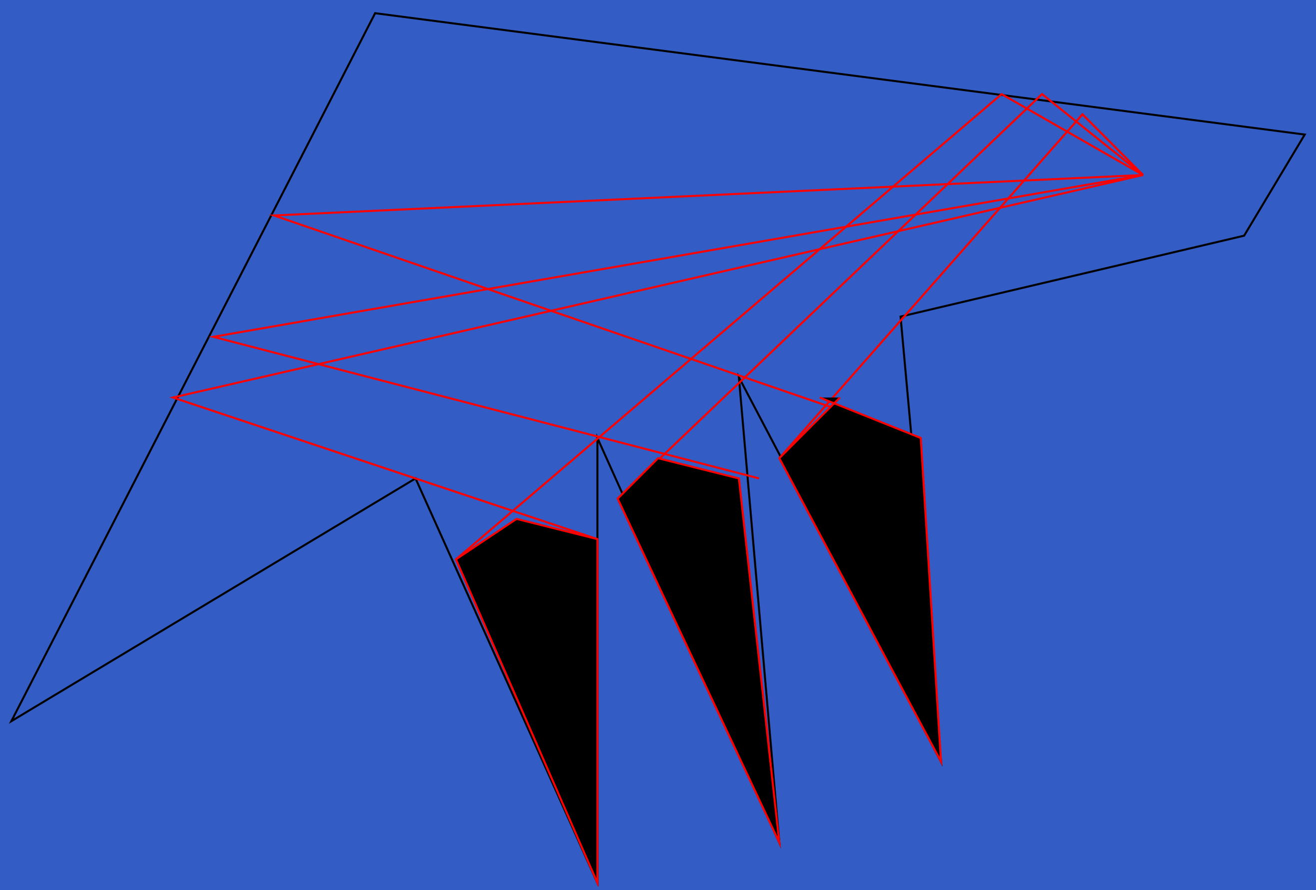
Computing region visible by specular reflection



- For any edge (or part of edge) e_i of P on $V(S)$, add the triangle Δ^i, with virtual source S^i to create a triangle with e_i.
- Add this triangle Δ_i to P glued at e_i and compute $V_i(S) = P \cap V(S^i, P \cup \Delta^i)$.

