

Introduction to Robotics



Amitabha Mukerjee

IIT Kanpur, India

What is a Robot?

Robot properties:

- Flexibility in Motion
- Mobile robots

daksh ROV: de-mining robot
20 commissioned in Indian
army 2011.

100+ more on order
built by R&D Engineers, Pune

daksh platform derived
gun mounted robot (GMR)



Want your personal robot?



Roomba vacuum
Cleaning robot

By i-robot
Price: ~ rs. 15-30K

How to vacuum a space?



Roomba vacuum
Cleaning robot

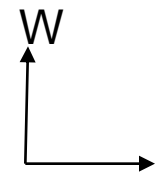
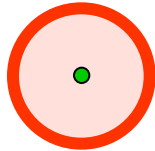
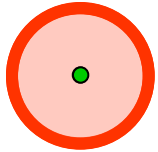
By i-robot
Price: ~ rs. 30K

<https://www.youtube.com/watch?v=dweVBqei9L>

△

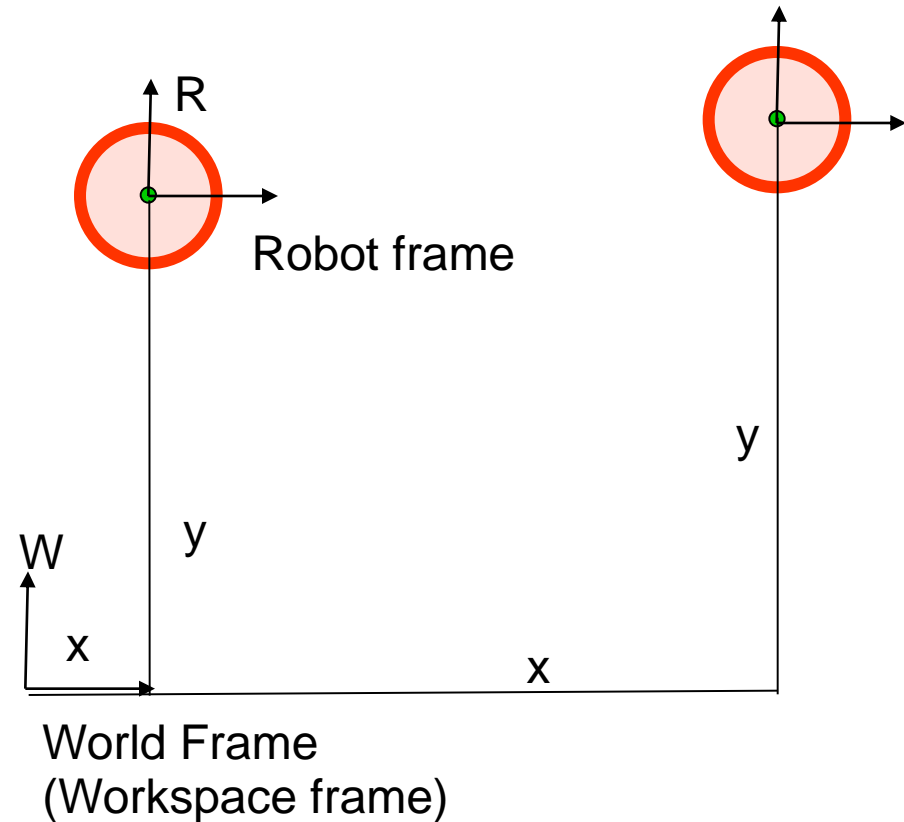
Models of Robot Motion

Circular robot



World Frame
(Workspace frame)

Models of Robot Motion



DEFINITION:

NOTE:

degrees of freedom:

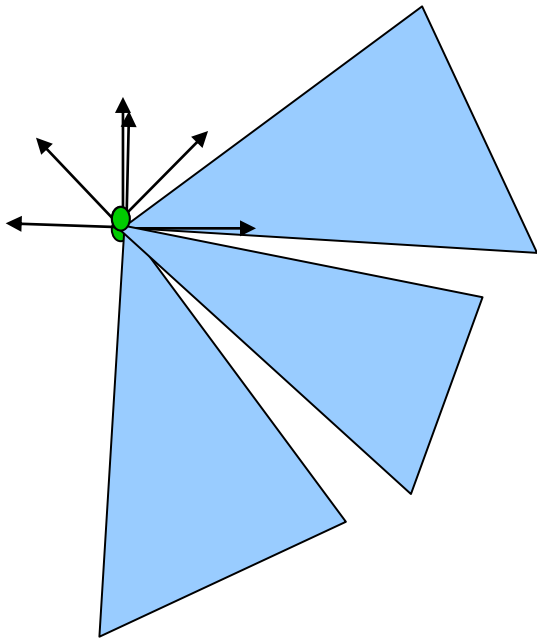
Given robot frame R , every point on the robot is known

number of parameters needed to fix the robot frame R in the world frame W

$(x,y) =$ **configuration**
(vector \mathbf{q})

given configuration \mathbf{q}
for a certain pose of the robot, the set of points on the robot is a function of the configuration: say $R(\mathbf{q})$

Non-Circular Robot



DEFINITION:

degrees of freedom:

number of parameters needed
to fix the robot frame R
in the world frame W

How many parameters needed to fix
the robot frame if it can only translate?

How many if it can rotate as well?

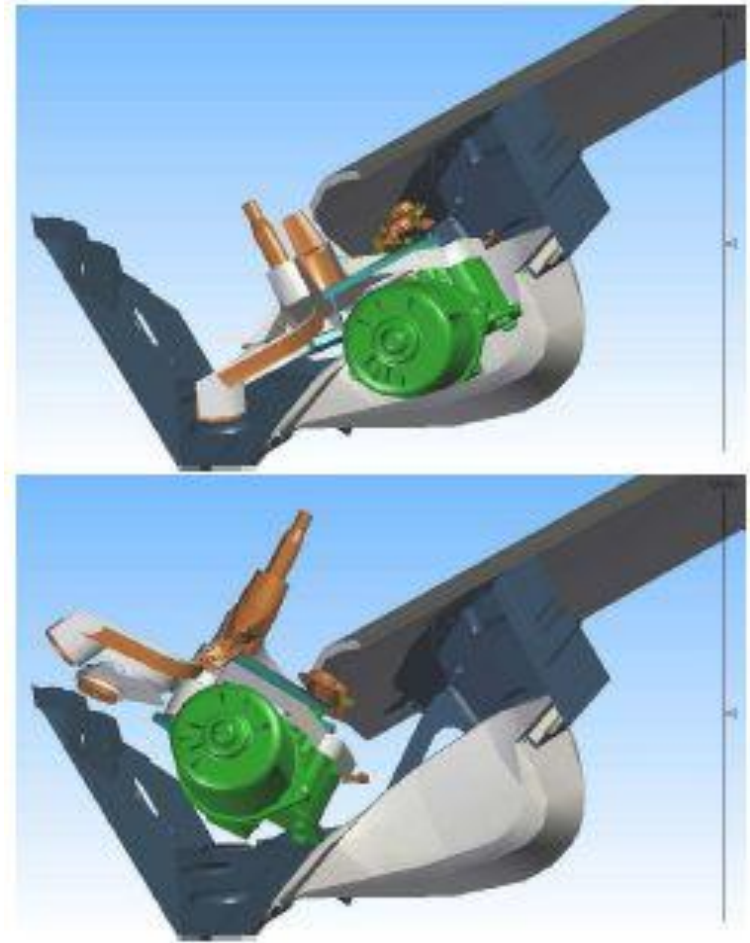
Motion in 3-D: Piano movers problem

General 3D motion:

How many parameters
needed to fix the pose?

Can a design be
assembled?

Test based on CAD models



Research mobile robot



Turtlebot

Based on i-robot (roomba) platform
(with kinect RGB-D sensor)

ROS (open-source) software

Price: ~ 75K

Articulated robots

What is a Robot?

Robots properties:

- Flexibility in Motion
 - Mobile robots
 - Articulated robots

SCARA 4-axis arm
(4 degrees-of-freedom)

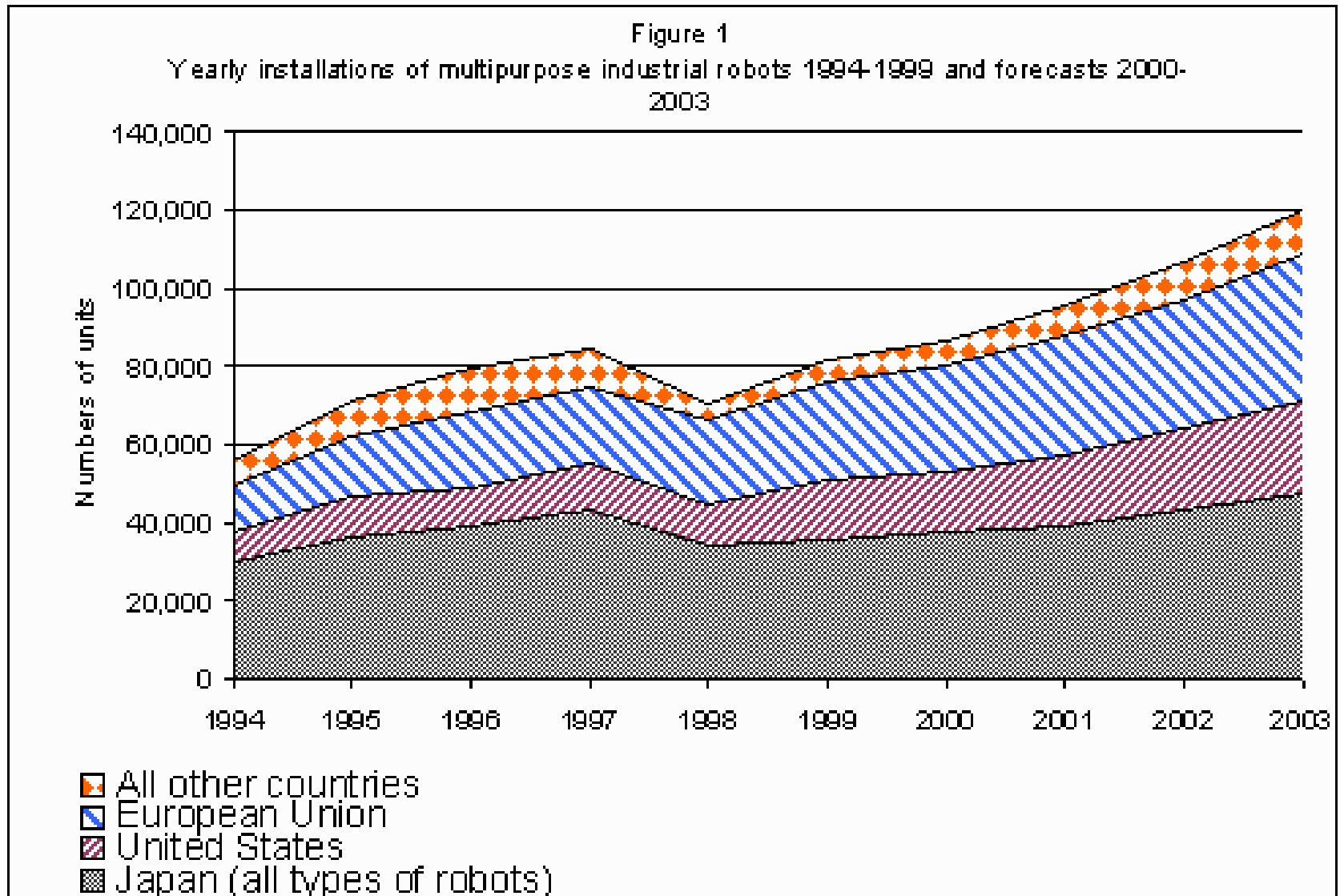
by Systemantics
Bangalore



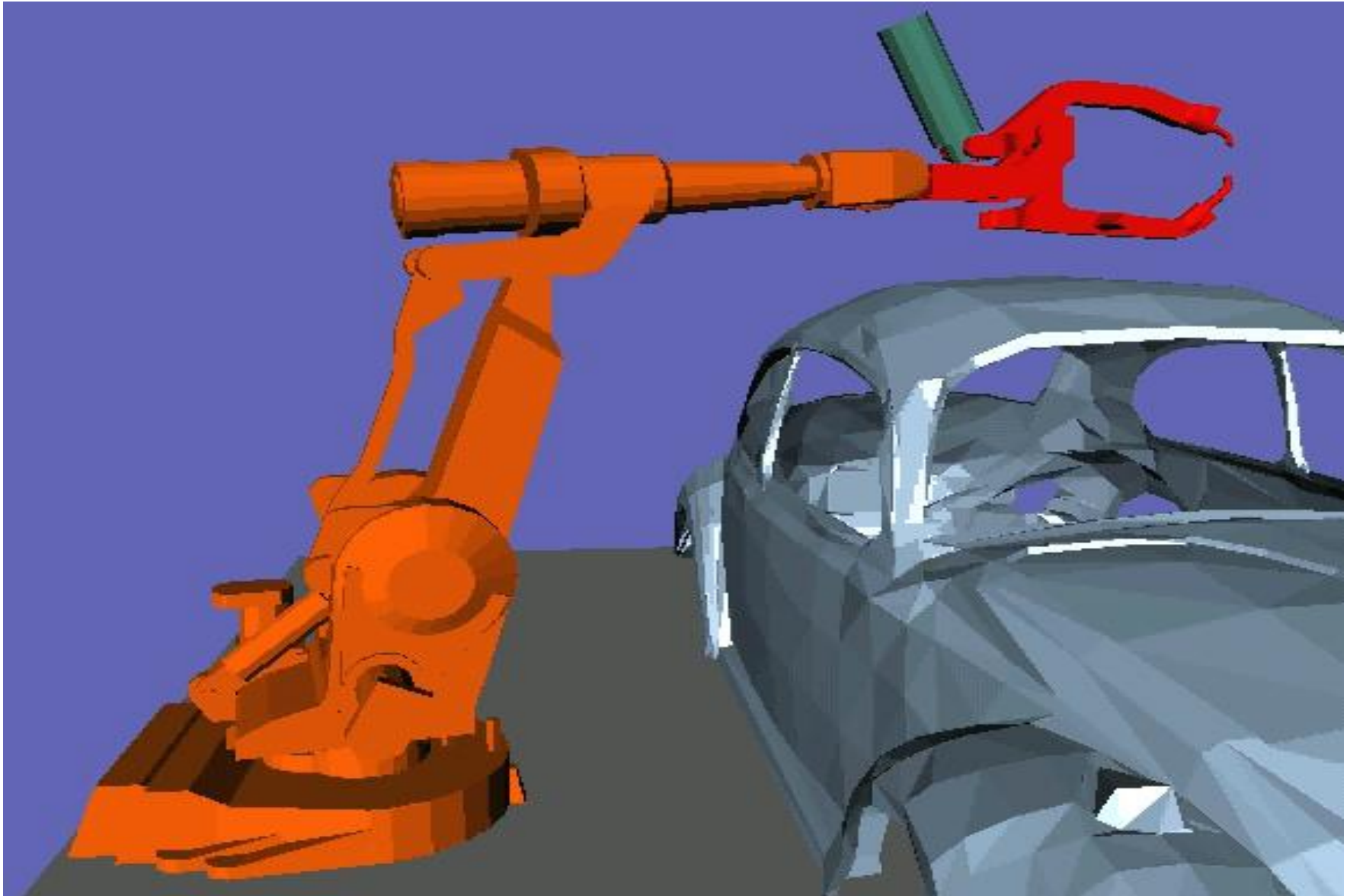
Industrial Robot



Industrial Robots



How to program a welding robot?



