Geometric Data Structures

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Outline I

Outline II

Introduction

Scope of the lecture

- *Range queries:* We consider 1-d and 2-d range queries for point sets.

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- *Interval trees:* Interval trees and segment trees for reporting all intervals on a line containing a given query point on the line.

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- *Range trees and Kd-trees:* Improved 2-d orthogonal range searching with range trees and kd-trees.
- *Interval trees:* Interval trees and segment trees for reporting all intervals on a line containing a given query point on the line.
- *Paradigm of Sweep algorithms:* For reporting intersections of line segments, and for computing visible regions.

Not in the Scope yet relevant

- *Point location Problem:* The elegant algorithm makes use of traditional data structures such as height balanced trees which are augmented and modified to suite the purpose.

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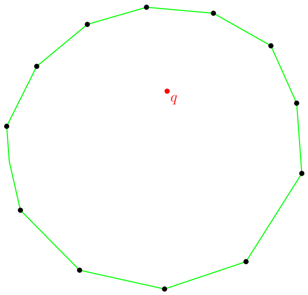
- *Point location Problem:* The elegant algorithm makes use of traditional data structures such as height balanced trees which are augmented and modified to suite the purpose.
- *BSP trees:* Trees are usually normal binary trees again (not even height balanced), so we skip it, even though it is quite interesting and needs a lecture by itself to properly treat the subject.

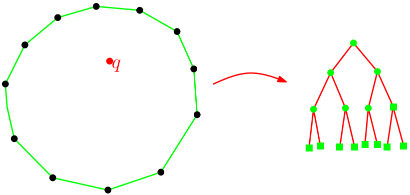
Motivation

Point Inclusion in Convex Polygons



We know a linear time algorithm to determine whether a point is inside a simple polygon.
Can we do it efficiently if polygon is known to be convex? Perhaps in *sub-linear* time.

Point Inclusion in Convex Polygons — Pre-processing



Sub-linear algorithm Impossible (!!) without storing the points in a suitable data structure beforehand.
This is called *pre-processing* of data.

Preprocessing of data

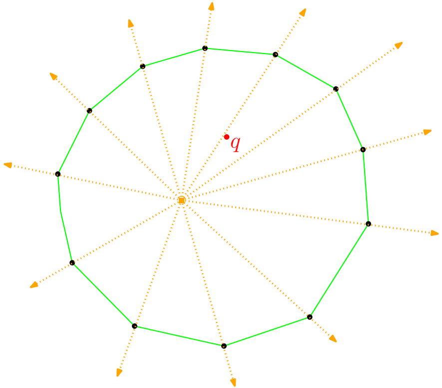
The idea is we pre-process only once for potentially many queries, such that overall efficiency is increased.
The pre-processing step might be costly.

Example

Preprocessing in $O(n \log n)$ acceptable if we can do $O(n)$ queries (or more) in $O(\log n)$ time.

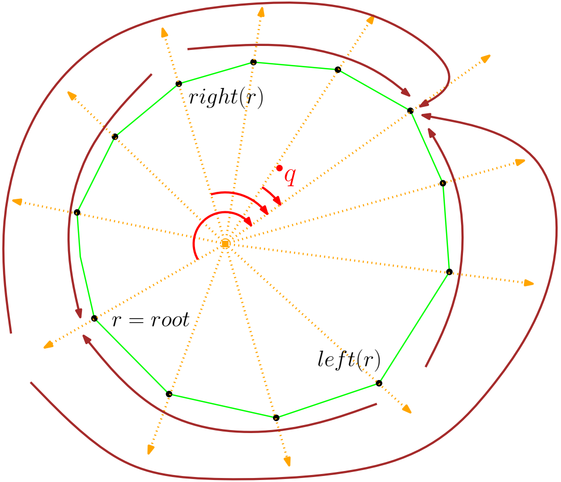
Actually, the break away point is $O(\log n)$ queries if the naive algorithm is linear.

Point Inclusion in Convex Polygons — Binary Search



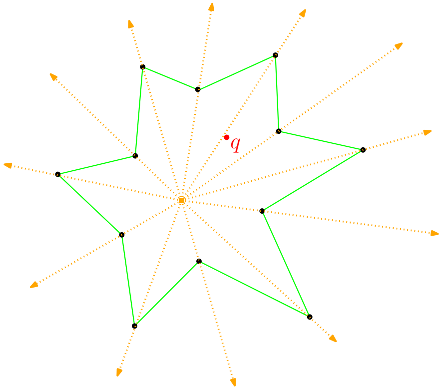
We divide the convex polygon in chain of cones and do a binary search.

Point Inclusion in Convex Polygons — Binary Search



Number of searches = 3.

Point Inclusion in Star-shaped Polygons



Same algorithm also works for star shaped polygons. Convex polygons are special cases of star-shaped polygons.

Range Searching

Example of Range Searching

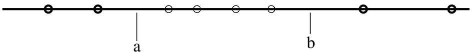
Problem

Given a fixed sequence of numbers $x_1, x_2, \ldots, x_n$ report all numbers x such that $a \leq x < b$ for finitely many varying a's and b's.

I.e. Given, say, 1, 2, 4, 6, 8, 10, we are asked to output numbers between 3 and 7.

In the next slide puts the points on a real line.

1-dimensional Range searching

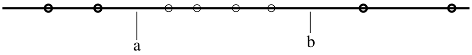


Problem

Given a set P of n points $\{p_1, p_2, \cdots, p_n\}$ on the real line, report points of P that lie in the range $[a, b]$, $a \leq b$.

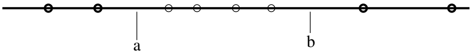
How do we do it?

1-dimensional Range searching



- Using binary search on an array, we can answer such a query in $O(\log n + k)$ time where k is the number of points of P in $[a, b]$.

1-dimensional Range searching



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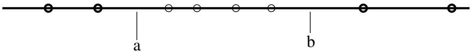

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- We therefore use AVL-trees, B^*-trees, Red-black trees, etc.