B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with one reflection, Discrete. Comput. Geom. 19 (1998), 553-574.

Computing region visible by specular reflection



- The entire visible region $V_s(S)$ is the union, of all $P_i's$, where $P_i = V(S) \cup V_i(S), 0 \leq i \leq a-1$.
- The number of edges of the entire visible region can be shown to be $O(n + a^2 + an)$.

Computing region visible by specular reflection

- $V(S)$ has $O(n)$ vertices.
- Each of the a mirrors of $V(S)$ gives $O(n)$ edges in $V_i(S)$.
- Each pair of mirror visible regions gives at most a constant number of vertices of $V_s(S)$ for complex invisible regions. For simple invisible regions adjacent to the boundary, we charge the costs to the $O(n^2)$ vertices of $V_s(S)$ on $bd(P)$.
- This size is at most $C(n + a^2 + an)$ for some constant $C > 0$.
- Between all $P_i's$, there can be $\Theta(n^3)$ intersections ! [The $O(n^2)$ pairs of $P_i's$ can have $O(n)$ intersections each.]
- So computing the union is done in $o(n^3)$ time by divide-and-conquer, in $\log a$ rounds of merging of the a polygons, $P_i's, 0 \leq i \leq a - 1$.

Computing region visible by specular reflection

- Merging using Bentley-Ottmann [1] place-sweep algorithm in $O(\log a)$ levels for a polygons requires time $T(a)$ where:
- $T(k) \leq 2T(k/2) + (v+u)\log v$, where $v = 2C(n + (\frac{k}{2})^2 + nk/2)$ and $u = C(n + k^2 + nk)$.
- Since $a = O(n)$, we have the total cost $T(a) = O(n^2 \log^2 n)$.
- So, the entire region visible by one specular reflection can be computed in $O(n^2 \log^2 n)$ time [2].

[1] J. L. Bentley and T. A. Ottmann, Algorithms for reporting and counting geometric intersections, IEEE Trans. Comput., 28:643-647, 1979.

[2] *B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with one reflection, Discrete. Comput. Geom. 19 (1998), 553-574.*

Extensions to general geometries for specular reflections

- Davis [1] deals with extensions of multiple specular reflections for general polygons, multiple light sources and in the presence of obstacles or holes.
- Multiple specular reflections for simple polygons were studied in [2] by Aronov et al., establishing asymptotically tight bounds of $O(n^{2k})$ for k specular reflections for constant $k \geq 2$.
- Open: Extensions to three and multiple dimensions with applications in graphics.

[1] A. R. Davis, Visibility with reflections in triangulated surfaces, Ph. D. thesis, Dept. of Comput. Inf. Sci. Polytechnic University, Brooklyn, NY.

[2] B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with multiple specular reflections, Discrete Comput. Geom. 20 (1998), 61-78.

More open questions for visibility with reflection

- Like the link *centre*, can we find the set of points whose maximum *reflection distance* to points in the polygon is minimized?
- The link-k neighbourhoods intersect giving the link *centre* if k is the link *radius* [1,2].
- Such a property does not hold for one specular reflection [3], but holds for diffuse reflections [4].

[1] W. Lenhart, R. Pollack, J.-R. Sack, R. Seidel, M. Sharir, S. Suri, G. T. Toussaint, S. Whitesides and C. K. Yap, Computing the link centre of a simple polygon, Discrete Comput. Geom. 3:281-293, 1988.

[2] H. N. Djidjev, A. Lingas and J-R. Sack, An $O(n \log n)$ algorithm for computing the link center of a simple polygon, Discrete. Comput. Geom. 8: 131-152, 1992.

[3] B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with one reflection, Discrete Comput. Geom. 19 (1998), 553-574.