What is a Robot?

Robot properties

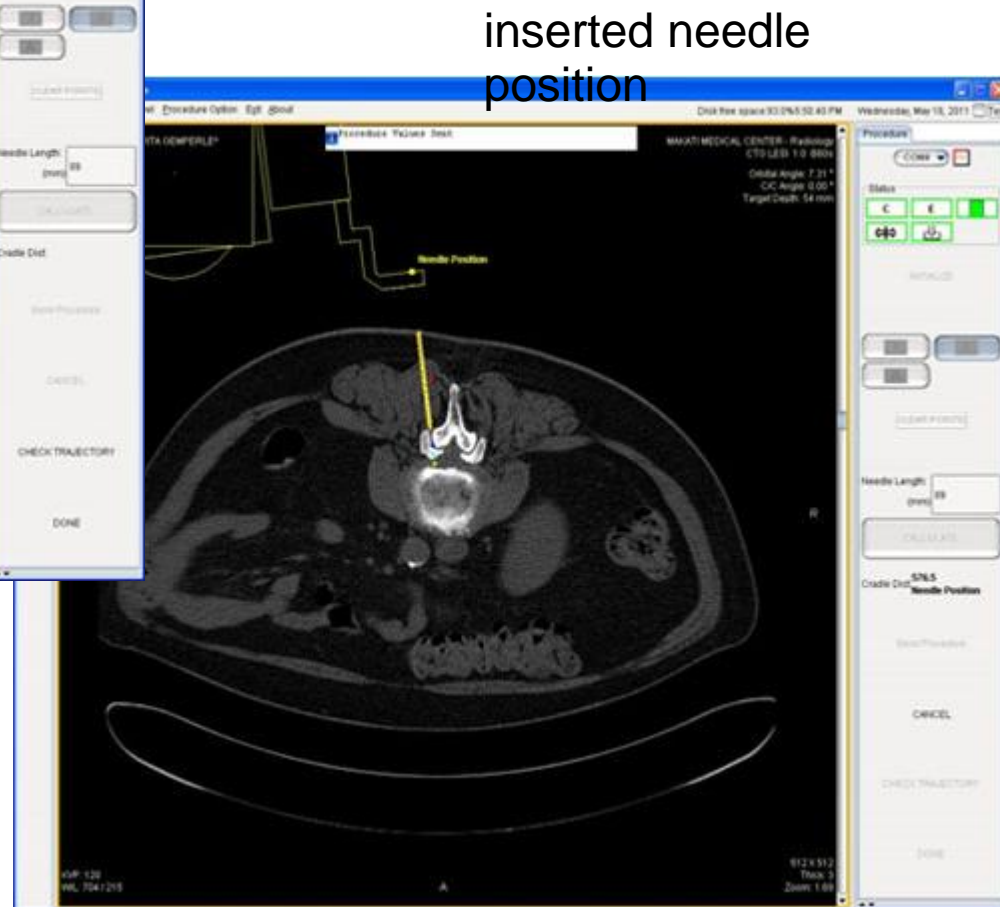
- Flexibility in M
- Mobile robot
- Articulated r
- Industrial r
- Surgical robots



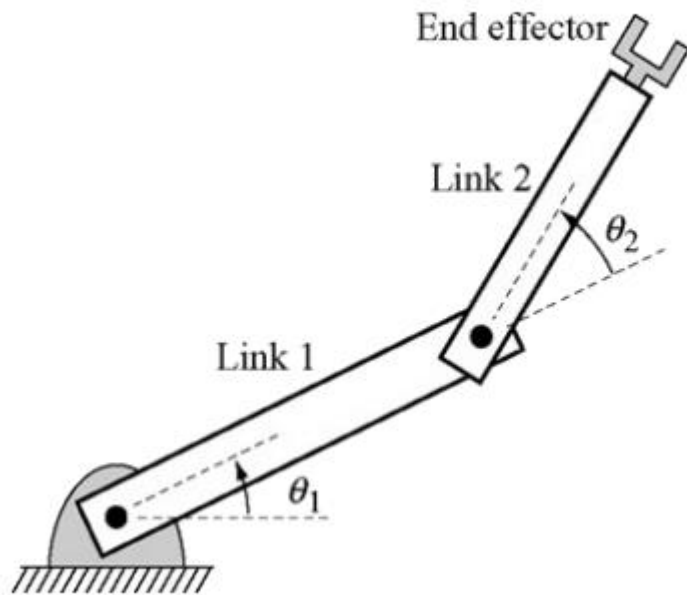
Surgical Robot : Lumbar biopsy



needle path as planned on CAT scan



Modeling Articulated Robots



Kinematic chain:

Pose of Link n depends on the poses of Links $1 \dots (n-1)$

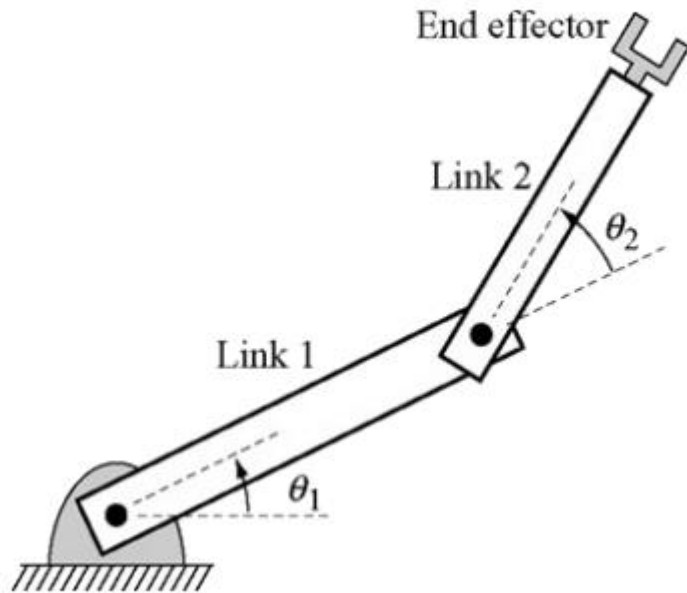
Transformation between frame of link $(n-1)$ and link n , depends on a single motion parameter, say θ_n

Exercise:

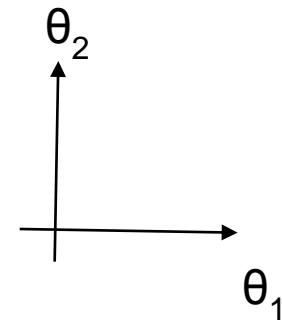
What are the coordinates of the origin of the end-effector center?

Modeling Articulated Robots

workspace



configuration space



Exercise:

Sketch the robot pose for the configuration $[0, -90]$

Modeling Articulated Robots

Forward kinematics

Mapping from configuration \mathbf{q} to robot pose, i.e. $R(\mathbf{q})$

Usually, $R()$ is the product of a sequence of transformations from frame i to frame $i+1$.

Note: Must be very systematic in how frames are attached to each link

Inverse kinematics

a. Given robot pose, find \mathbf{q}

Or

b. Given end-effector pose, find \mathbf{q}

Q. Is the answer in (b) unique?

What is a Robot?

Robots properties

- Flexibility in Motion
 - Mobile robots
 - Articulated robots
 - Digital actors



Reality: limited functionality



Mobility isnt everything

What is a Robot?

Robots involve

- Flexibility in Motion
 - Dentists cradle?
 - Washing machine?
- Intentionality
 - Measure : not default probability distribution
 - e.g. Turn-taking (contingent behaviour)
 - Goal : intrinsic or extrinsic

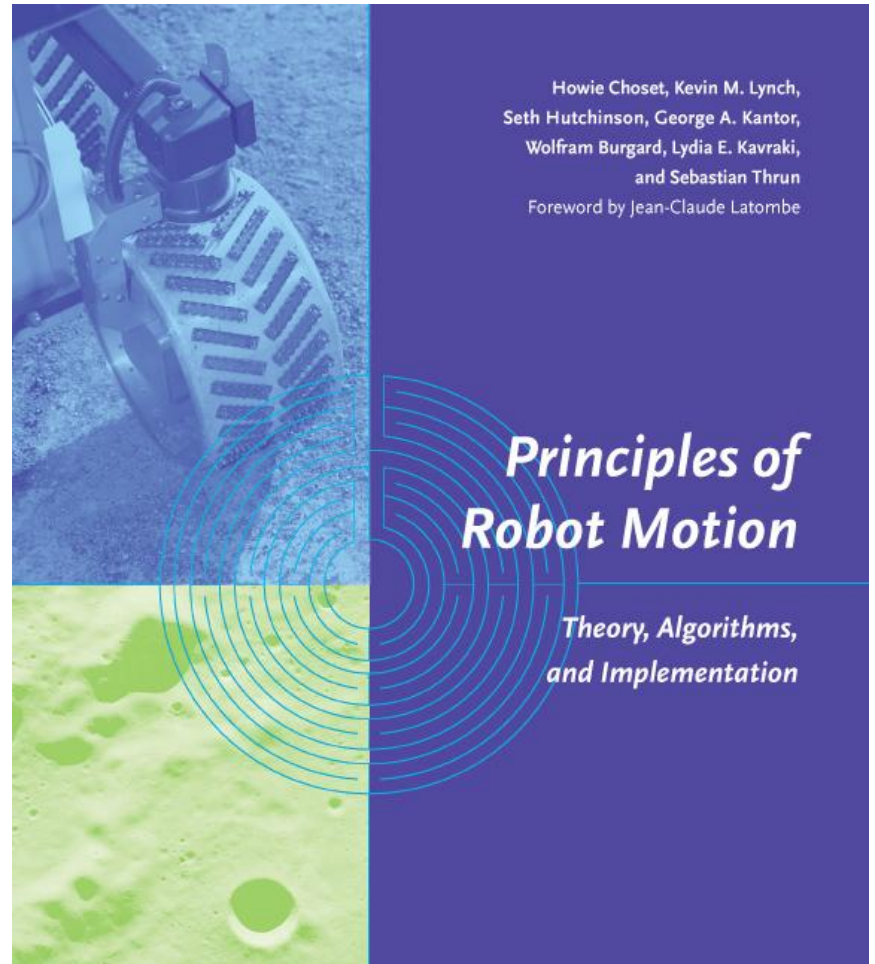
Robot Motion Planning



Amitabha Mukerjee

IIT Kanpur, India

indian edition
rs 425

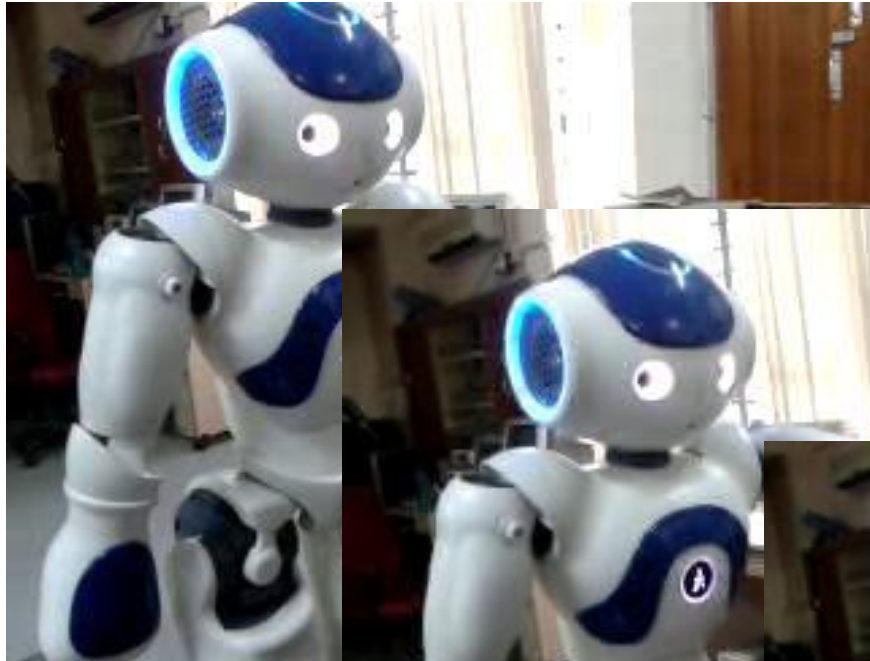


Sensing and Motion Planning



[bohori venkatesh singh mukerjee 05]
Bohori/Venkatesh/Singh/Mukerjee:2005

Programming a robot



Aldebaran Nao

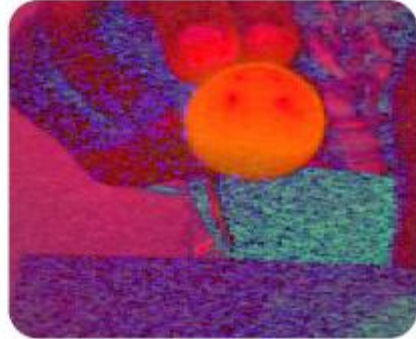
Grasping an offered ball

Programming a robot

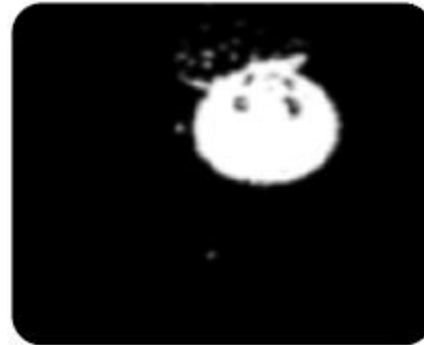
1. detect ball using colour:



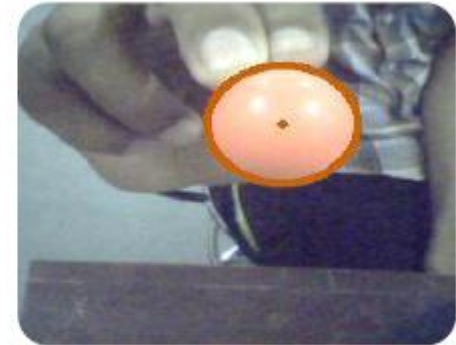
image captured by nao



HSV



binarized



contour detected

2. estimate distance of ball (depth)
from image size

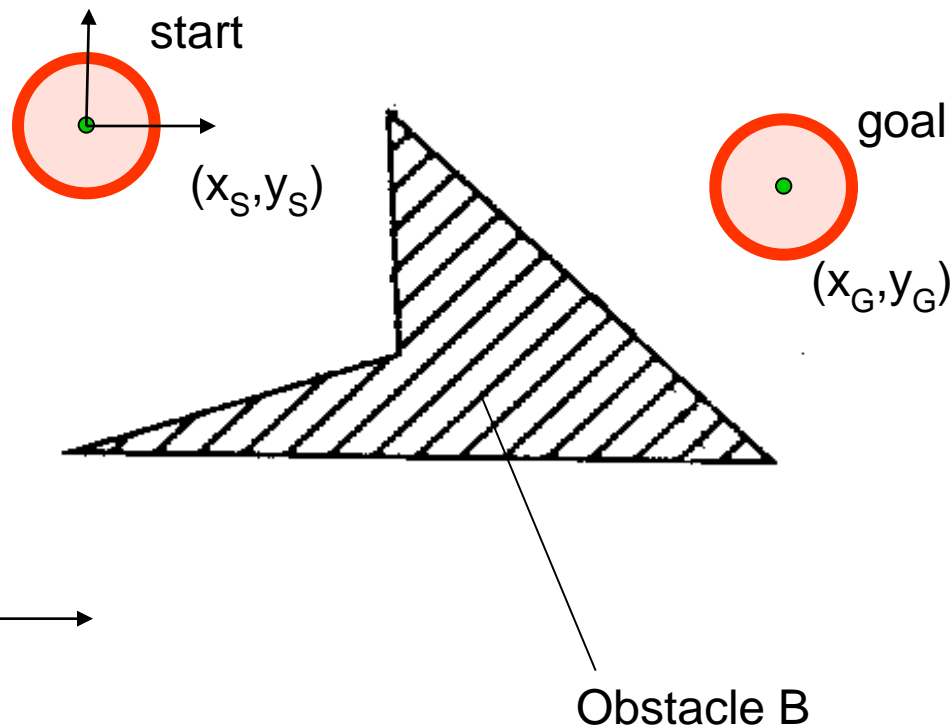
3. Inverse kinematics to grasp ball

Sensing in the workspace

Motion planning in C-space

Configuration Space

Robot Motion Planning



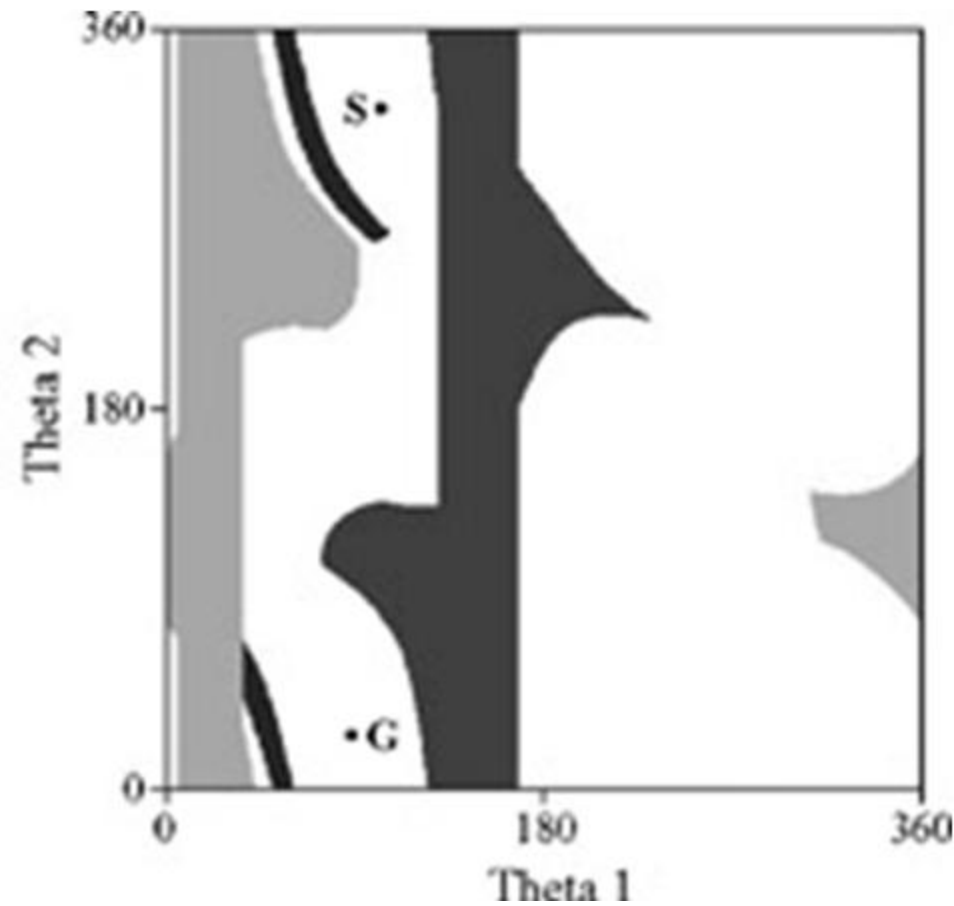
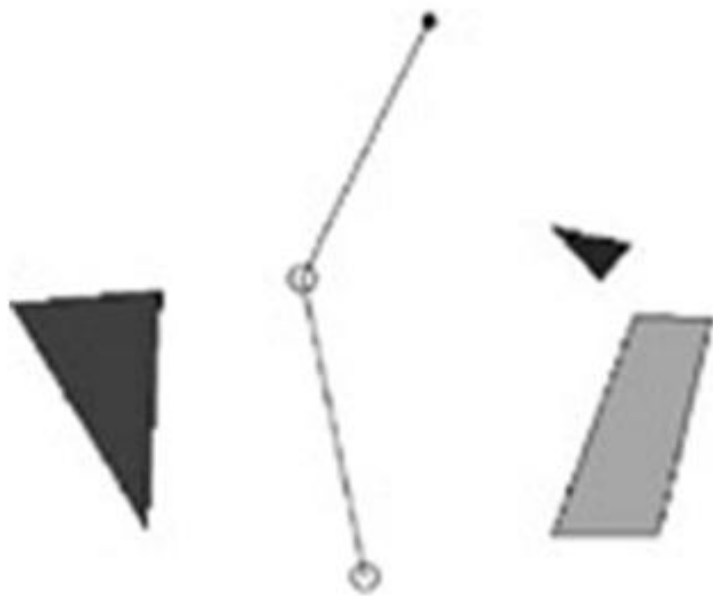
Valid paths will lie among those where the robot does not hit the obstacle

find path P from start to goal s.t.

$$\text{for all } t, R(t) \cap B = \emptyset$$

How to characterize the set of poses for which the robot does not hit the obstacle B?

Robot Motion Planning

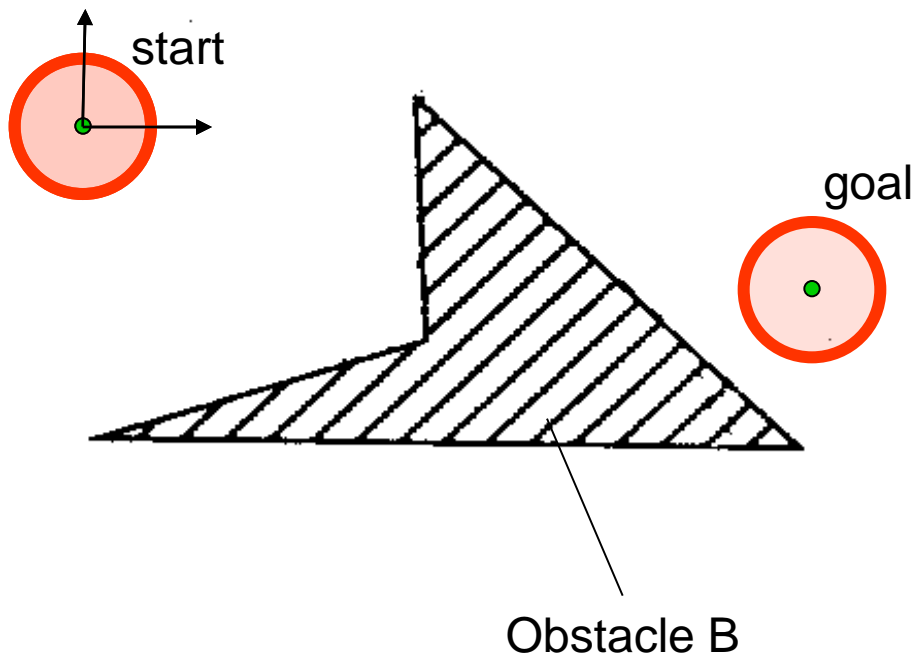


Continuum approaches vs Discretization

Two approaches to Robot motion planning:

- **continuum:**
treat motion space as single continuum
→ optimization
- **discretization:**
decompose motion space into regions / segments
→ graph-search

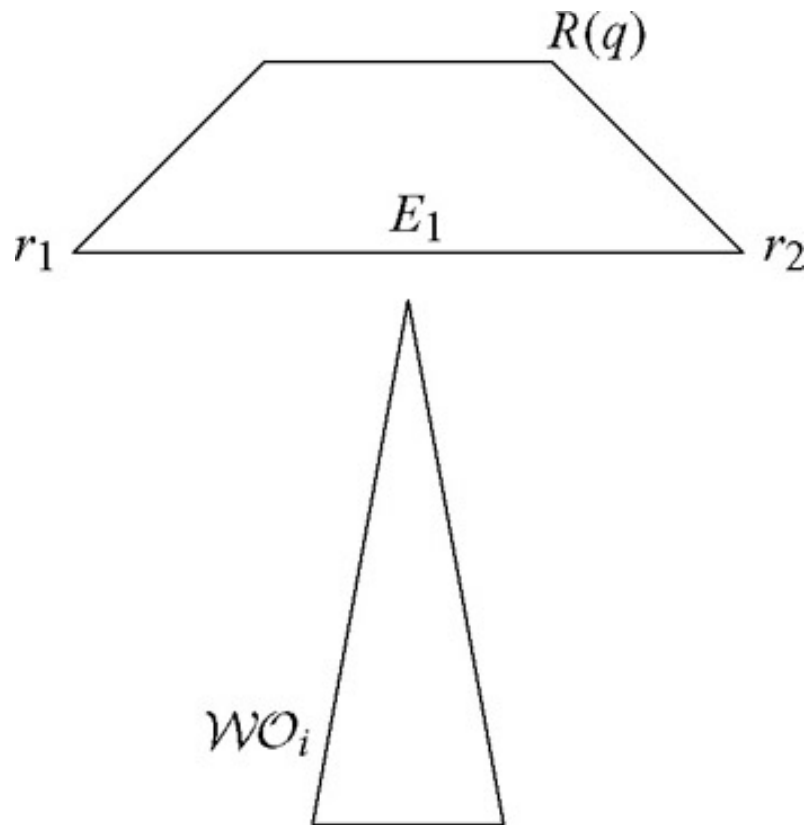
Potential fields



Potential fields

1. Goal: negative (attractive) potential
Obstacles: positive (repulsive) potential
2. Robot moves along gradient
3. Problems:
 - need to integrate the potential over the area of robot
 - problem of local minima

Finite area robots



Instead of integrating over robot area, restrict to a set of *control* points

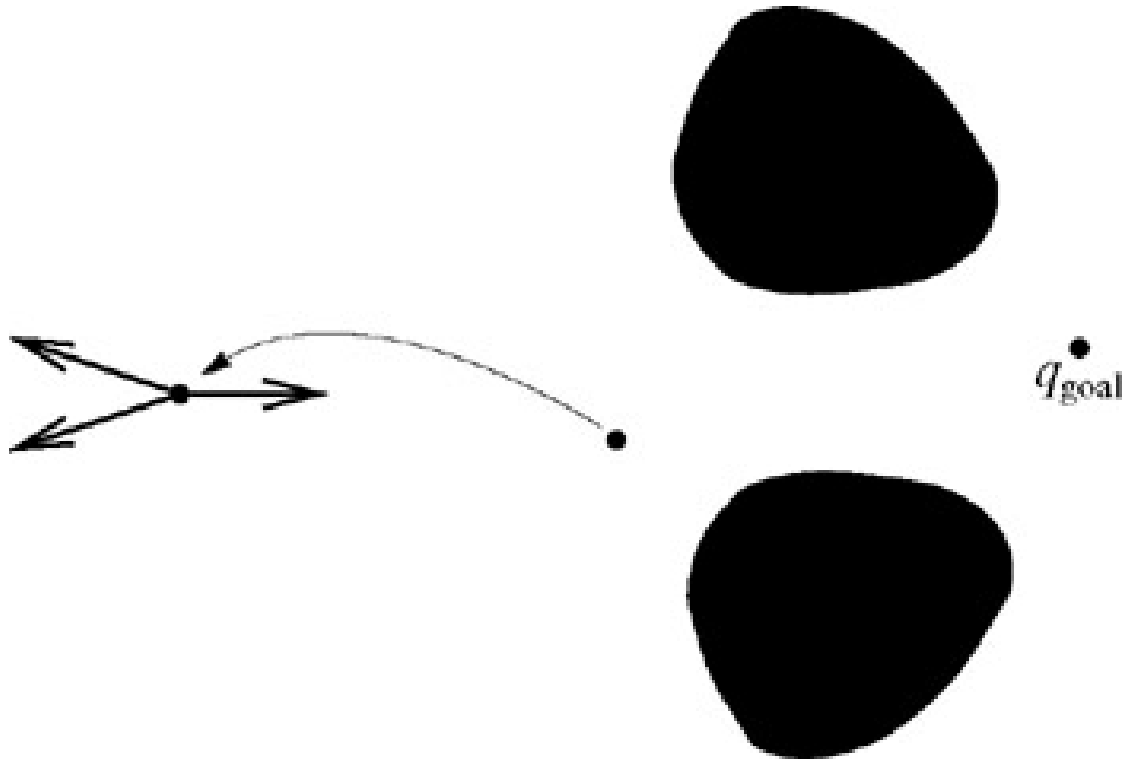
e.g. vertices

Problem:

With control points r_1 and r_2 on robot $R(q)$, edge E_1 may still hit Obstacle.