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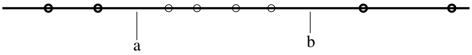

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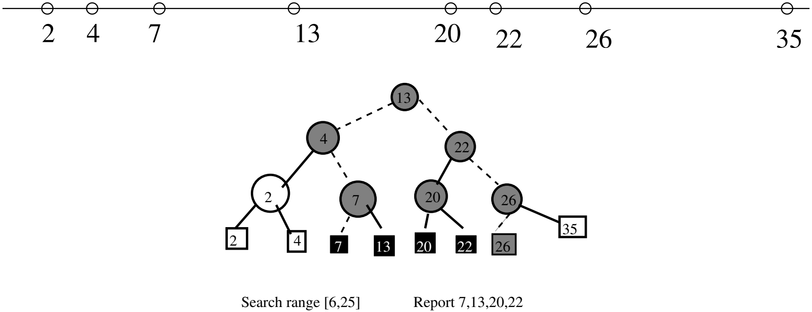
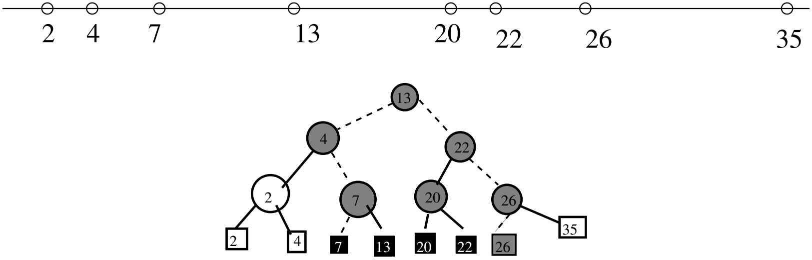
- However, when we permit insertion or deletion of points, we cannot use an array data structure so efficiently.
- We therefore use AVL-trees, B^*-trees, Red-black trees, etc.
- How? By putting all the data on the leaves and linking them by a linked list.

1-dimensional Range searching



- We use a *binary leaf search tree* where leaf nodes store the points on the line, sorted by x-coordinates.

1-dimensional Range searching



Search range [6,25]   Report 7,13,20,22

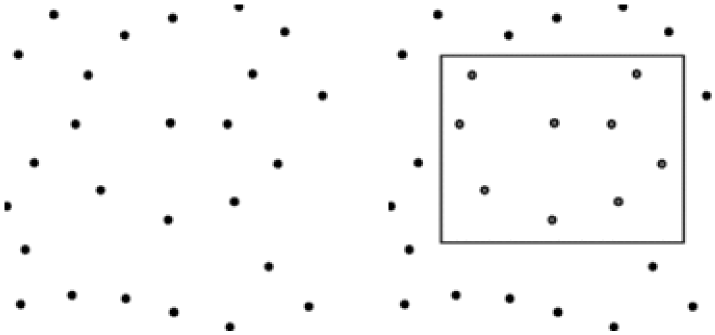
- We use a *binary leaf search tree* where leaf nodes store the points on the line, sorted by x-coordinates.
- Each internal node stores the x-coordinate of the rightmost point in its left subtree for guiding search.

Notion of Output-sensitive Algorithm

- Note the k in the query time i.e. $O(k + \log n)$. The time to report the points depends on the number of output points, k.
- Can be potentially very large, $O(n)$, or very small, $O(1)$, i.e. zero points.
- We would always like such a proportional kind of *Output sensitive* algorithm.

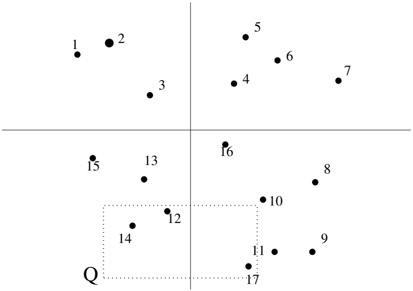
Problem of Rangesearching

Our featured problem: Report points inside a rectangle.



For obvious reasons, this problem is known as 2D/3D range searching.

2-dimensional Range Searching



Problem

Given a set P of n points in the plane, report points inside a query rectangle Q whose sides are parallel to the axes.

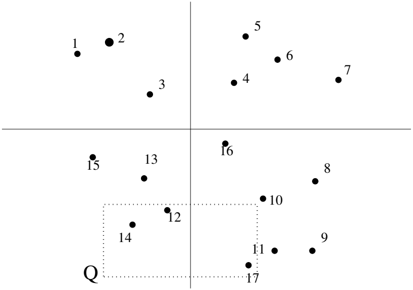
Here, the points inside Q are 14, 12 and 17.
How do we find these?

2-dimensional Range Searching: Simple method

Naive Method

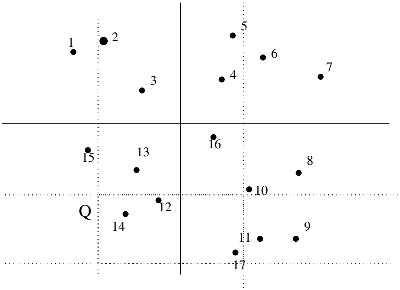
- Check for each point!! If inside the rectangle, output it.
- Neither efficient for multiple searches,
- nor is output sensitive.

2-dimensional Range Searching



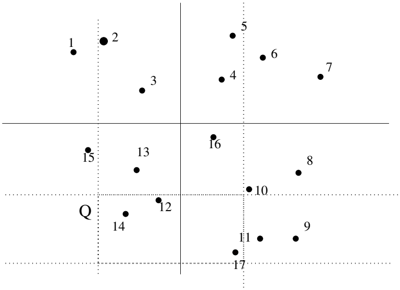
How can we do it efficiently in sub-linear time with an output-sensitive algorithm?

2-dimensional Range Searching - using 1-dimensional Range Searching



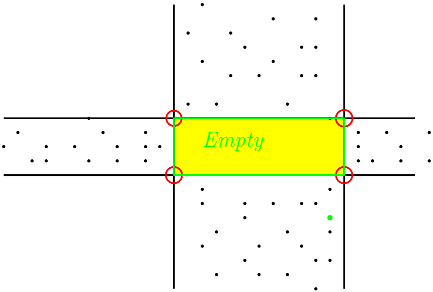
- Using two 1-d range queries, one along each axis, solves the 2-d range query. Output intersection of sets.

2-dimensional Range Searching - using 1-dimensional Range Searching



- Using two 1-d range queries, one along each axis, solves the 2-d range query. Output intersection of sets.
- The cost incurred may exceed the actual output size of the 2-d range query (worst case is $O(n)$, $n = |P|$).

2-dimensional Range Searching — disadvantages in using 1-dimensional Range Searching



- Algorithm is sub-linear but not output-sensitive.
- We are checking points which are not in the desired output. This was not the case in 1-d range searching.

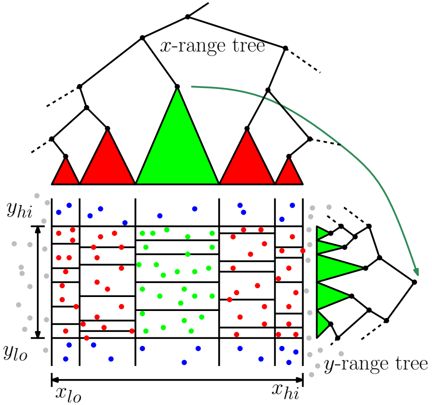
2D Range Searching: Using Range trees and Kd-trees

- Can we do better?
- *Yes!* We can do better using *Kd-tree* (from worst-case $O(n)$ to worst-case $O(\sqrt{n}+k)$).
- *Yes!!* At the cost of further pre-processing time and space we can do still better using *range trees* ($O(\log^2 n + k)$).