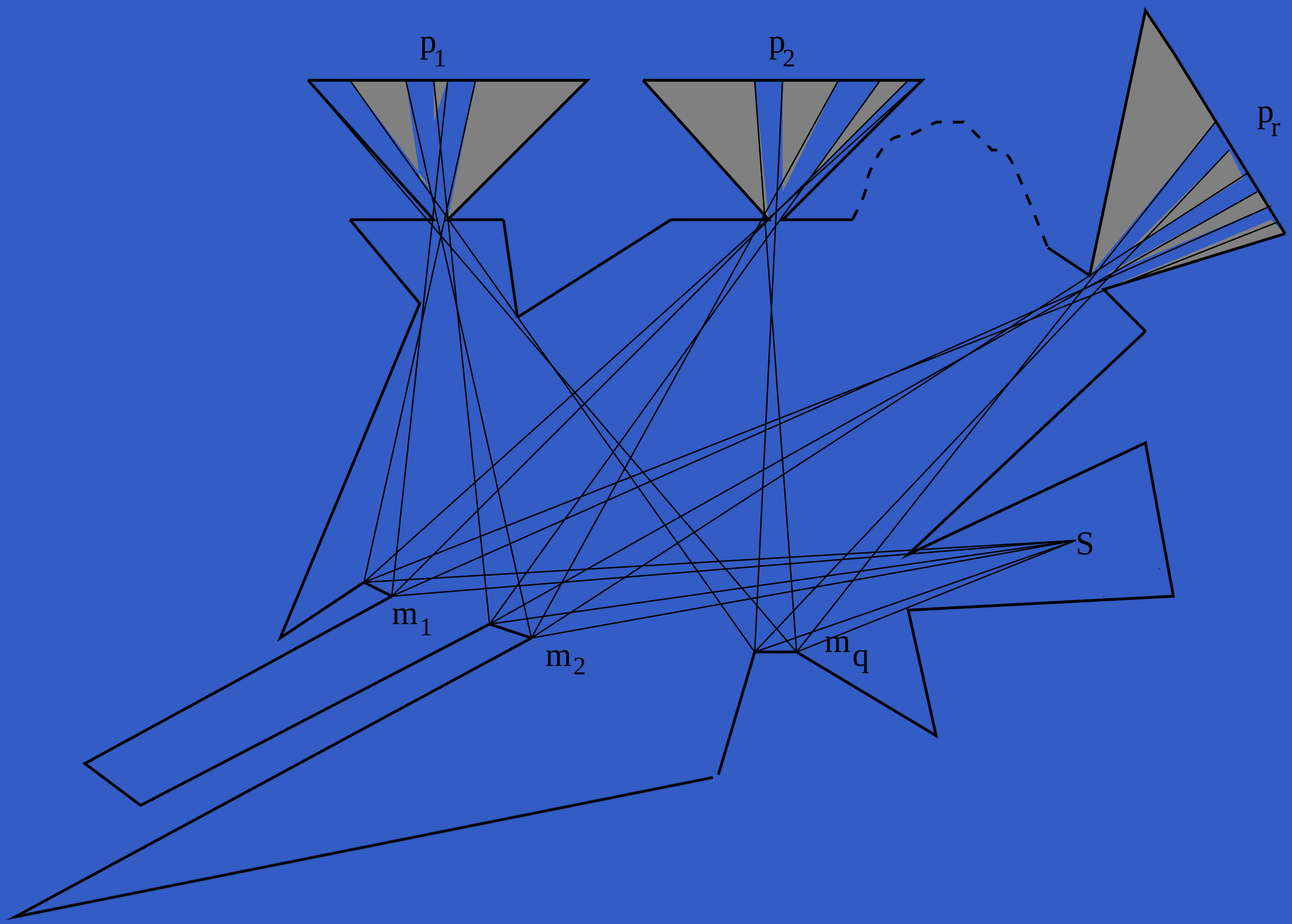
[4] D. Chithra Prasad, New visibility problem: Combinatorial and computational complexitiess, Ph. D. thesis, Dept. of Computer Sc. and Engg., IIT Kharagpur, India.

