Review of
Visibility Algorithms in the Plane[7]
by Subir Kumar Ghosh
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1 Overview

Computational Geometry is a young field, tracing its beginnings to the PhD thesis of Michael Shamos [6] in 1978. Its many applications include, to name a few, computer graphics and imaging, robotics and computer vision, and geometric information systems. Topics studied within computational geometry include arrangements, convex hulls, partitions, triangulation of polygons, Voronoi diagrams, and visibility.

One of the most famous theorems in computational geometry is the *Art Gallery Theorem*, and this theorem also serves as an example of the focus of the book under review. Posed by Victor Klee in 1973, it asks how many stationary guards are required to see all points in an art gallery represented by a simple, n-sided polygon. The answer, given first by Chvátal [3] and later by Fisk [5], using an elegant graph-theoretic proof, is that $\lfloor n/3 \rfloor$ guards are always sufficient and may be necessary. Questions such as this one, of visibility within a polygon, are the subject of **Visibility Algorithms in the Plane**, by S. Ghosh.

2 Summary of Contents

The book, which contains eight chapters, is a thorough and detailed investigation of theorems and algorithms for a variety of types of polygon visibility problems. It is aimed at an audience of graduate students and researchers who already have a background in computational geometry, and it also assumes that the reader has a general knowledge of algorithms and data structures. The first chapter is introductory, and each of the other seven chapters focuses on a particular type or aspect of visibility. Each chapter begins by reviewing relevant theorems and problems; then algorithms and other results are presented. Each chapter ends with a discussion of other issues related to the chapter's topic, including on-line and parallel algorithms.

- *Chapter 1 Background* defines the several notions of visibility that are subsequently discussed in the book. Properties of polygons and triangulations are covered, and the complexity model *real RAM*, which is used in the rest of the book, is introduced. Chap. 1 also covers the Art Gallery Problem.

- *Chapter 2 Point Visibility* considers the problem, given a polygon P and a point $q \in P$, of computing the visibility polygon $V(q)$, which is the polygonal sub-region of P consisting of all points visible from q. In other words, $V(q)$ is the set of all points $p \in P$ such that the straight line from q to p lies entirely within P. Results and algorithms are given for simple and non-simple polygons, and for non-winding and winding polygons.

- *Chapter 3 Weak Visibility and Shortest Paths* considers three variations of visibility along an edge v_iv_{i+1} of a polygon P. P is *completely visible* from v_iv_{i+1} if every point $p \in P$ is visible from every point $w \in v_iv_{i+1}$. P is *strongly visible* from v_iv_{i+1} if v_iv_{i+1} contains a point w such that every point $p \in P$ is visible from w. P is *weakly visible* from v_iv_{i+1} if each point $z \in P$ is visible from at least one point $w_z \in v_iv_{i+1}$. Further, the *weak visibility polygon* of P from v_iv_{i+1} is the set of all points $z \in P$ such that z is visible to at least one point of v_iv_{i+1}. Weak visibility polygons are characterized in terms of Euclidean shortest paths between vertices, and algorithms for computing weak visibility polygons are given. Algorithms are also given to compute Euclidean shortest paths between points in P.

- *Chapter 4 LR-Visibility and Shortest Paths:* P is an *LR*-visibility polygon if its boundary contains two points s and t such that every point on the clockwise boundary from s to t is visible to at least one point on the counterclockwise boundary from s to t. *LR*-visibility polygons, which generalize weak visibility polygons, are characterized in terms of Euclidean shortest paths. Algorithms to recognize *LR*-visibility polygons, and to compute shortest paths within *LR*-visibility polygons, are presented.

- *Chapter 5 Visibility Graphs:* The *visibility graph* of a polygon P has as its vertices the vertices of P, with two vertices adjacent if they are visible to each other in P. Algorithms are given to compute visibility graphs of simple polygons and polygons with holes. Algorithms are also given to compute the *tangent visibility graph* of P, which contains those edges of the visibility graph that are relevant for computing Euclidean shortest paths.

- *Chapter 6 Visibility Graph Theory* considers questions complementary to those of Chap. 5, namely, given a graph G, determine whether it is the visibility graph of some polygon, and if so, construct such a polygon. These two questions are called, resp., the graph *recognition* and graph *reconstruction* problems. These are both unsolved problems, and their complexity is unknown as well, except it is known that the reconstruction problem is in *PSPACE* [4]. Necessary conditions for a graph to be a visibility graph are given, and testing algorithms are also given for these conditions. In addition, the chapter gives algorithms for recognizing special classes of visibility graphs.

- *Chapter 7 Visibility and Link Paths:* In contrast to a Euclidean shortest path in a polygon P, a *minimum link path* minimizes the number of line segments in a piecewise linear path joining two points of P, and the *link diameter* of P is the maximum number of links in any minimum link path in P. Algorithms are given to find minimum link paths between two points of P, and also to compute the link diameter and related parameters.

- *Chapter 8 Visibility and Path Queries:* Query problems in computational geometry are problems that require a large number of similar computations within a common polygonal domain. For example, given an arbitrary polygon P with n vertices and a point $q \in P$, there is an algorithm to compute the visibility polygon of q in time $O(n \log n)$ [1]; thus this problem for m points, $\{q_1, q_2, ..., q_m\} \subseteq P$, can be solved in time $O(mn \log n)$. But with $O(n^2)$ preprocessing time, the question for m points in P can be answered in time $O(mn)$ time [2]. This query algorithm is presented, as well as query algorithms for the ray-shooting problem (given $q \in P$, find the point on the boundary of P closest to q), Euclidean shortest path, and minimum link path.

3 Comments and conclusion

The book has ample exercises interspersed in the text. They vary in difficulty from brief thought exercises to research level investigations. The text is conversational in tone, yet clear and rigorous in its exposition. It has an index and an extensive bibliography; when using the former while preparing this review, I found it was missing a few fundamental terms, my only criticism of the book. In general, this text accomplishes its intended purpose well – providing a graduate-level text on visibility algorithms that can also serve as a useful reference.

References

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