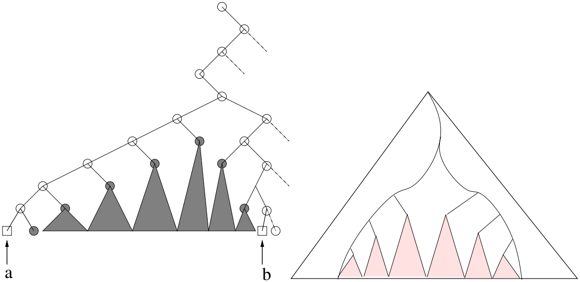
First we take up range-trees (somewhat easier to understand).

Range Trees

Range trees: Idea



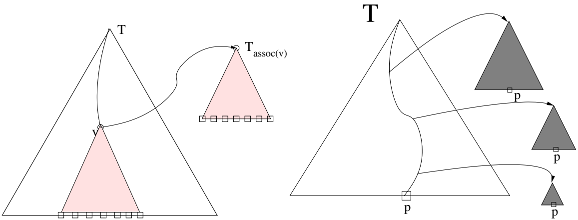
Range trees: Concepts



Given a 2-d rectangle query $[a, b] \times [c, d]$, we can identify subtrees whose leaf nodes are in the range $[a, b]$ along the x-direction.

Only a subset of these leaf nodes lie in the range $[c, d]$ along the y-direction.

Range trees: Storage Requirement?

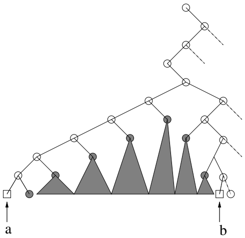


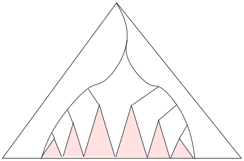
$T_{assoc(v)}$ is a binary search tree on y-coordinates for points in the leaf nodes of the subtree tooted at v in the tree T.

The point p is duplicated in $T_{assoc(v)}$ for each v on the search path for p in tree T.

The total space requirements is therefore $O(n \log n)$.

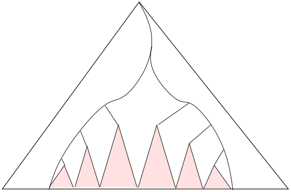
Range trees: Method





We perform 1-d range queries with the y-range $[c, d]$ in each of the subtrees adjacent to the left and right search paths for the x-range $[a, b]$ in the tree T.

Complexity of range searching using range trees



Since the search path is $O(\log n)$ in size, and each y-range query requires $O(\log n)$ time, the total cost of searching is $O(\log^2 n)$. The reporting cost is $O(k)$ where k points lie in the query rectangle.

Range searching with Range-trees

- Given a set S of n points in the plane, we can construct a range tree in $O(n \log n)$ time and space, so that rectangle queries can be executed in $O(\log^2 n + k)$ time.

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- The query time can be improved to $O(\log n + k)$ using the technique of *fractional cascading*. We won't discuss this, some deep constructions are involved.
- *For Audience: How can you construct range tree in* $O(n \log n)$ *time.* $O(n \log^2 n)$ *is easy.*

Kd Trees

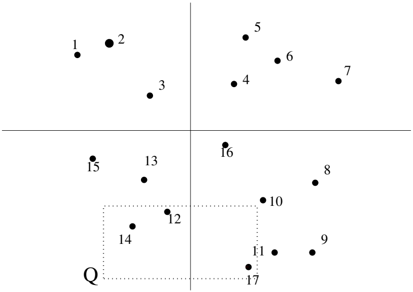
Kd-trees for Range Searching Problem

There is another beast called Kd-trees.

We use a data structure called Kd-tree to solve the problem some-what efficiently in efficient space.

What are Kd-trees?

2-dimensional Range Searching



Problem (Review: What are we doing?)

Given a set P of n points in the plane, report points inside a query rectangle Q whose sides are parallel to the axes.

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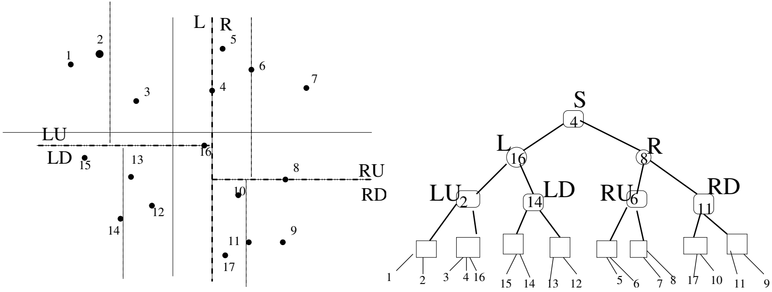
Kd-trees



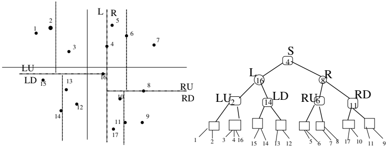
For the given point set, this is how the tree looks like.

Kd-trees for Range Searching Problem

What are Kd-trees?

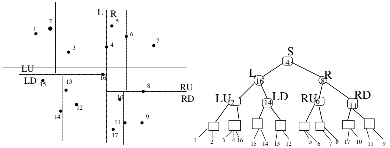
- *Kd-trees* are basically *binary* trees where we divide data first on x-coordinates, then y-coordinates, then z-coordinates, etc. and then cycle.
- In 2D case, alternate levels of Kd-tree are sorted along x-axis and y-axis.

Kd-trees: Some concepts



- The tree T is a perfectly height-balanced binary search tree with alternate layers of nodes spitting subsets of points in P using x- and y- coordinates, respectively as follows.

Kd-trees: Some concepts



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- The point r stored in the root vertex T splits the set S into two roughly equal sized sets L and R using the median x-coordinate $xmedian(S)$ of points in S, so that all points in L (R) have coordinates less than or equal to (strictly greater than) $xmedian(S)$.

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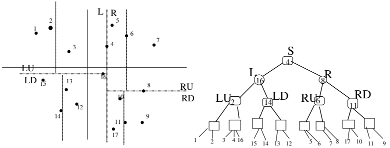

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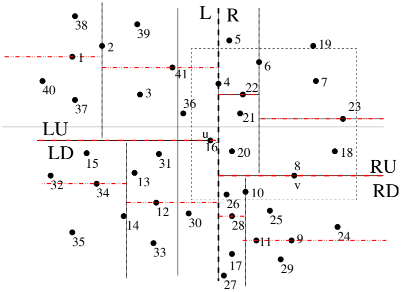
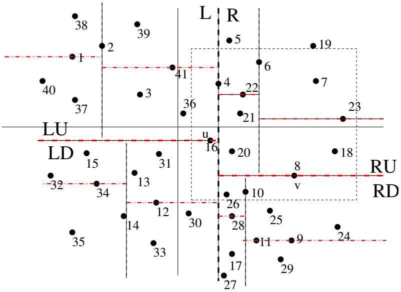
- The entire plane is called the $region(r)$. Better to keep the extents of $region(r)$ as well with r.