Computing regions visibile with diffuse reflections

1. Compute the direct point visible region $V(S)$ from source S.
2. Let m_i be (a part of) an edge of P in $V(S)$.
3. Compute the region $V(S, m_i)$, visible from the diffuse mirror m_i. This region would be weakly visible from the mirror m_i.
4. Compute the union of all these visible regions as $V_d(S)$, using divide-and-conquer, as we did for the specular case.
5. Using the same recurrence relation as in the specular case, we get the running time as $O(n^2 \log^2 n)$ [1].

[3] B. Aronov, A. R. Davis, T. K. Dey, S. P. Pal and D. Chithra Prasad, Visibility with one reflection, Discrete Comput. Geom. 19 (1998), 553-574.

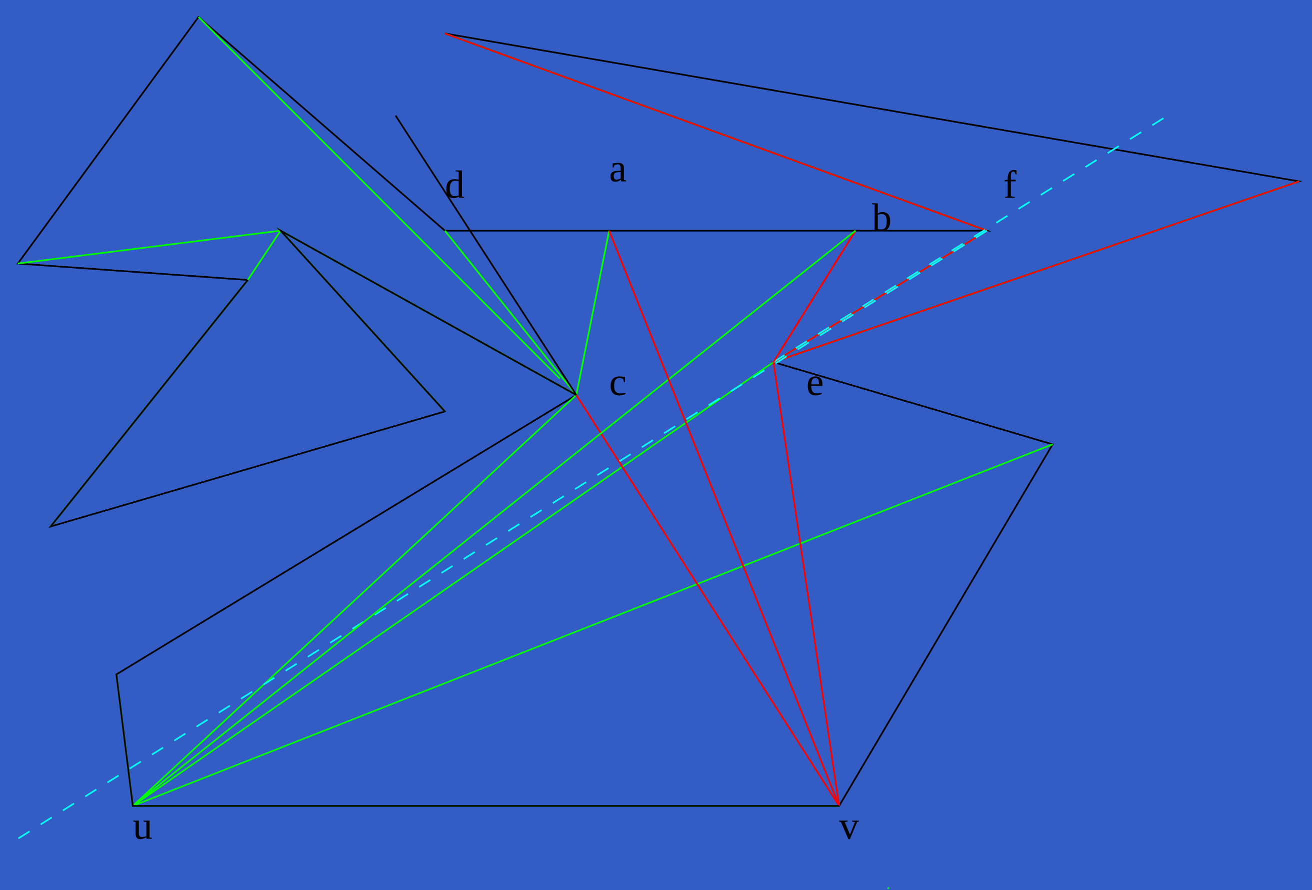
$\Omega(n^2)$ edges for visible region with diffuse reflection