→ Attempt to reduce computation to points

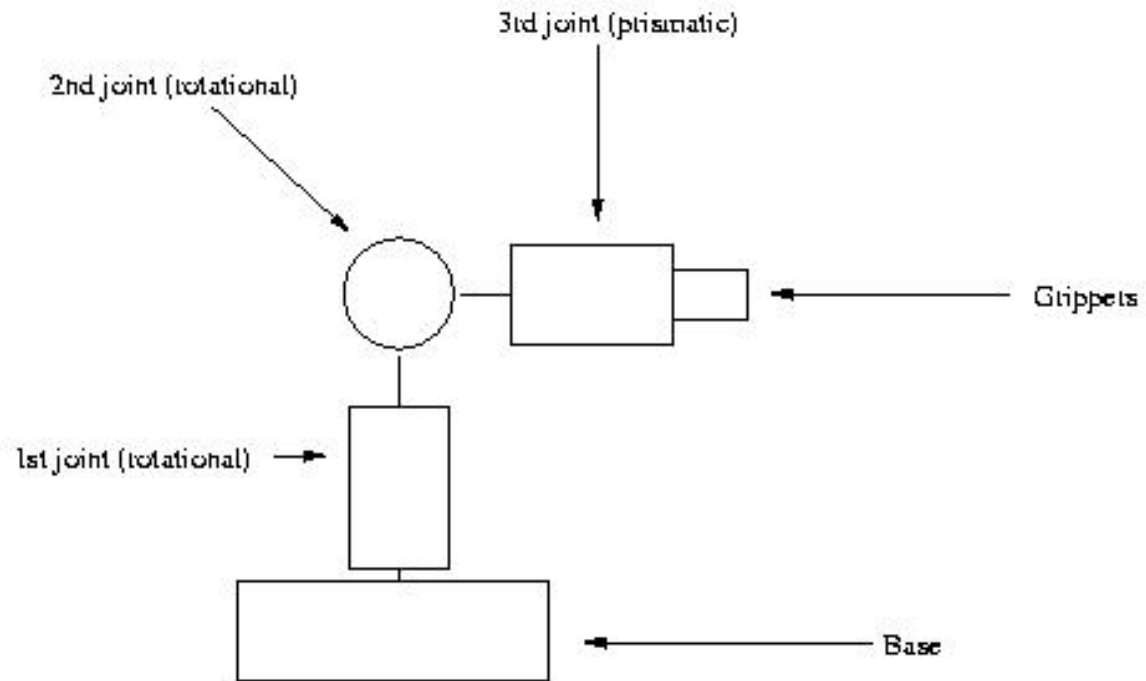
Local Minima



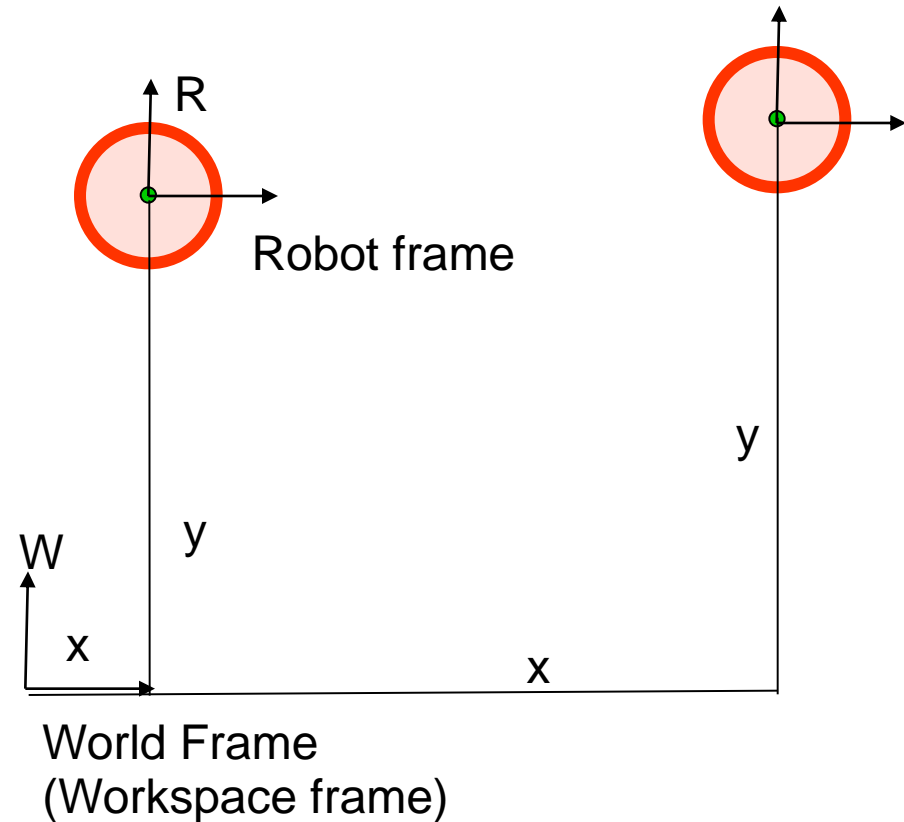
persists even for point robots

Nature of Configuration spaces

Robot Model



Models of Robot Motion



DEFINITION:

NOTE:

degrees of freedom:

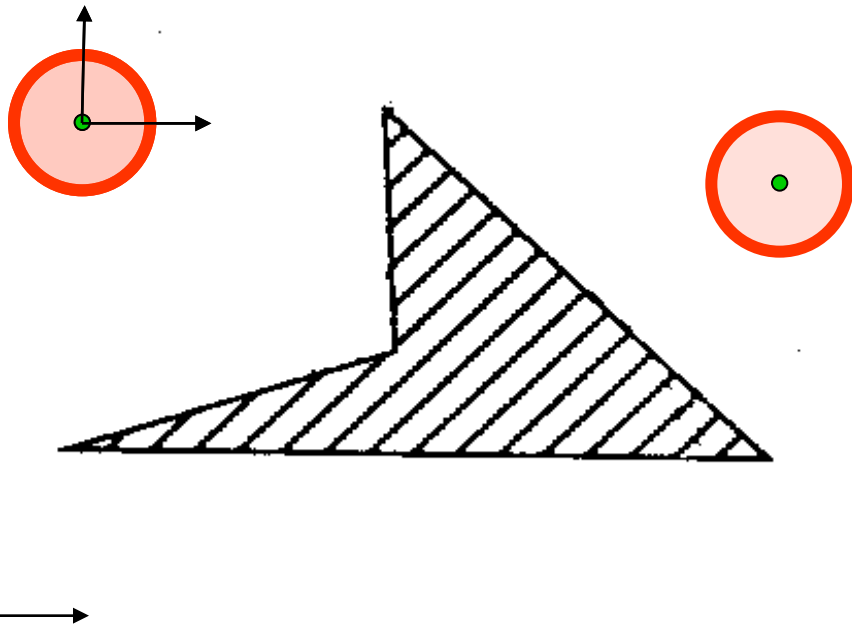
Given robot frame R, every point on the robot is known

number of parameters needed to fix the robot frame R in the world frame W

$(x,y) =$ **configuration**
(vector \mathbf{q})

given configuration \mathbf{q}
for a certain pose of the robot, the set of points on the robot is a function of the configuration: say $R(\mathbf{q})$

Robot Motion Planning

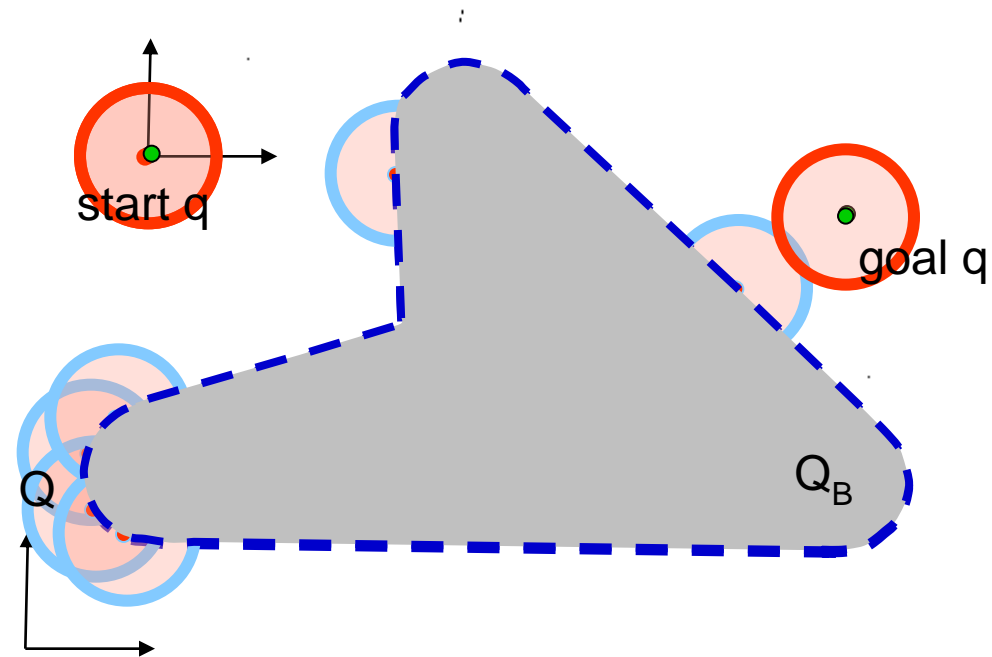


find path P from q_s to q_G s.t. for all $q \in P, R(q) \cap B = \emptyset$

? generate paths and check each point on every path?

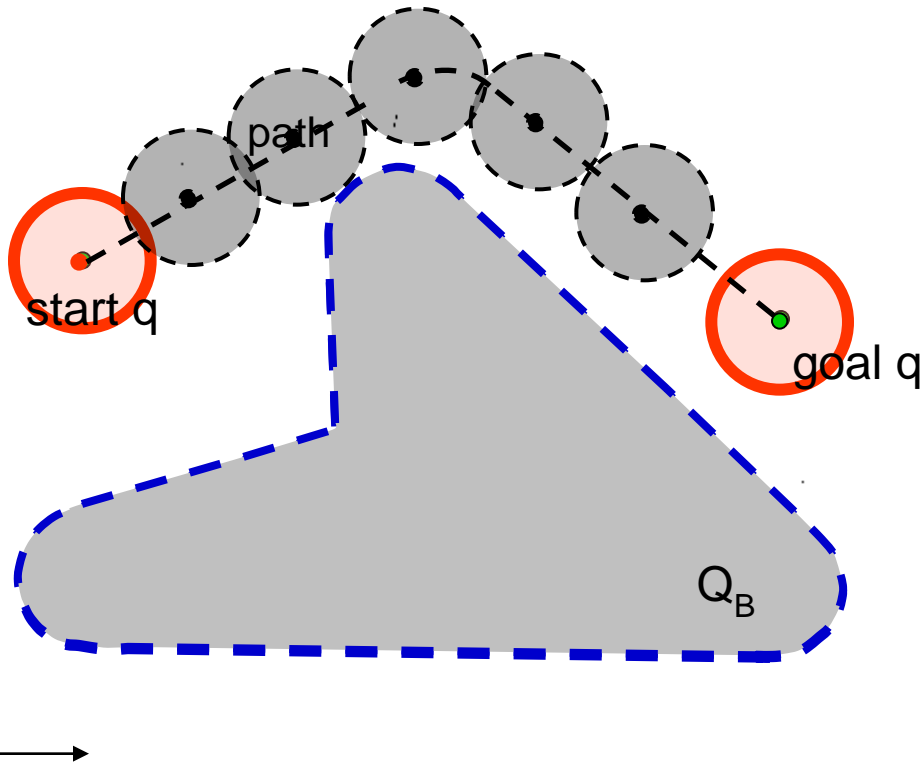
Would it be easier to identify Q_{free} first?

Robot Motion Planning



$$Q_B = [\mathbf{q} \mid R(\mathbf{q}) \cap B \neq \emptyset]$$

Motion Planning in C-space

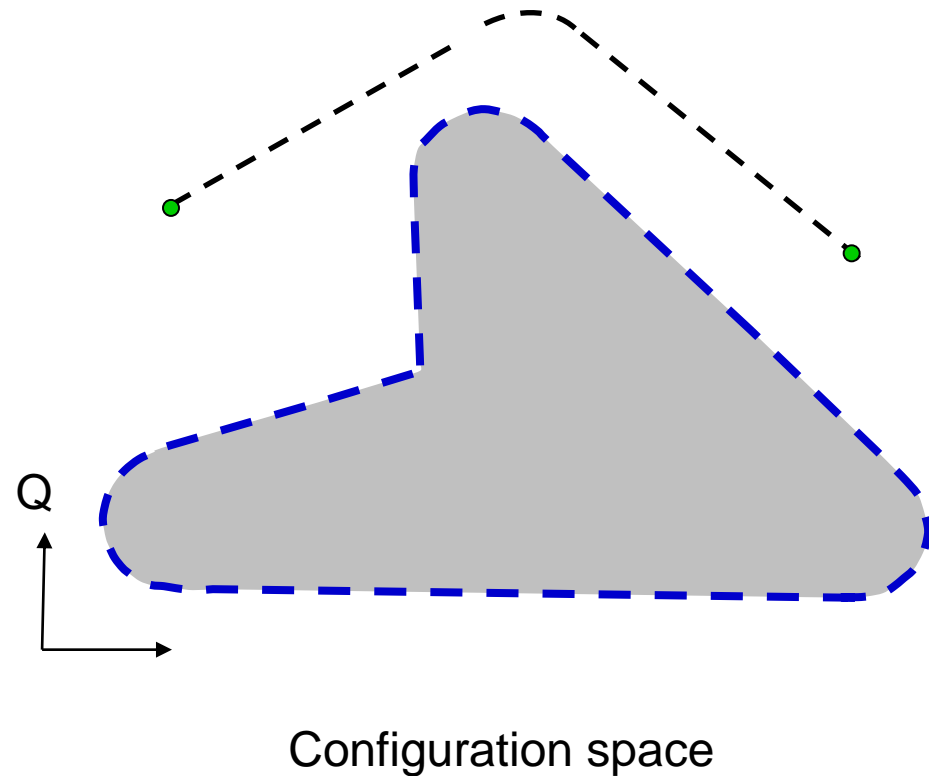
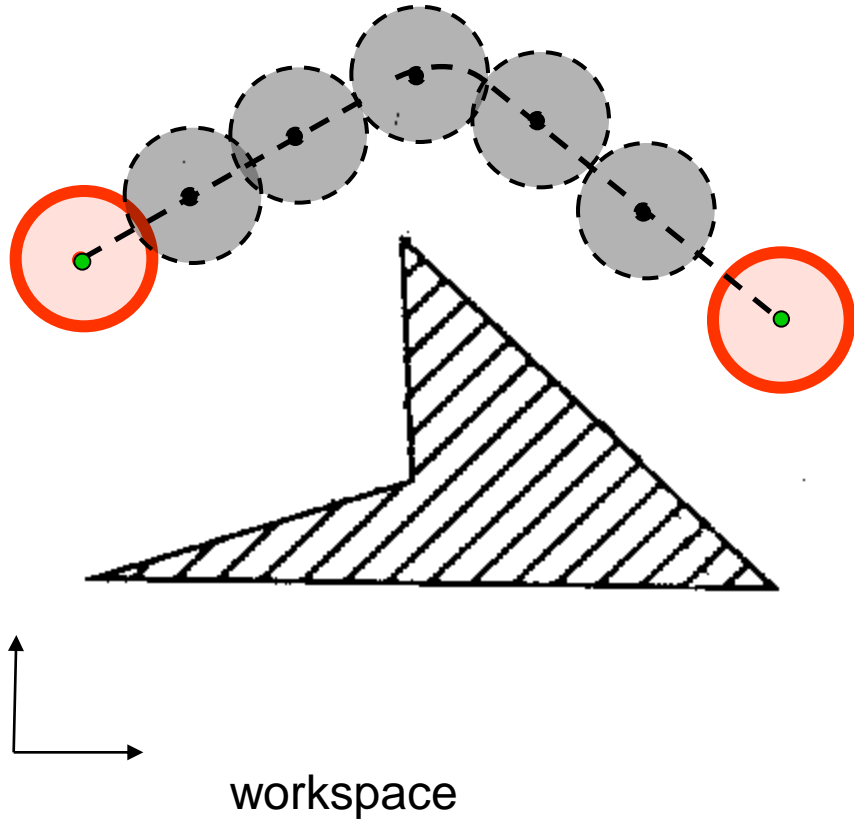