Kd-trees: Some Concepts



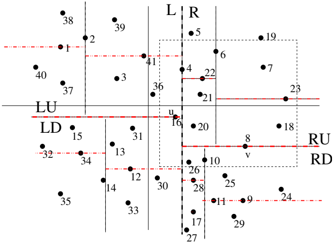
- The set L (R) is split into two roughly equal sized subsets LU and LD (RU and RD), using point u (v) that has the median y-coordinate in the set L (R), and including u in LU (RU).

Kd-trees: Some Concepts



- The set L (R) is split into two roughly equal sized subsets LU and LD $(RU$ and $RD)$, using point u (v) that has the median y-coordinate in the set L (R), and including u in LU (RU).
- The entire halfplane containing set L (R) is called the $region(u)$ $(region(v))$.

Answering rectangle queries using Kd-trees



- A query rectangle Q may partially overlap a region, say $region(p)$, completely contain it, or completely avoids it.

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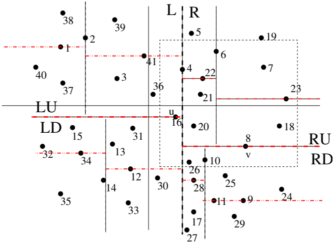
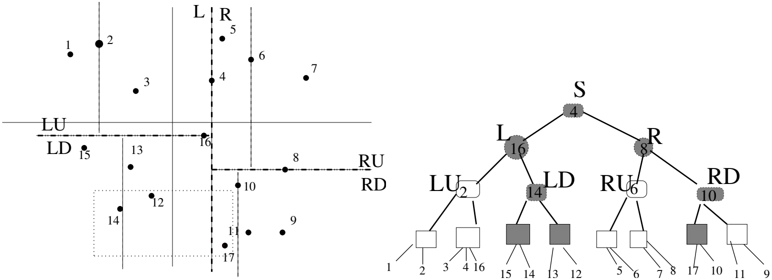


- A query rectangle Q may partially overlap a region, say $region(p)$, completely contain it, or completely avoids it.
- If Q contains an entire bounded $region(p)$ then report all points in $region(p)$.
- If Q partially intersects $region(p)$ then descend into the children.
- Otherwise skip $region(p)$.

Time complexity of rectangle queries



The nodes of the Kd-tree we visit are (1) the internal nodes representing regions partially intersecting Q and (2) all the descendant nodes for regions fully inside Q.

Time complexity of output points reporting

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Time complexity of output points reporting

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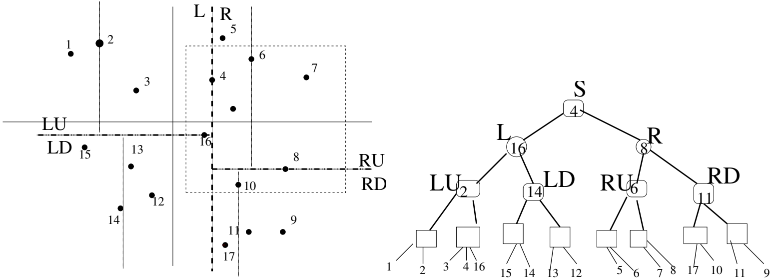
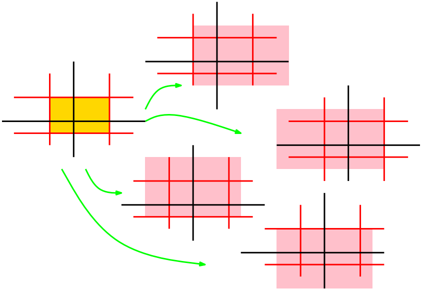
- No leaf level region in T has more than 2 points.
- So, the cost of inspecting points outside Q but within the intersection of leaf level regions of T can be charged to the internal nodes traversed in T, i.e. it will not be more.
- This cost is borne for all leaf level regions intersected by Q. This also takes care of fully contained regions.

Time complexity of traversing the tree



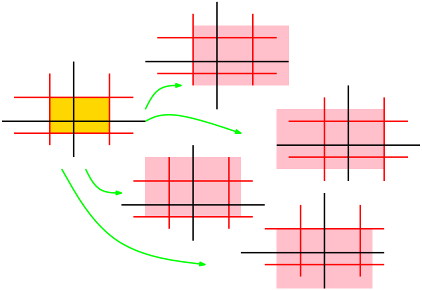
Now we need to bound the number of internal nodes of T traversed for a given query Q corresponding to partial intersections.

Time complexity of traversing the tree



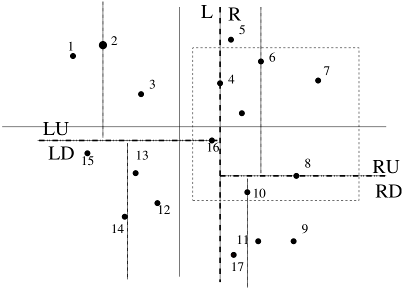
- It is sufficient to determine the upper bound on the number of (internal) nodes whose regions are intersected by a single vertical (horizontal) line. Why?

Time complexity of traversing the tree



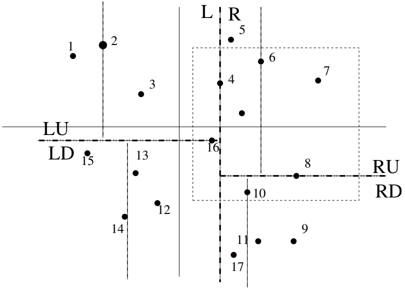
- It is sufficient to determine the upper bound on the number of (internal) nodes whose regions are intersected by a single vertical (horizontal) line. Why?
- Because the internal nodes encountered in case of rectangle are subset of internal nodes encountered for four straight-line (two horizontal and two vertical) queries.

Time complexity of traversing the tree



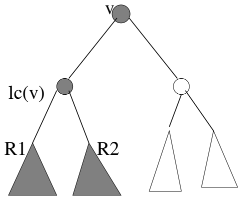
- Any vertical line intersecting S can intersect either L or R but not both, but it can meet both RU and RD (LU and LD).