D. Chithra Prasad, S. P. Pal and T. K. Dey, Visibility with multiple diffuse reflections, CGTA vol. 10, pp. 187-196, 1998.

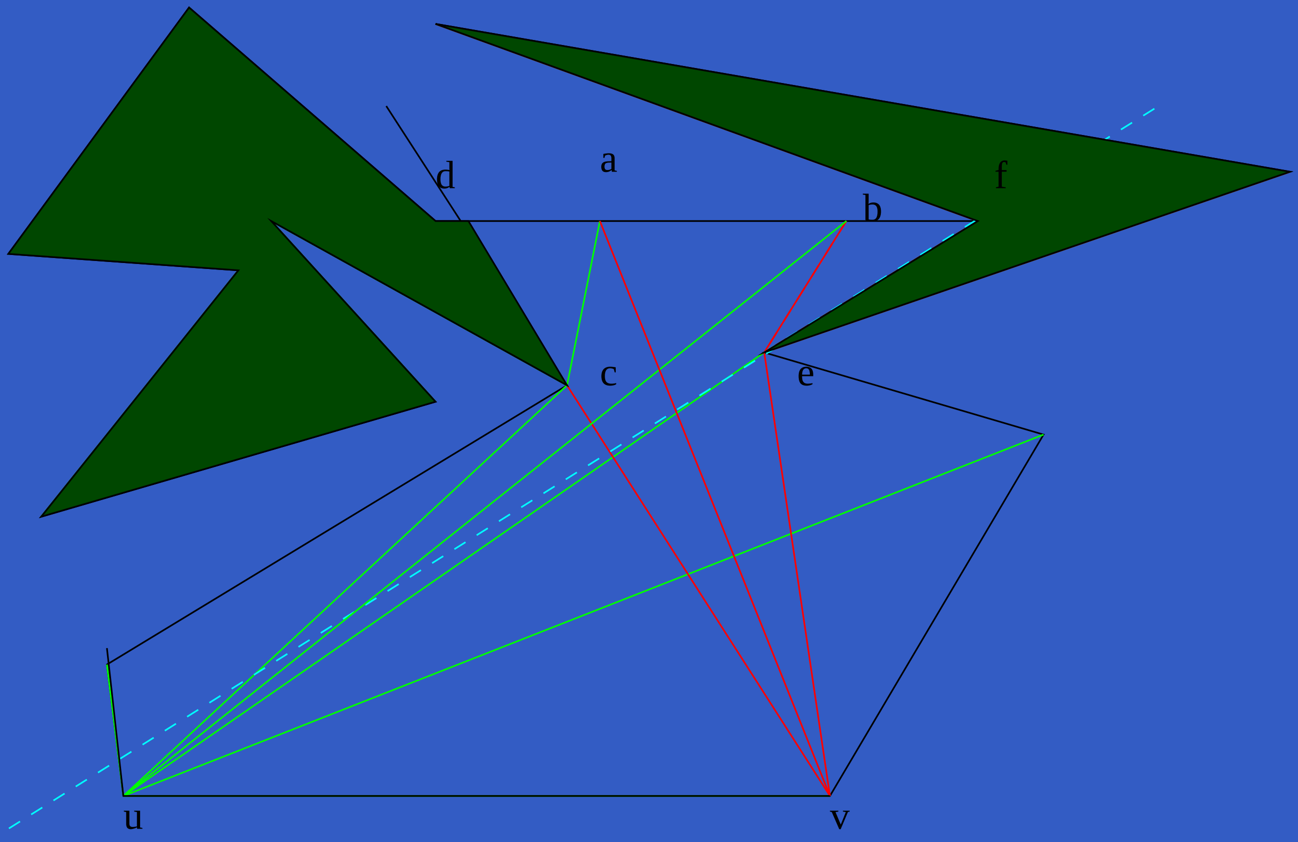
Is the number of edges always $O(n^2)$ for all simple polygons and for any number $k \geq 1$ of diffuse reflections?