configurations are points in C-space

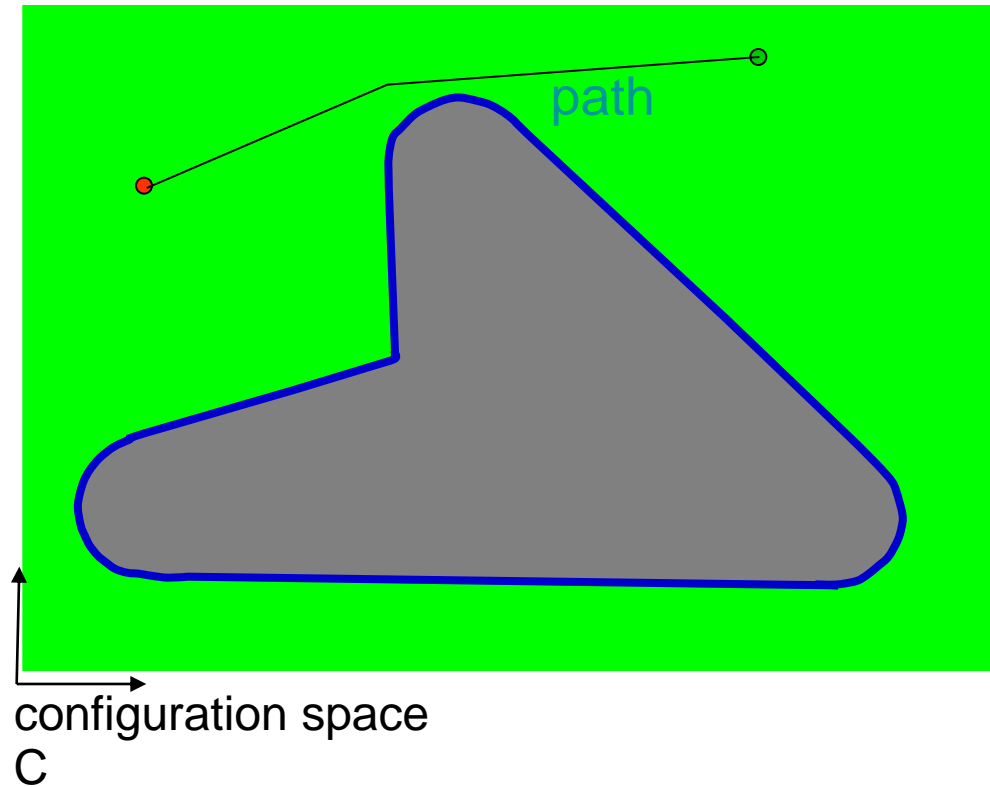
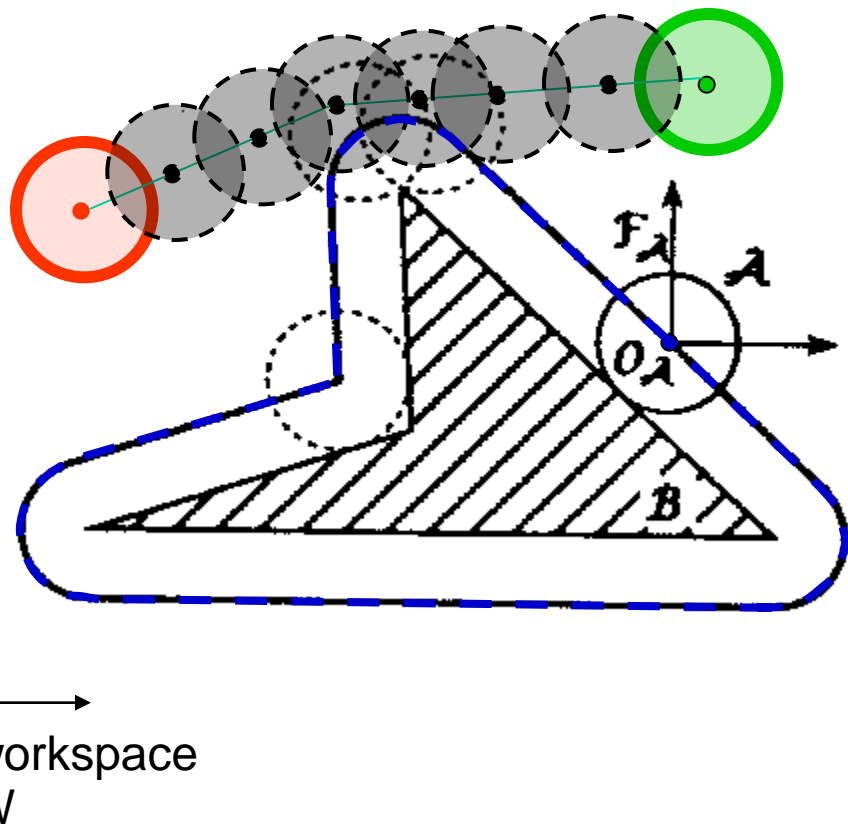
path P is a line

if $P \cap Q_B = \emptyset$, then path is in Q_{free}

Motion Planning in C-space



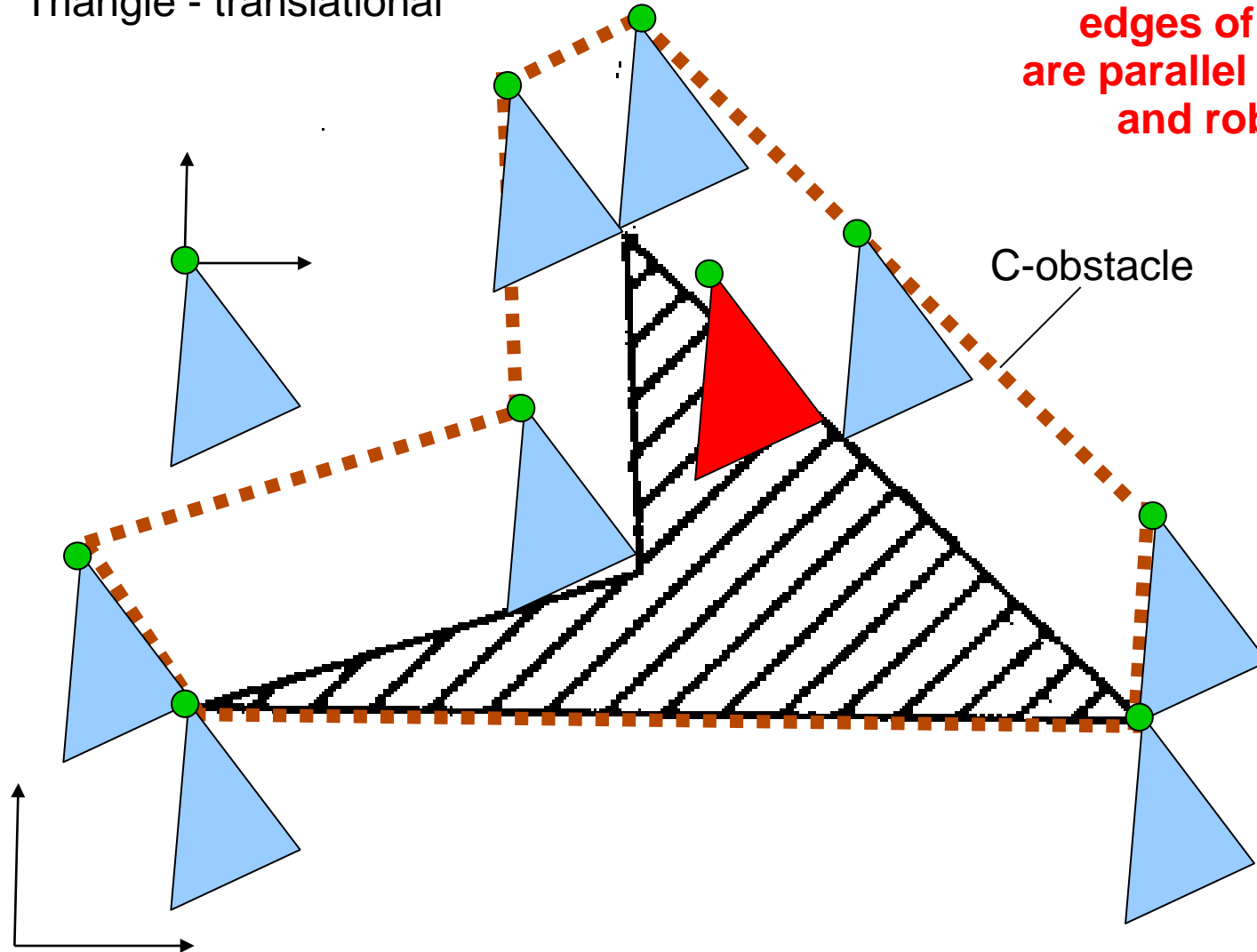
Robot Motion Planning



Non-circular mobile robots

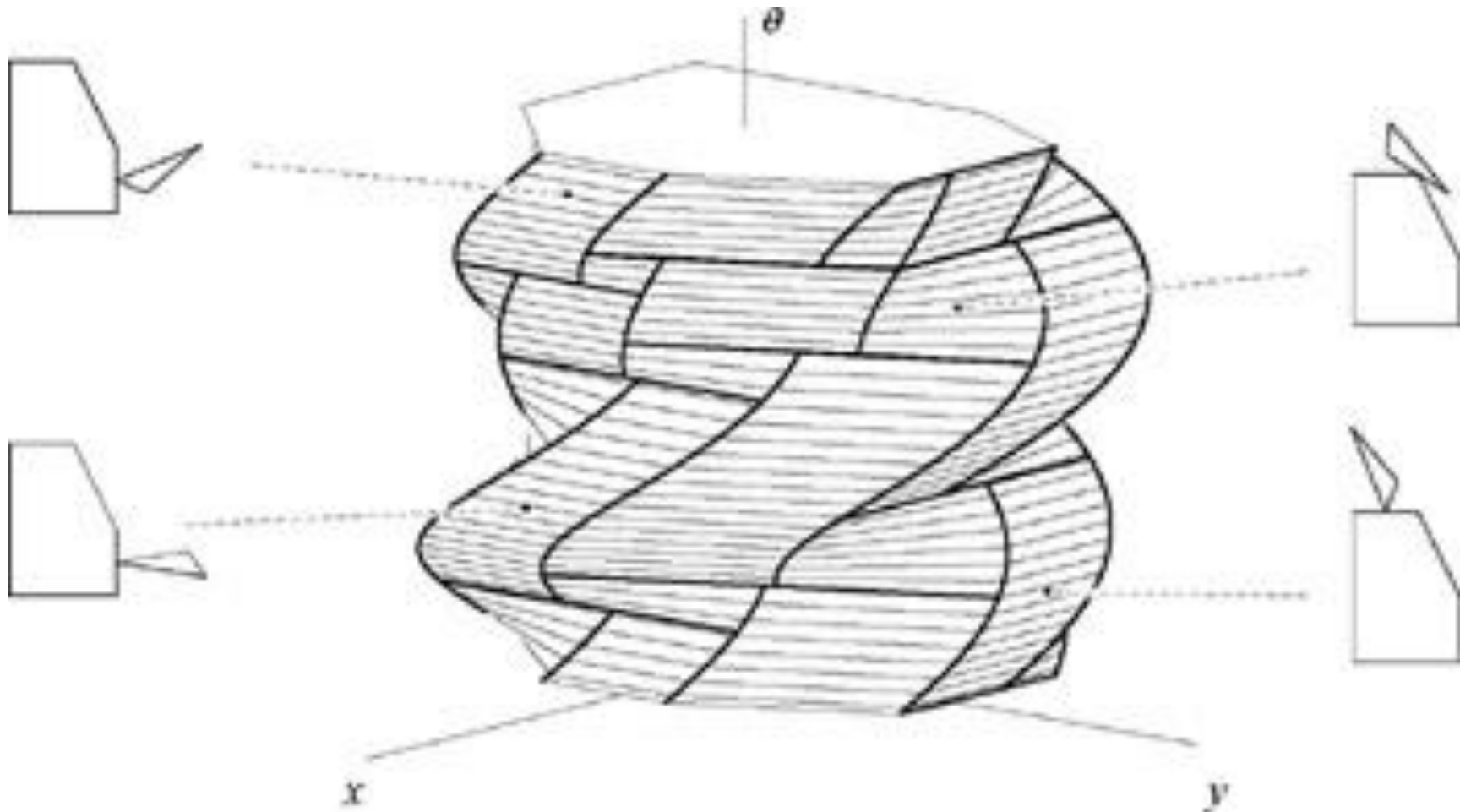
Triangle - translational

**edges of C-obstacle
are parallel to obstacle
and robot edges...**

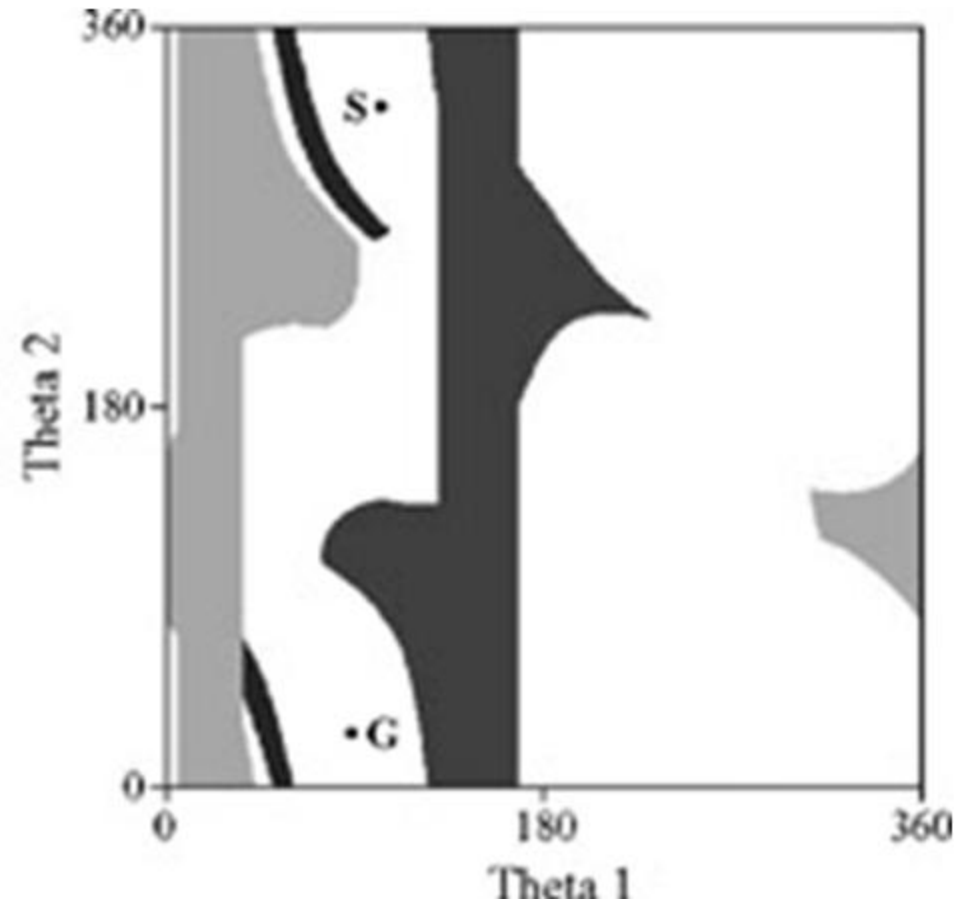
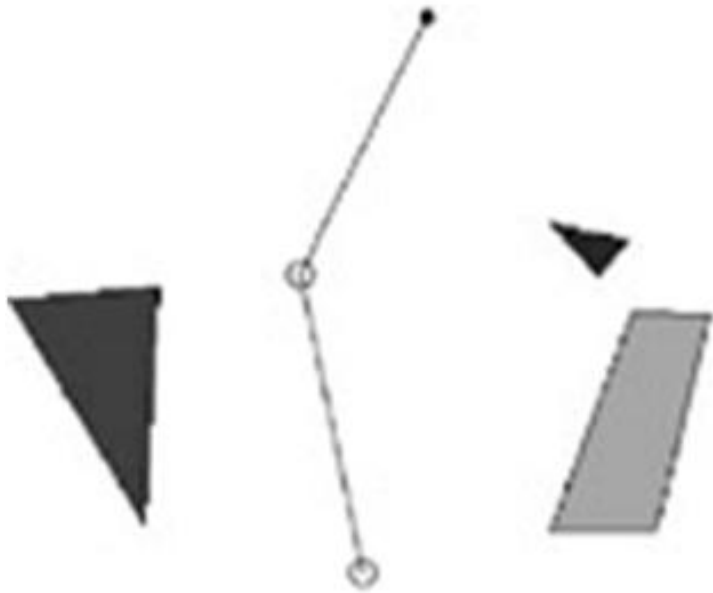