Time complexity of traversing the tree



- Any vertical line intersecting S can intersect either L or R but not both, but it can meet both RU and RD (LU and LD).
- Any horizontal line intersecting R can intersect either RU or RD but not both, but it can meet both children of RU (RD).

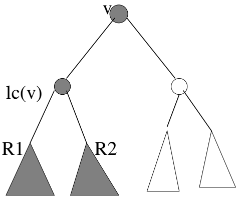
Time complexity of rectangle queries



- Therefore, the time complexity $T(n)$ for an n-vertex Kd-tree obeys the recurrence relation

$$T(n) = 2 + 2T(\frac{n}{4}), \quad T(1) = 1$$

Time complexity of rectangle queries



- Therefore, the time complexity $T(n)$ for an n-vertex Kd-tree obeys the recurrence relation

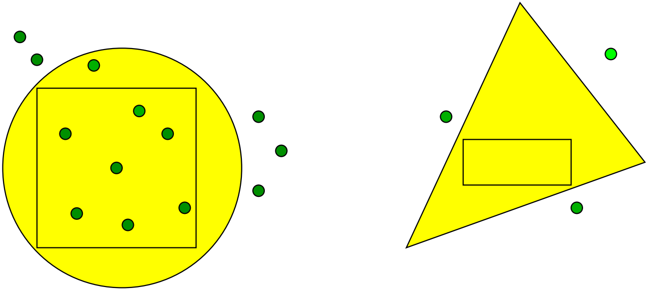
$$T(n) = 2 + 2T(\frac{n}{4}), \quad T(1) = 1$$

- The solution for $T(n)$ is $O(\sqrt{n})$ (an exercise for audience!, direct substitution does not work!!).
- The total cost of reporting k points in Q is therefore $O(\sqrt{n} + k)$.

Summary:Range searching with Kd-trees

Given a set S of n points in the plane, we can construct a Kd-tree in $O(n \log n)$ time and $O(n)$ space, so that rectangle queries can be executed in $O(\sqrt{n} + k)$ time. Here, the number of points in the query rectangle is k.

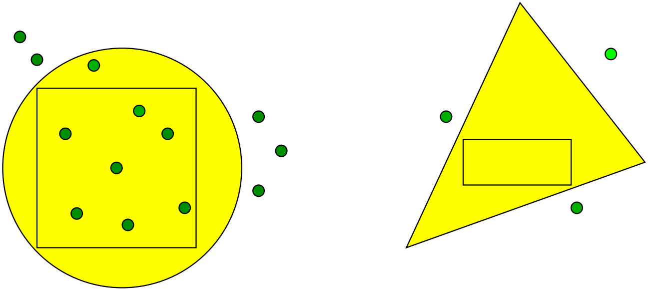
More general queries



General Queries: Points inside triangles, circles, ellipses, etc.

- Triangles can simulate other shapes with straight edges.

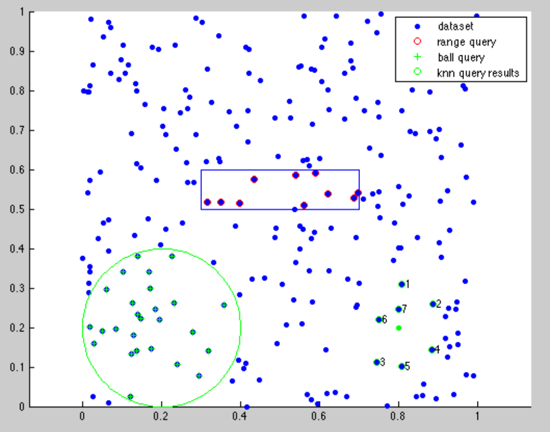
More general queries



General Queries: Points inside triangles, circles, ellipses, etc.

- Triangles can simulate other shapes with straight edges.
- Circle are different, cannot be simulated by triangles! (or any other straight edge figure!!).

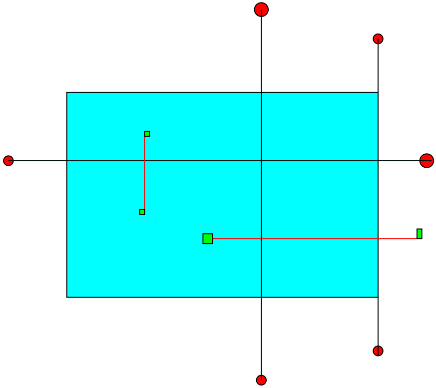
More general queries



Kd-trees can be used for all of these, rectangle query, circle query and nearest neighbour!!

Interval Queries

Motivation: Windowing Problem

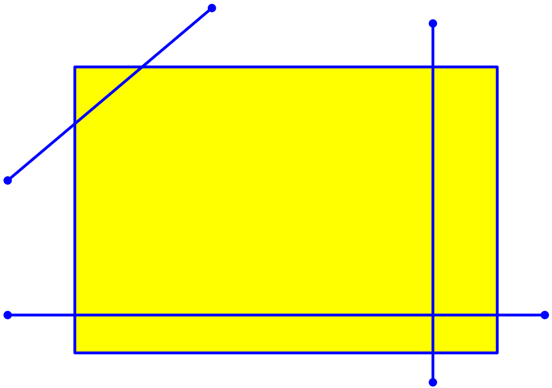


Problem

Report all the objects (may be partial) inside a window.

We already dealt with points. Here the objects are line-segments.

Motivation: Windowing Problem



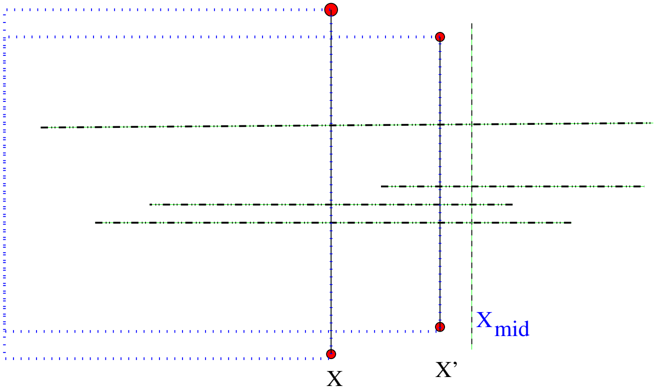
Windowing problem is not a special case of rangequery. Even for orthogonal segments.

Segments with endpoints outside the window can intersect it.

Simple Algorithm

- Report line segments if both end-points inside the window, i.e. full line-segments.
- Report line segments intersecting the four boundaries, i.e., partial line segments.

Special Treatment: Windowing Problem



We solve the special case. We take the window edges as infinite lines for segments partially inside (we report extra).