Computing edge-visible or weakly visible regions



Compute left and right shortest path trees and traverse the polygon boundary to cut off regions not weakly visible from uv. A point is weakly visible from uv if and only if its parents in the two trees are distinct.

Computing edge-visible or weakly visible regions



Computing regions visibile with diffuse reflections

1. Invisible regions are only within shadows of $V(S)$, where reflected rays move away from the shadows.
2. So, any invisible region has an internal convex chain as it is cut out on one side of a number of reflected rays.
3. The outer side is simply a connected part of the polygonal boundary.
4. Two mirrors m_i and m_j can give rise to only a single invisible region vertex.
5. So, there are at most $O(n^2)$ vertices in $V_d(S)$ due to such vertices and due to vertices of weak visible polygons from $O(n)$ mirrors.

Visibility with multiple diffuse reflections

1. We know from [1] that $V_d(S)$ has $O(n^k)$ edges where k is the number of diffuse reflections permitted on the path of light.
2. In [2], Aronov et al. show that $O(n^9)$ edges bound the region visible due to an indefinite number $k \geq 2$ of diffuse reflections.

[1] D. Chithra Prasad, S. P. Pal and T. K. Dey, Visibility with multiple diffuse reflections, CGTA vol. 10, pp. 187-196, 1998.

[2] Boris Aronov, Alan R. Davis, John Iacono and Albert Siu Cheong Yu, The Complexity of Diffuse Reflections in a Simple Polygon, LATIN 2008.

Open problems for diffuse reflections

1. It is conjectured that $O(n^2)$ is the real upper bound for $k \geq 2$ reflections in [1].
2. In [2], the upper bound is claimed as $O(n^9)$ for all $k \geq 2$. Can this be further improved?
3. The minimum link path bewteen two points in a simple polygon can be found in linear time. We wish to efficiently compute the path suffering the minimum number of diffuse reflections. These are link paths with turns in the interiors of boundary edges.

[1] D. Chithra Prasad, S. P. Pal and T. K. Dey, Visibility with multiple diffuse reflections, CGTA vol. 10, pp. 187-196, 1998.

[2] Boris Aronov, Alan R. Davis, John Iacono and Albert Siu Cheong Yu, The Complexity of Diffuse Reflections in a Simple Polygon, LATIN 2008.

Open problems for diffuse reflections

1. We conjecture that the diffuse reflection diameter is at most $\lceil \frac{n}{2} \rceil$ [1]. The snake-like monotone polygon has a minimum reflection path, shortest path and a minimum link path with $\lceil \frac{n}{2} \rceil$ links.
2. The shortest path and the minimum link path are related [2,3]; there must be a link for each ‘eave’ in the shortest path.
3. A minimum link path lies inside the complete visibility polygons of convex pieces of the shortest path [2,3]. It is not clear whether similar approaches will also help computing the minimum reflection path efficiently.