Non-circular mobile robots

C-space with rotation θ (polygonal obstacle)



Configuration Space for Articulated Robots

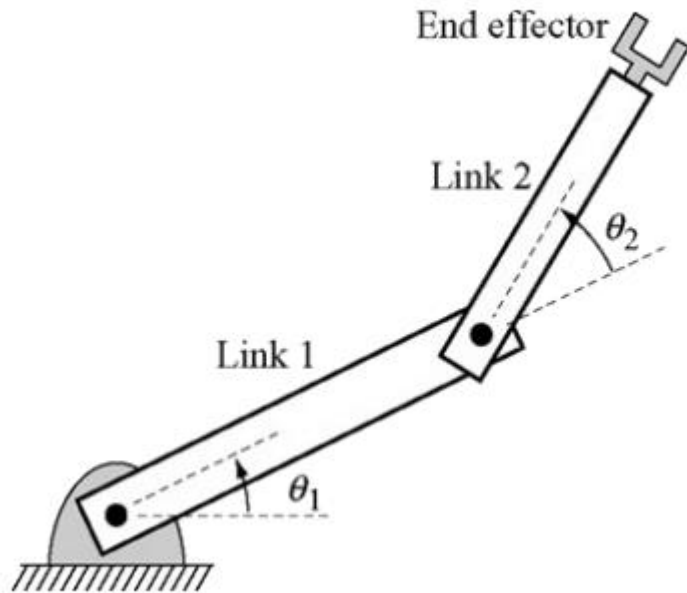


Configuration Space Analysis

Basic steps (for ANY constrained motion system):

1. determine degrees of freedom (DOF)
2. assign a set of configuration parameters \mathbf{q}
e.g. for mobile robots, fix a frame on the robot
3. identify the mapping $R : Q \rightarrow W$, i.e. $R(\mathbf{q})$ is the set of points occupied by the robot in configuration \mathbf{q}
4. For any \mathbf{q} and given obstacle B , can determine if $R(\mathbf{q}) \cap B = \emptyset$. \rightarrow can identify Q_{free}
Main benefit: The search can be done for a point
5. However, computation of C-spaces is not needed in practice; primarily a conceptual tool.

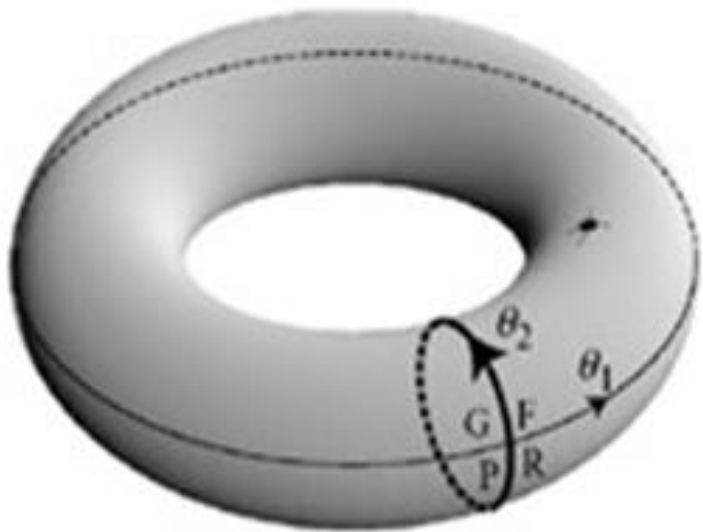
Articulated Robot C-space



How many parameters needed to fix the robot pose ?

What may be one assignment for the configuration parameters?

C-space as manifolds



Topology of C-space: Torus ($S^1 \times S^1$)

Choset, H et al 2007, Principles of robot motion: Theory, algorithms, and implementations, chapter 3

C-space as manifolds

- **manifold:** generalization of curves / surfaces

every point on manifold has a neighbourhood homeomorphic to an open set in \mathbb{R}^n

- Mapping $\Phi : S \rightarrow T$ is bijective (covers all of T and mapping is unique)

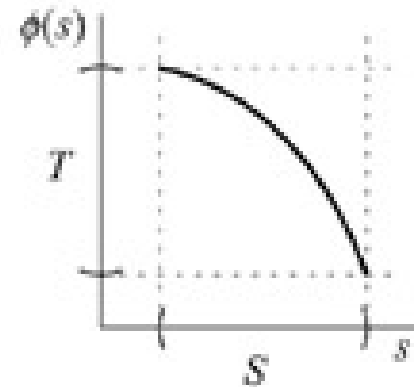
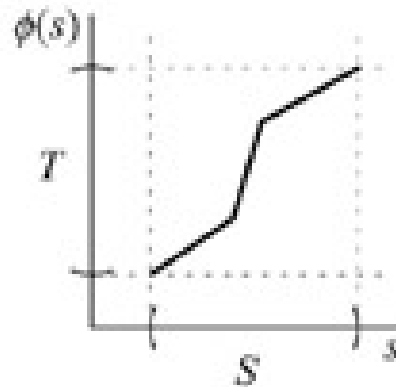
Φ is

homeomorphic:

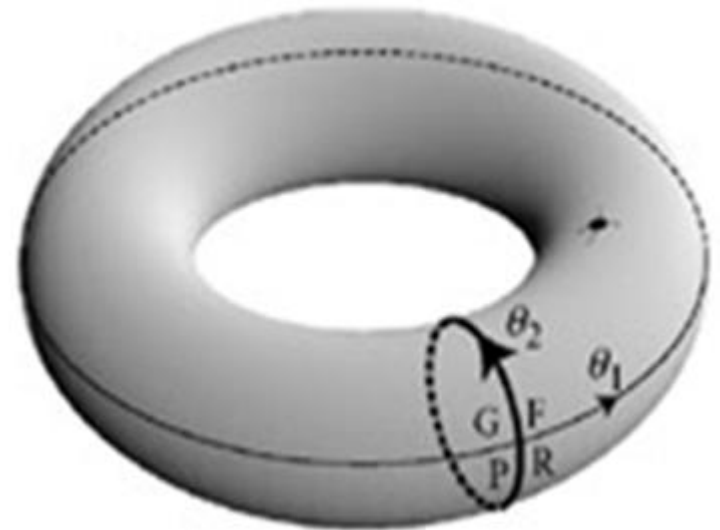
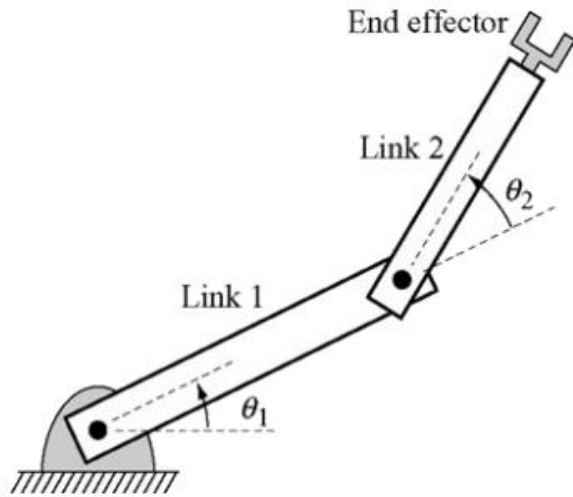
(f / f^{-1} are continuous)

diffeomorphic :

(f / f^{-1} are C^∞ smooth)



C-space as manifolds



Neighbourhood of q is mappable to \mathbb{R}^2

global topology is not \mathbb{R}^2 but $S^1 \times S^1$ (torus)

Map from C-space to W

Given configuration \mathbf{q} , determine volume occupied by $R(\mathbf{q})$ in workspace

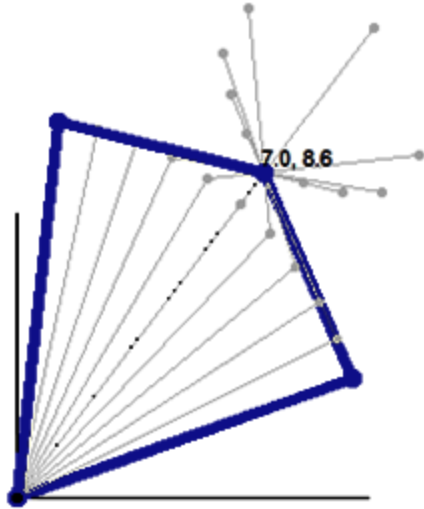
For multi-link manipulators, spatial pose of link $(n+1)$ depends on joint configuration \mathbf{q} for joints $1, 2, \dots, n$.