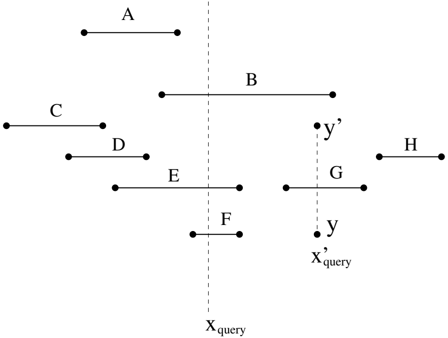
Problem concerning intervals

Problem

Given a set of intervals I and a query point x, report all intervals in I intersecting x.

Solution: Obviously $O(n)$, $n = |I|$, algorithm will work.

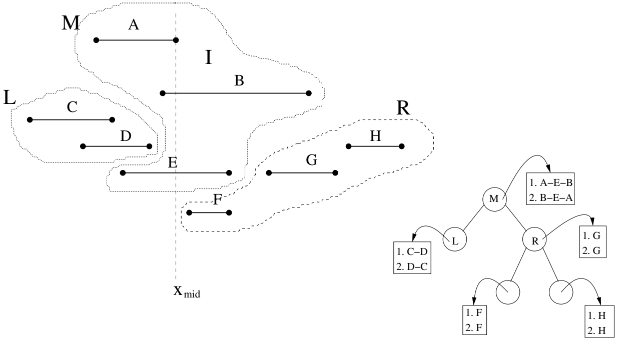
Finding intervals containing a query point



Simpler queries ask for reporting all intervals intersecting the vertical line $X = x_{query}$.
More difficult queries ask for reporting all intervals intersecting a vertical segment joining (x'_{query}, y) and (x'_{query}, y').

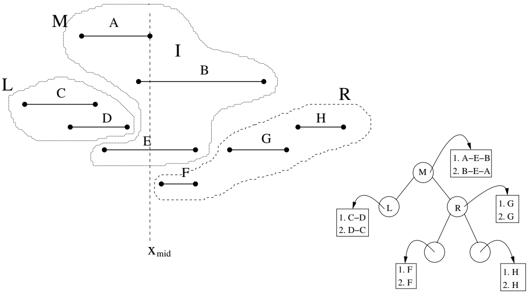
Interval Trees

Interval trees: What are these?



The set M has intervals intersecting the vertical line $X = x_{mid}$, where x_{mid} is the median of the x-coordinates of the $2n$ endpoints. The root node has intervals M sorted in two independent orders (i) by right end points (B-E-A), and (ii) left end points (A-E-B).

Interval tree: Computing and Space Requirements



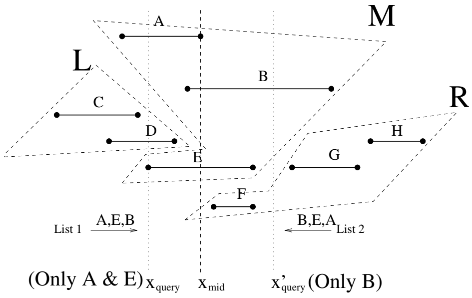
The set L and R have at most n endpoints each.

So they have at most $\frac{n}{2}$ intervals each.

Clearly, the cost of (recursively) building the interval tree is $O(n \log n)$.

The space required is linear.

Answering queries using an interval tree



For $x_{query} < x_{mid}$, we do not traverse subtree for subset R.

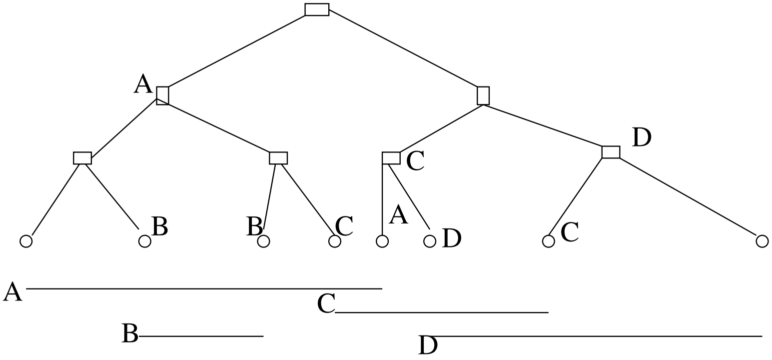
For $x'_{query} > x_{mid}$, we do not traverse subtree for subset L.

Clearly, the cost of reporting the k intervals is $O(\log n + k)$.

Another solution using segment trees

There is yet another beast called *Segment Trees!*.
Segment trees can also be used to solve the problem concerning intervals.

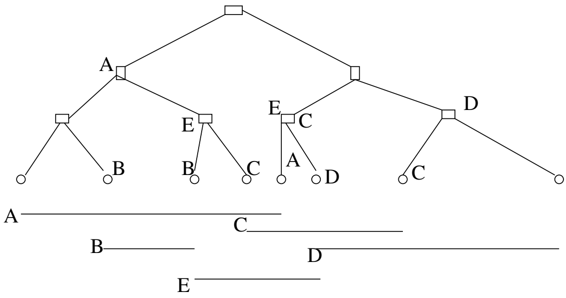
Introducing the segment tree



For an interval which spans the entire range $inv(v)$, we mark only internal node v in the segment tree, and not any descendant of v.

We never mark any ancestor of a marked node with the same label.

Representing intervals in the segment tree

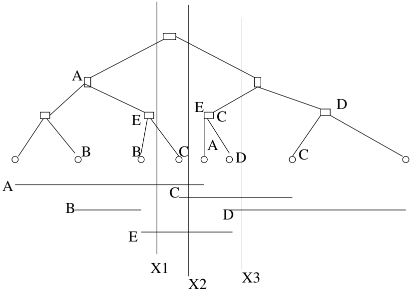


At each level, at most two internal nodes are marked for any given interval.

Along a root to leaf path an interval is stored only once.

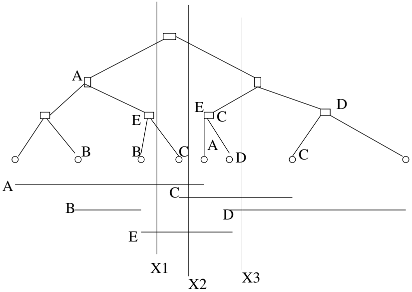
The space requirement is therefore $O(n \log n)$.

Reporting intervals containing a given query point



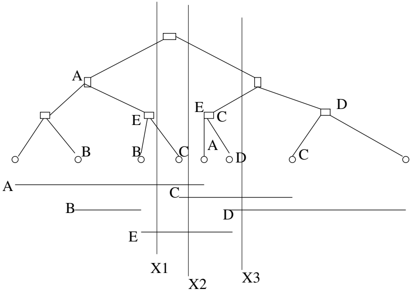
- Search the tree for the given query point.

Reporting intervals containing a given query point



- Search the tree for the given query point.
- Report against all intervals that are on the search path to the leaf.