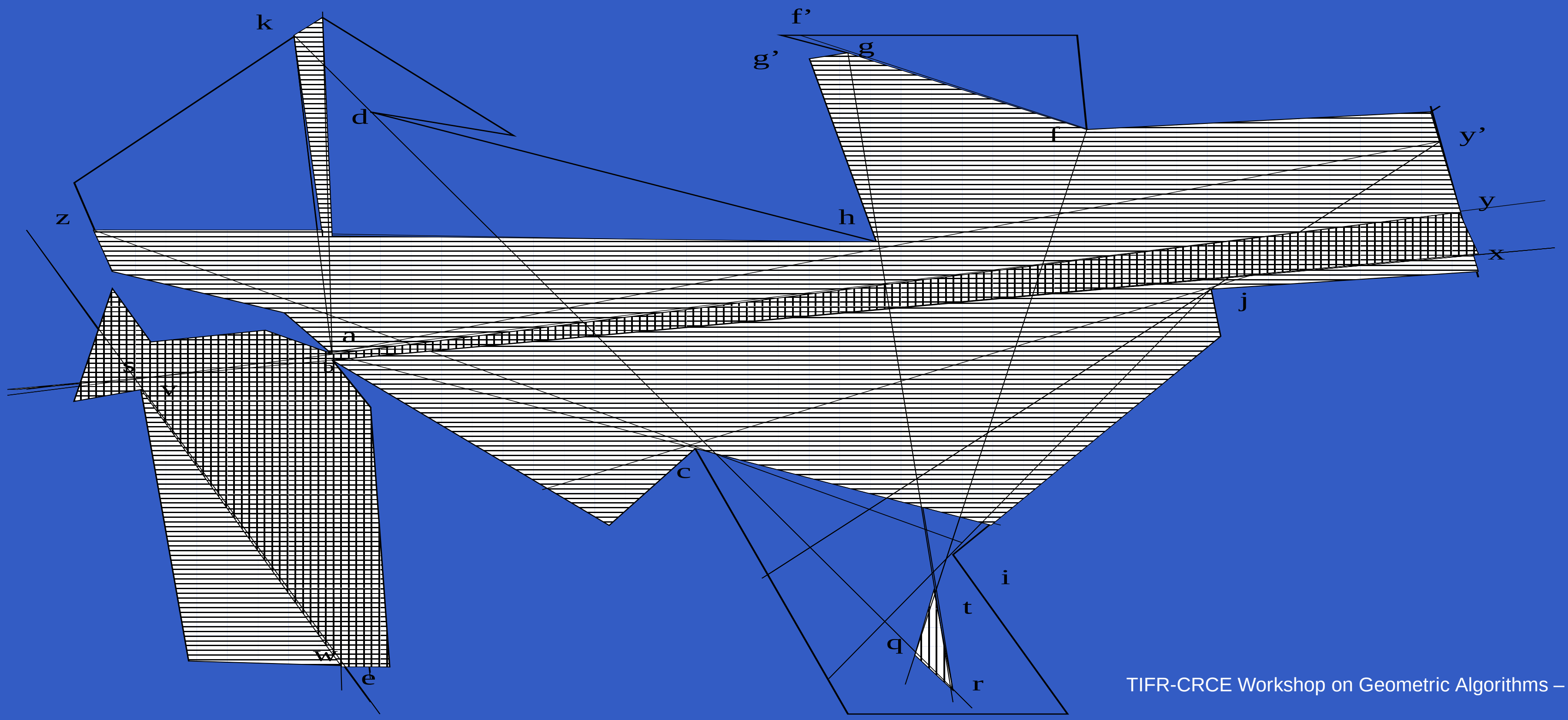
[1] *M. Aanjaneya, S. P. Pal and A. Bishnu, Directly visible pairs and illumination by reflections in orthogonal polygons, 24th European Workshop on Computational Geometry, Nancy, France, 2008.*

[2] *S. K. Ghosh, Visibility algorithms in the plane, Cambridge University Press, 2007.*

[3] *S. K. Ghosh, Computing visibility polygons from a convex set and related problems, Journal of Algorithms, 12:75-95,*

Holes possible with two diffuse reflections

S. P. Pal, S. Brahma and D. Sarkar, A linear worst-case lower bound on the number of holes in regions visible due to multiple diffuse reflections, Journal of Geometry, Vol. 81, no. 1-2, December 2004, Birkhauser-Verlag..



Holes possible with two diffuse reflections

Construct an example with $\Omega(n)$ holes when the number of reflections $k = 2, 3$.