→ **Forward Kinematics**

Map from W to C-space: given pose in workspace, find \mathbf{q}

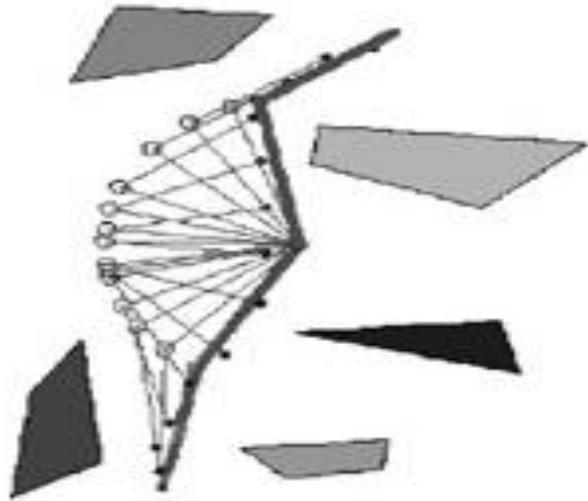
→ **Inverse Kinematics**

Mapping obstacles



Point obstacle in
workspace

Articulated Robot C-space



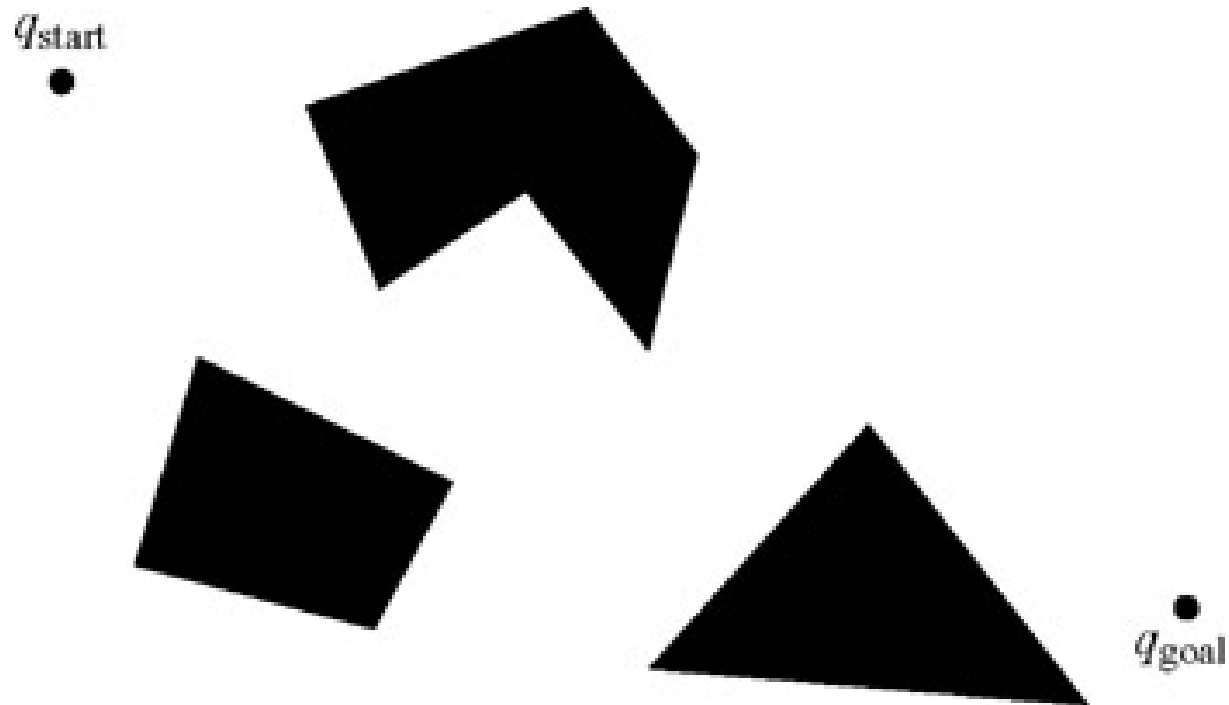
Path in workspace



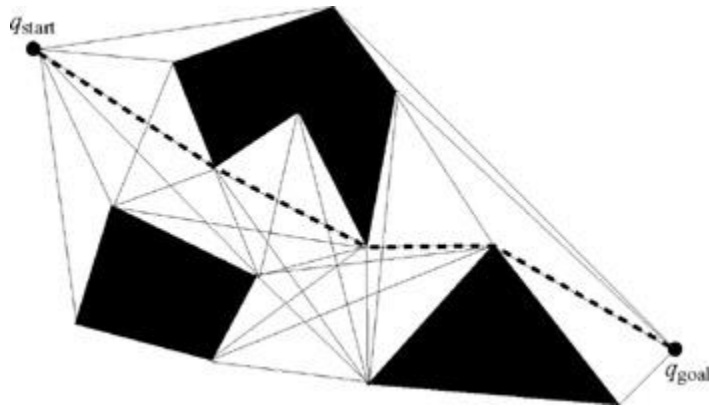
Path in Configuration Space

Graph-based approaches

Visibility Graph methods



Visibility Graph methods



Construct edges between visible vertices

Sufficient to use only **supporting** and **separating** tangents

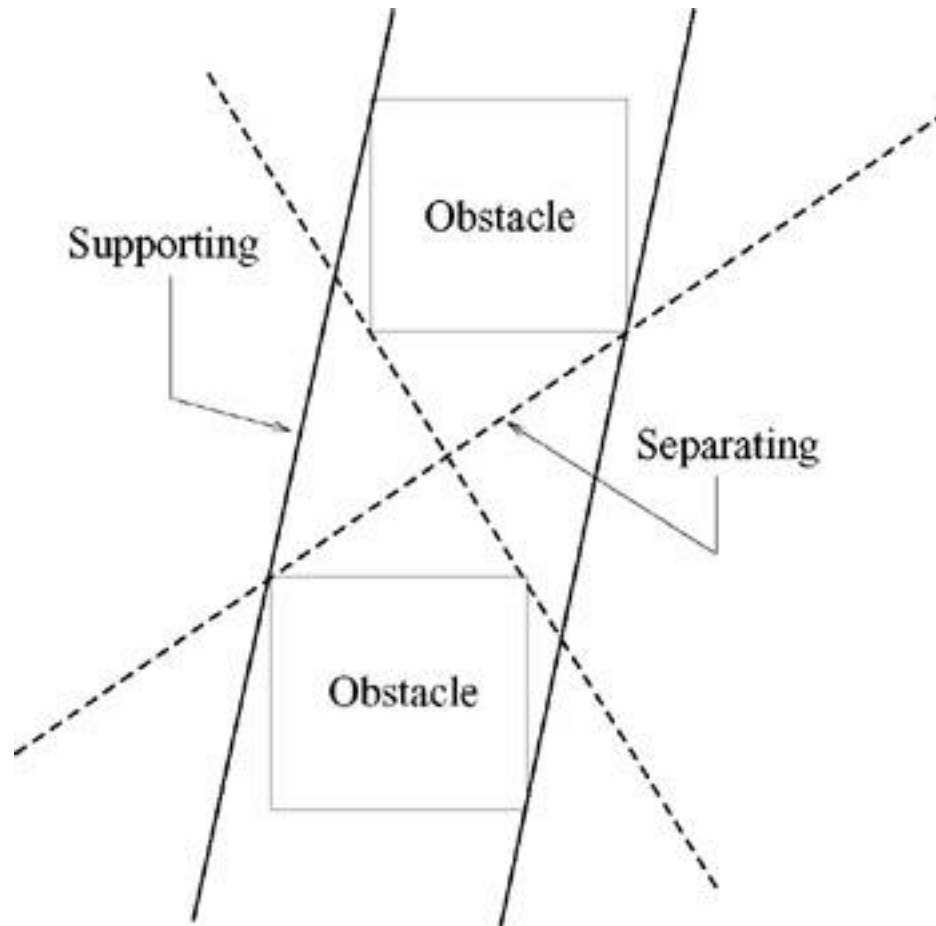
Complexity:

Direct visibility test: $O(n^3)$

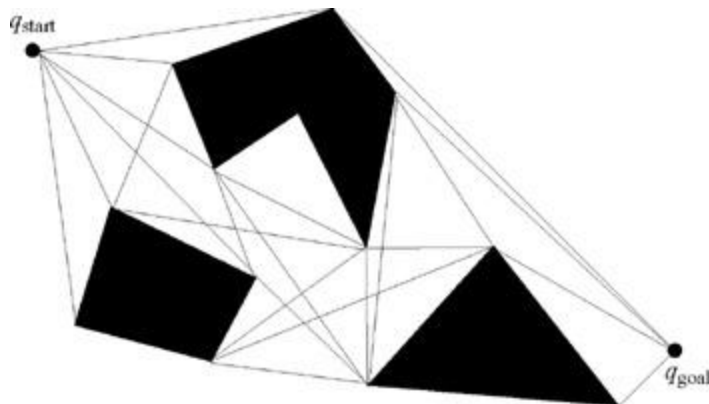
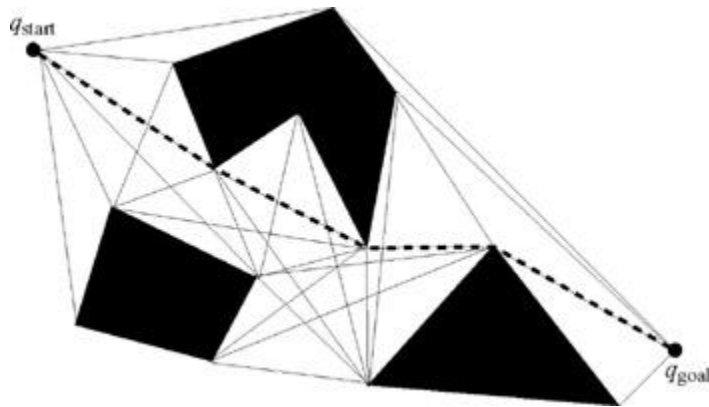
(tests for each vtx: $O(n)$ emanations
x $O(n)$ obst edges)

Plane sweep algorithm: $O(n^2 \log n)$

Visibility Graph methods



Visibility Graph methods



Sufficient to use only **supporting** and **separating** tangents

Finds “shortest” path – but too close to obstacles

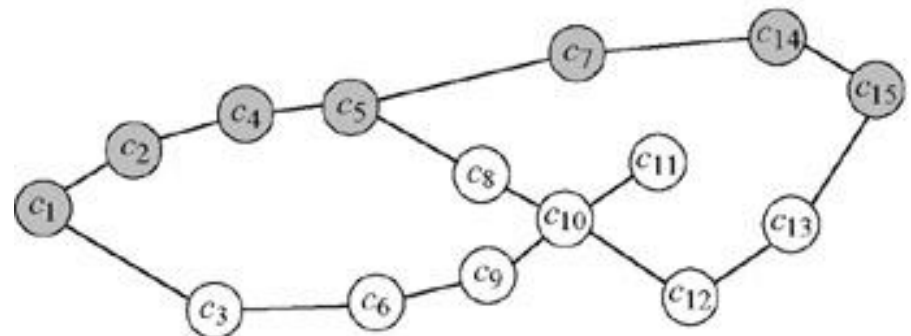
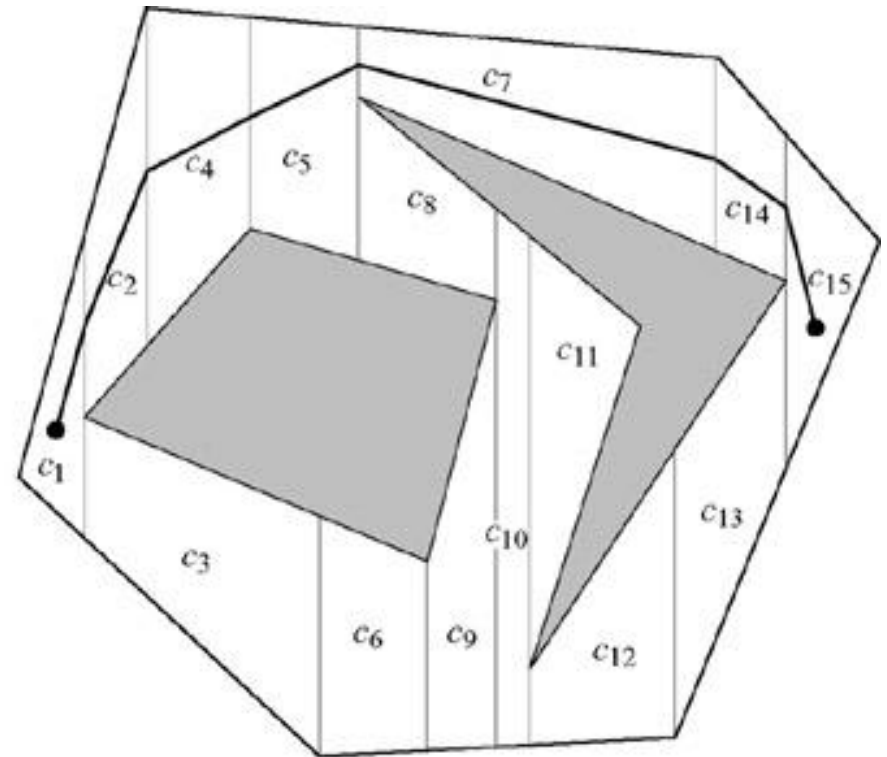
Cell decomposition methods

Trapezoidal decomposition:
Each cell is convex.

Sweep line construction:
 $O(n \log n)$

Graphsearch: $O(n \log n)$

Path: avoids obstacle
boundary but has high
curvature bends



Roadmap methods

Roadmaps

any roadmap RM must have three properties:

Connectivity:

path exists between any q'_{START} and q'_{GOAL} in RM

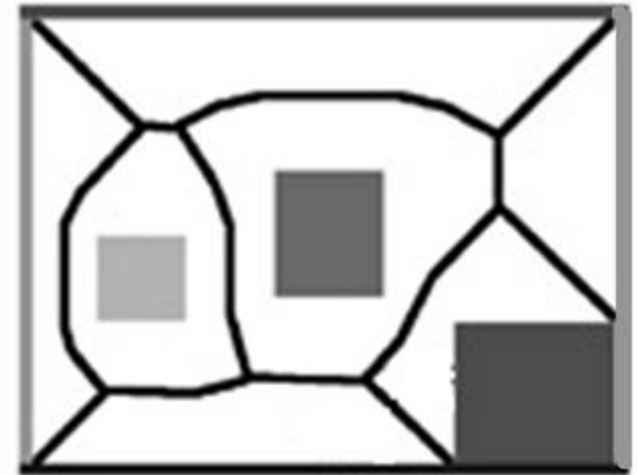
Accessibility:

exists a path from any $q_{START} \in Q_{free}$ to some $q'_{START} \in RM$

Departability:

exists a path from some $q'_{GOAL} \in RM$ to any $q_{GOAL} \in Q_{free}$

Staying away from Obstacles: Generalized Voronoi Graphs



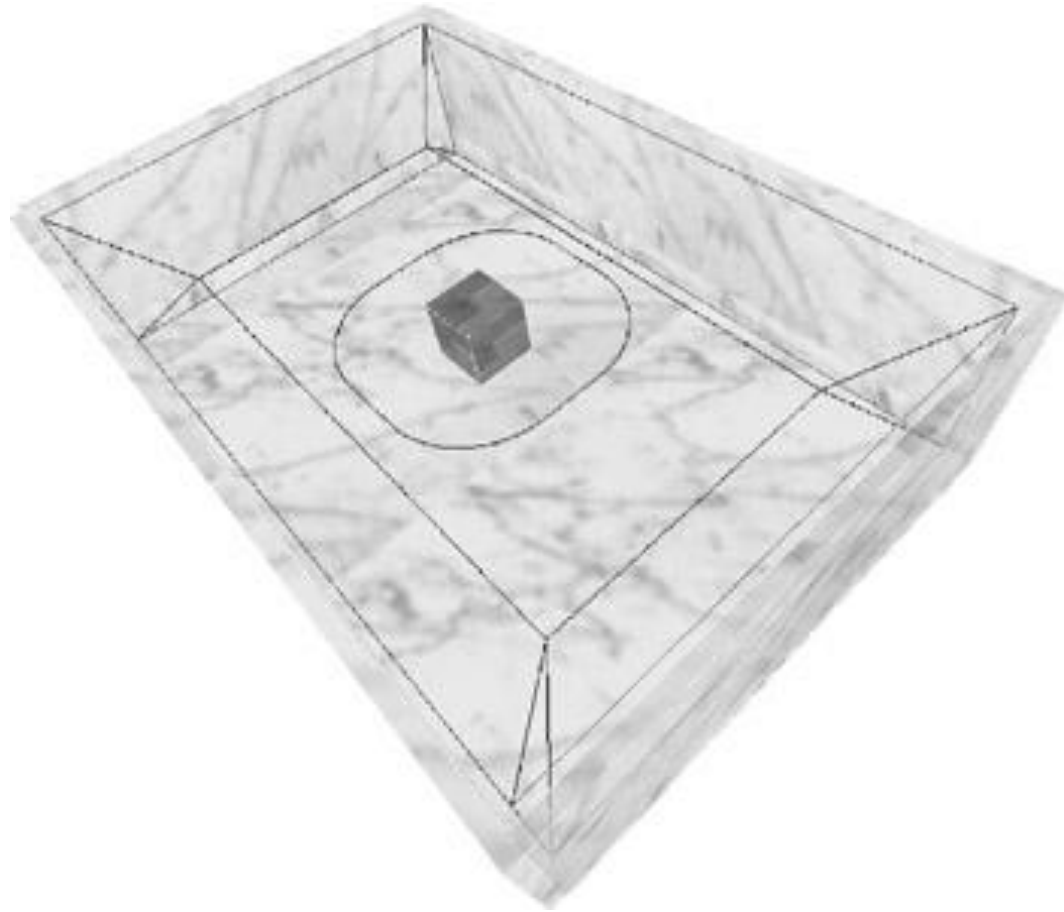
Voronoi Region of obstacle i :

$$\mathcal{F}_i = \{q \in \mathcal{Q}_{\text{free}} \mid d_i(q) \leq d_h(q) \quad \forall h \neq i\},$$

Voronoi diagram:

set of q equidistant from at least two obstacles

Generalized Voronoi Graphs



GVG Roadmaps

Accessibility / Deeparability:

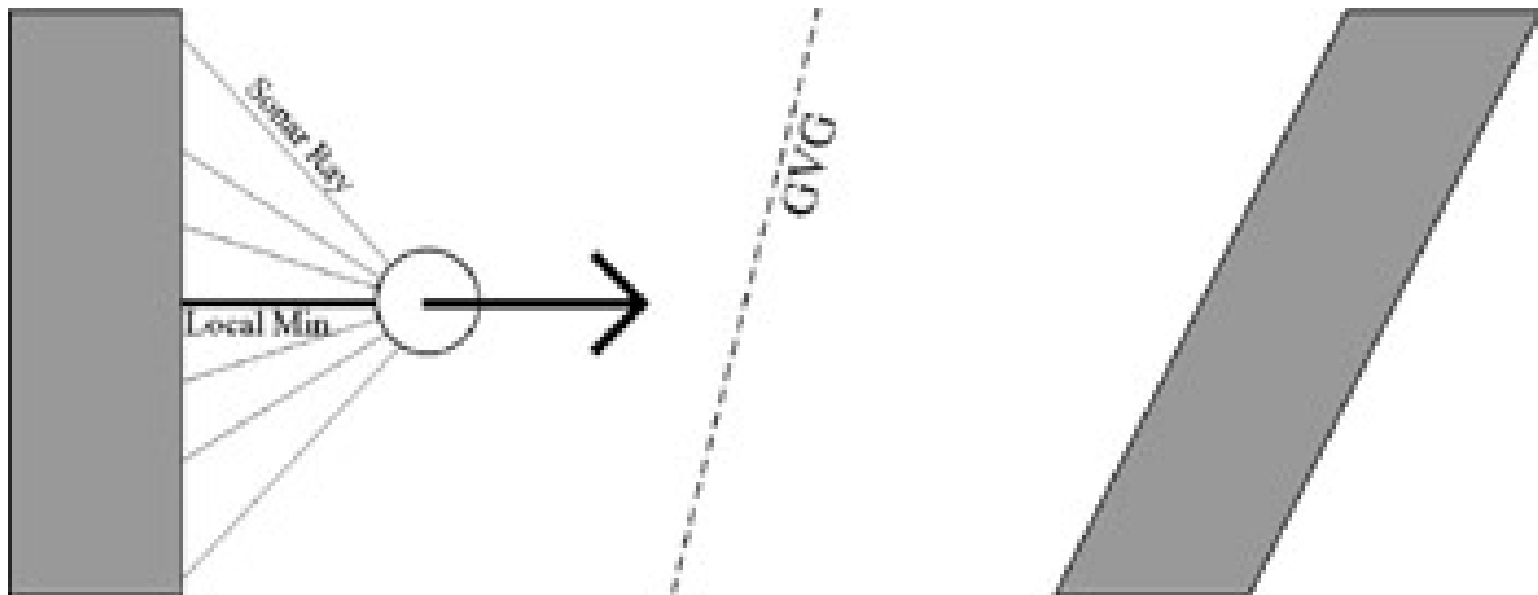
Gradient descent on distance from dominant obstacle :