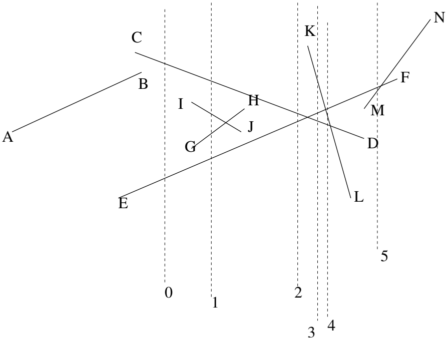
Reporting intervals containing a given query point



- Search the tree for the given query point.
- Report against all intervals that are on the search path to the leaf.
- If k intervals contain the query point then the time complexity is $O(\log n + k)$.

Line Sweep Technique

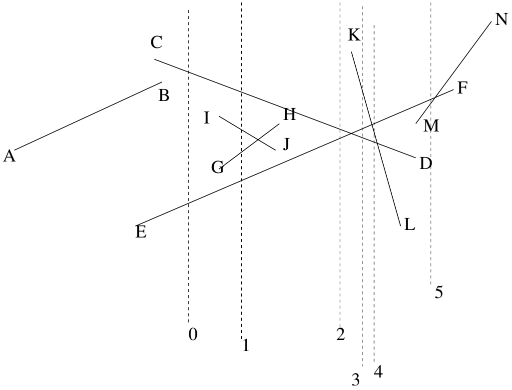
Problem to Exemplify Line Sweep



Problem

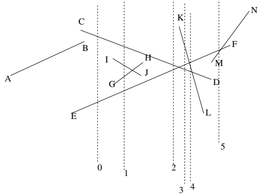
Given a set S of n line segments in the plane, report all intersections between the segments.

Reporting segments intersections



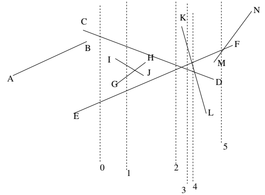
Easy but not the best solution: Check all pairs in $O(n^2)$ time.

Line Sweep: Some observations for Sweeping



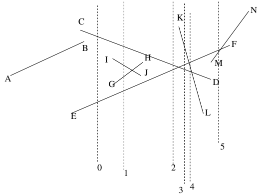
- A vertical line just before any intersection meets intersecting segments in an empty, intersection free segment, i.e. they must be consecutive.

Line Sweep: Some observations for Sweeping



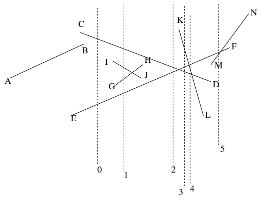
- A vertical line just before any intersection meets intersecting segments in an empty, intersection free segment, i.e. they must be consecutive.
- Detect intersections by checking consecutive pairs of segments along a vertical line.

Line Sweep: Some observations for Sweeping



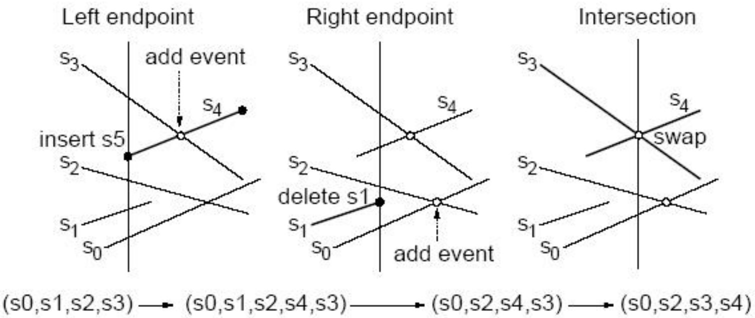
- A vertical line just before any intersection meets intersecting segments in an empty, intersection free segment, i.e. they must be consecutive.
- Detect intersections by checking consecutive pairs of segments along a vertical line.
- This way, each intersection point can be detected. How?

Line Sweep: Some observations for Sweeping



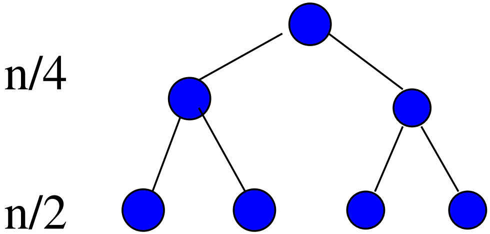
- A vertical line just before any intersection meets intersecting segments in an empty, intersection free segment, i.e. they must be consecutive.
- Detect intersections by checking consecutive pairs of segments along a vertical line.
- This way, each intersection point can be detected. How?
- We maintain the order at the sweep line, which only changes at event points.

Sweeping: Steps to be taken at each event



We use heap for event queue.
We use binary search trees (balanced) for segments in the sweep line.
Source (for image): http://research.engineering.wustl.edu/ pless/

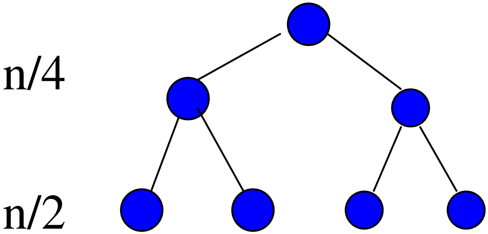
Reporting segments intersections



$$n = 2^k - 1$$

- The use of a heap does nor require apriori sorting.

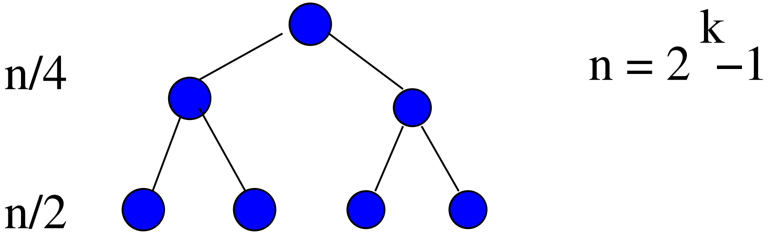
Reporting segments intersections



$$n = 2^k - 1$$

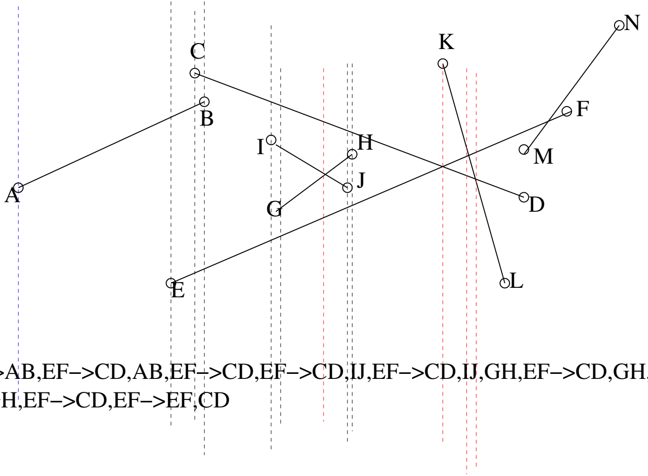
- The use of a heap does nor require apriori sorting.
- All we do is a heap building operation in linear time level by level and bottom-up.

Reporting segments intersections



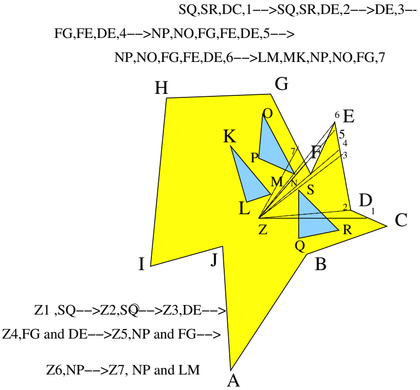
- The use of a heap does nor require apriori sorting.
- All we do is a heap building operation in linear time level by level and bottom-up.
- $1 \times \frac{n}{2} + 2 \times \frac{n}{4} + 4 \times \frac{n}{8} + \ldots$

Sweeping steps: Endpoints and intersection points



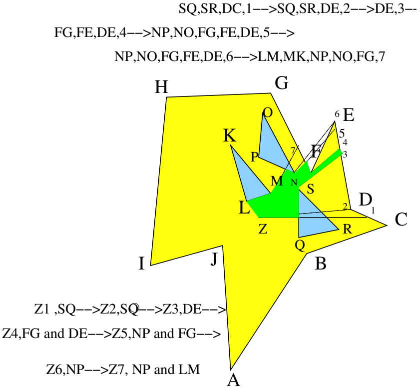
AB–>AB,EF–>CD,AB,EF–>CD,EF–>CD,IJ,EF–>CD,IJ,GH,EF–>CD,GH,IJ,EF
CD,GH,EF–>CD,EF–>EF,CD

Problem for Visibility: angular sweep



The problem is to determine edges visible from an interior point [6]. We can use a similar angular sweep method.

Visibility polygon computation



Final computed visibility region.

Application-I

- Why do we need special data structures for Computational Geometry?

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- Why do we need special data structures for Computational Geometry?
- Because objects are more complex than set of arbitrary numbers.
- And yet, they have geometric structure and properties that can be exploited.

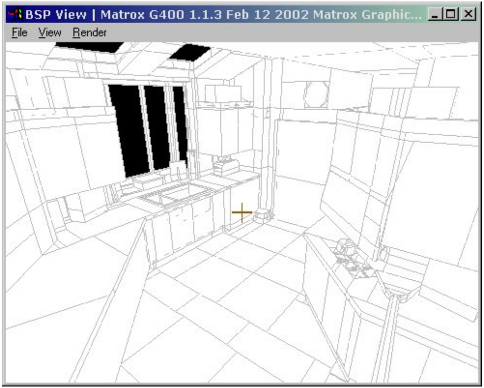
Application-I: Visibility in plane/space

Any first-person-shooter game needs to solve visibility problem of computational geometry which is mostly done by Binary Space Partitions (BSP) [4, 5]. (Software: BSPview)

Application-I: Visibility in rough terrain

We might not have enclosed space, or even nice simple objects.
(Software: BSPview)

Application-I: Visibility in a room



At every step, we need to compute visible walls, doors, ceiling and floor. (Software: BSPview)

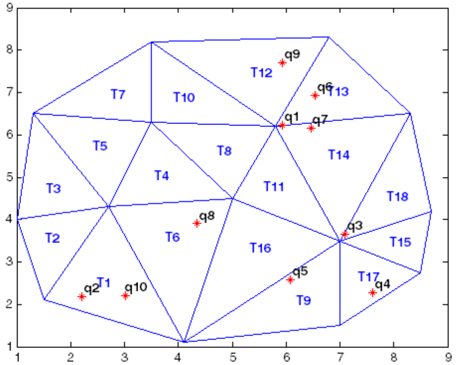
Application-I: Calculation of Binary Space Partitions

The data structure that is useful in this situation is known as *Binary Space/Planar Partitions*.
Almost every 3D animation with moving camera makes use of it in rendering the scene.

Application-II: Locating given objects is geometric subdivisions

Another problem, we might need to locate objects (the elephant) in distinct regions like trees, riverlet, fields, etc. (GPLed game: 0AD)

Application-II: Location of objects in subdivision

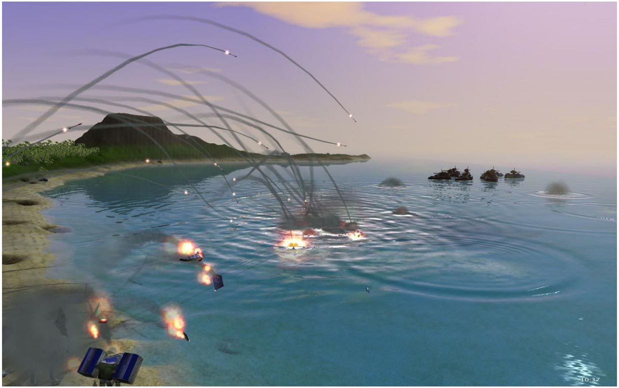


This problem is known as point location problem in its simplest special case.

Application-III: Finding objects in a window

Yet in another case, we need to find all objects in a given window that need to be drawn and manipulated. (GPLed game: 0AD)

Application-III: Finding intersections of objects



This is classical collision detection. Intersection of parabolic trajectories with a 3D terrain. (GPLed game: TA-Spring)

Application-III: Problem of Collision Detection/Finding Intersections

This problem is known as collision detection.
In the static case it is just the intersections computation problem.

Conclusion

Open Problems and Generalisations

- Generalizations of each of the problems in space and higher dimensions
- Counting points/objects inside different type of objects such as triangles, circles, ellipses, or, tetrahedrons, simplexes, ellipsoids in higher space and higher dimensions
- Good data structures for computing and storing visibility information in 3D

Summary

- We studied the concept of Kd-trees and Range trees.

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- We studied the concept of Kd-trees and Range trees.
- Next we saw how we can answer queries about intervals using interval trees and segment trees.
- We looked into line-sweep technique.
- Now we are for the final concluding remarks.

You may read

- The classic book by Preparata and Shamos [7], and
- The introductory textbook by Marc de Berg et al. [2],
- The book on algorithms by Cormen et al. [1] contains some basic geometric problems and algorithms,
- For point location Edelsbrunner [3], though a little hard to understand, is good,
- And lots of lots of web resources.

Acknowledgments

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At Last . . .

Thank You

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