- guaranteed to reach from any $q_{START} \in Q_{free}$ to some $q'_{START} \in RM$
- motion is along a “retract” or brushfire trajectory

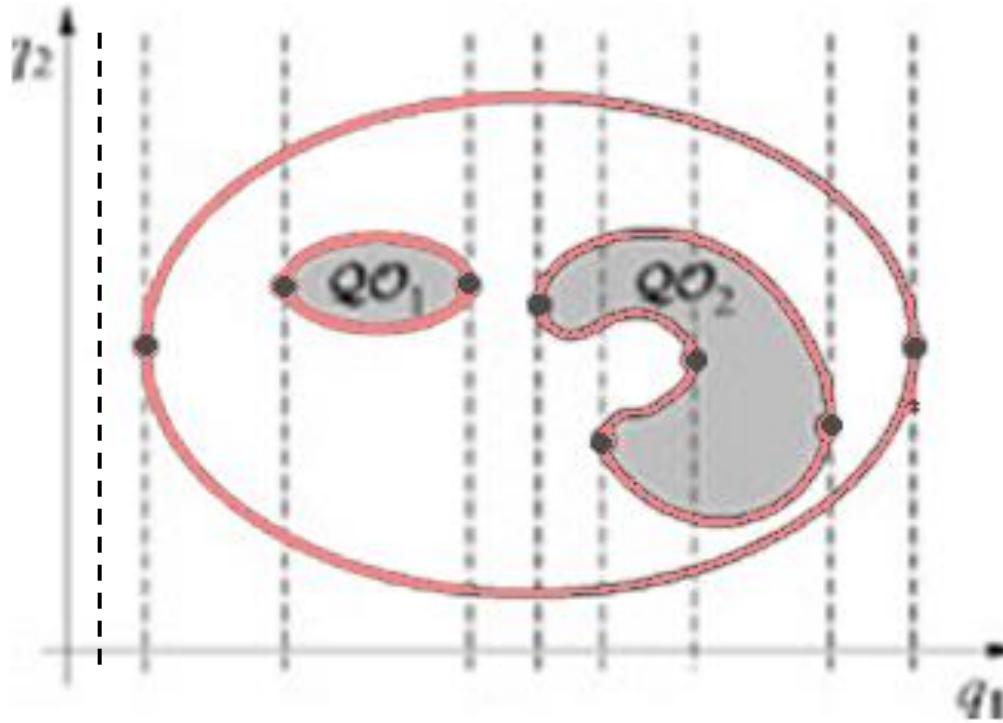
Connectivity:

GVG is Connected if path exists

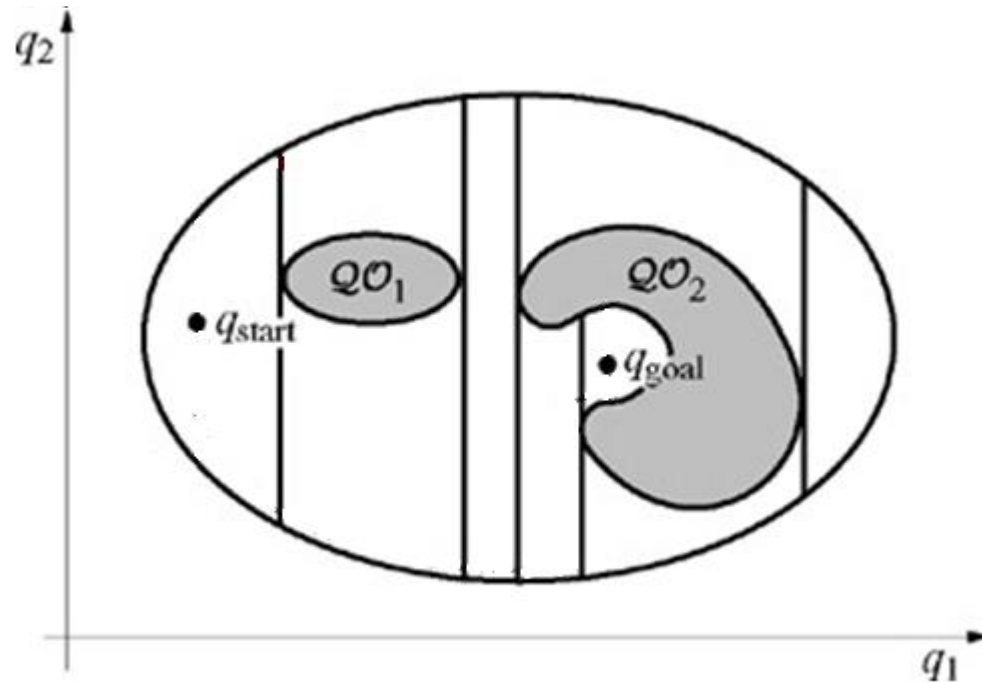
Sensor based Voronoi roadmap construction



Canny's Silhouette roadmap



Canny's Silhouette roadmap



Canny's Complexity Analysis

n : = degrees of freedom of robot (dim of C-space)

obstacles C-space boundaries represented as p polynomials of maximum degree w

Complexity:

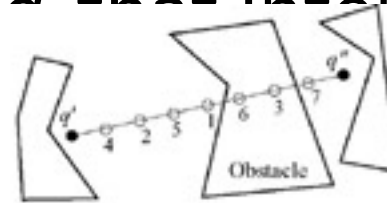
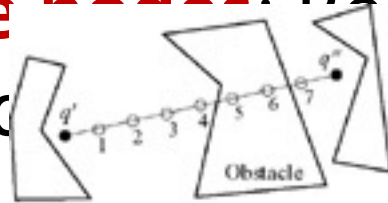
any navigation path-planning problem can be solved in $p^n(\log p)w^{O(n^4)}$ time

Probabilistic Roadmap (PRM)

Probabilistic Roadmap

Sample n poses $q_1 \dots q_n$ in the WORKSPACE

Free space nodes: Nodes that intersect with an obstacle in Q_{free}



Local planner: (a) Incremental: The algorithm returns failure after five collision checks.

(b) Subdivision: The algorithm returns failure after three collision checks.

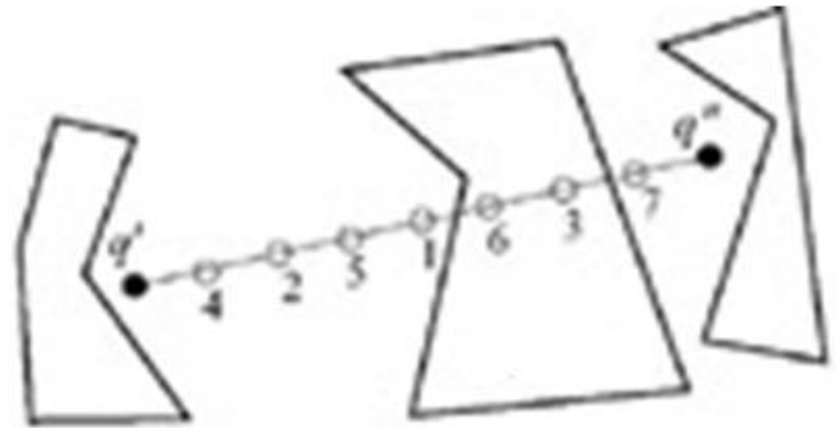
path $\langle q_i, q_j \rangle$ collision-free, add edge to graph

Resulting graph = *Probabilistic Roadmap*

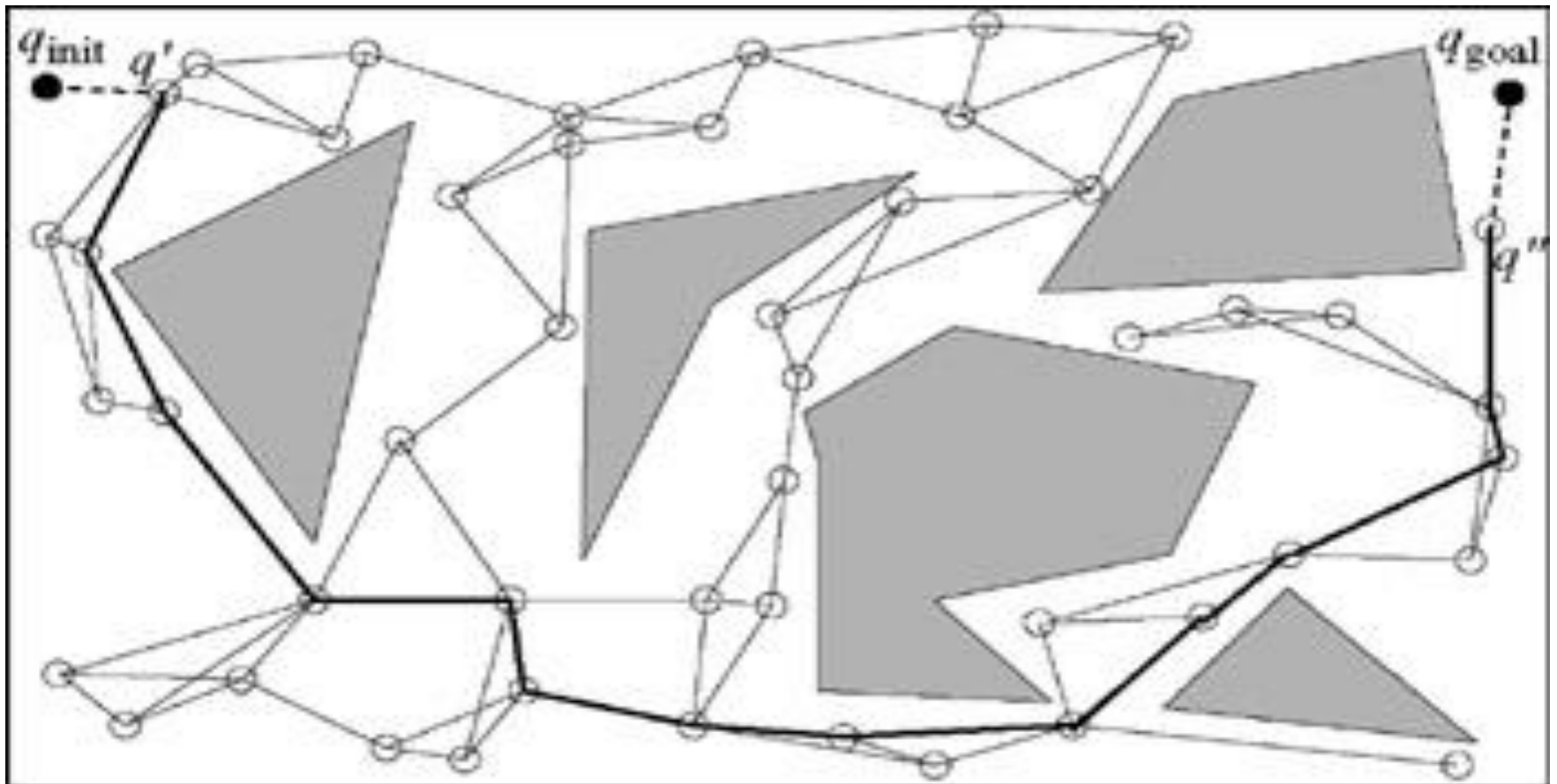
Local Planner

Objective: Test if path
 $\langle q_i, q_j \rangle$ is collision-
free

Linear Subdivision
algorithm: start at
midpoint(q_i, q_j) ;
subdivide
recursively until
desired precision



Probabilistic Roadmaps (PRM)



Sampling-based motion planning

Sample n poses $q_1 \dots q_n$ in the workspace

Reject q_n that overlap with an obstacle,
remaining poses are in Q_{free}

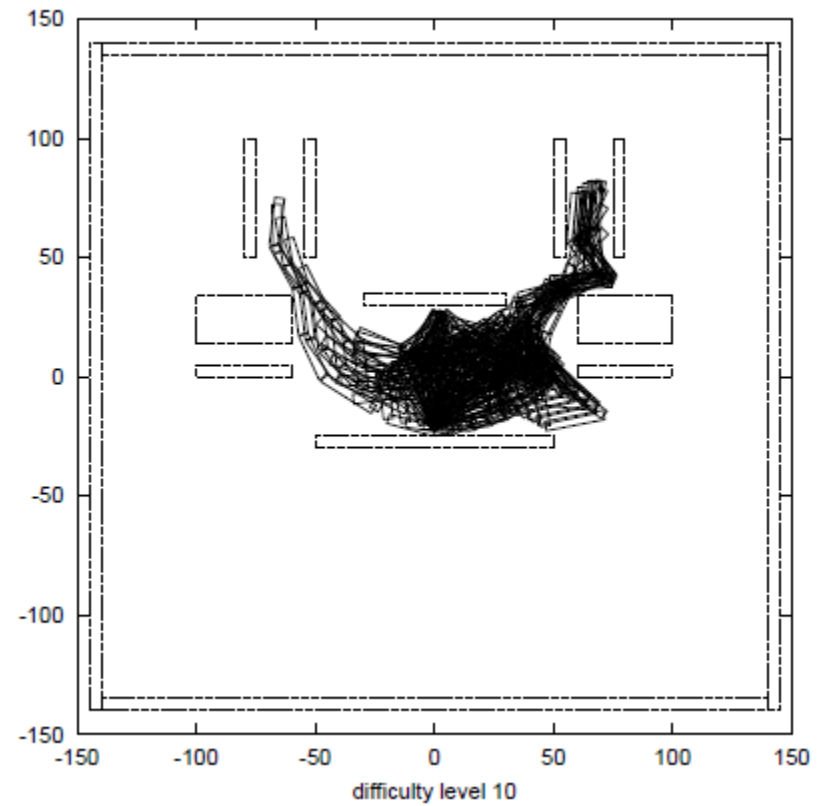
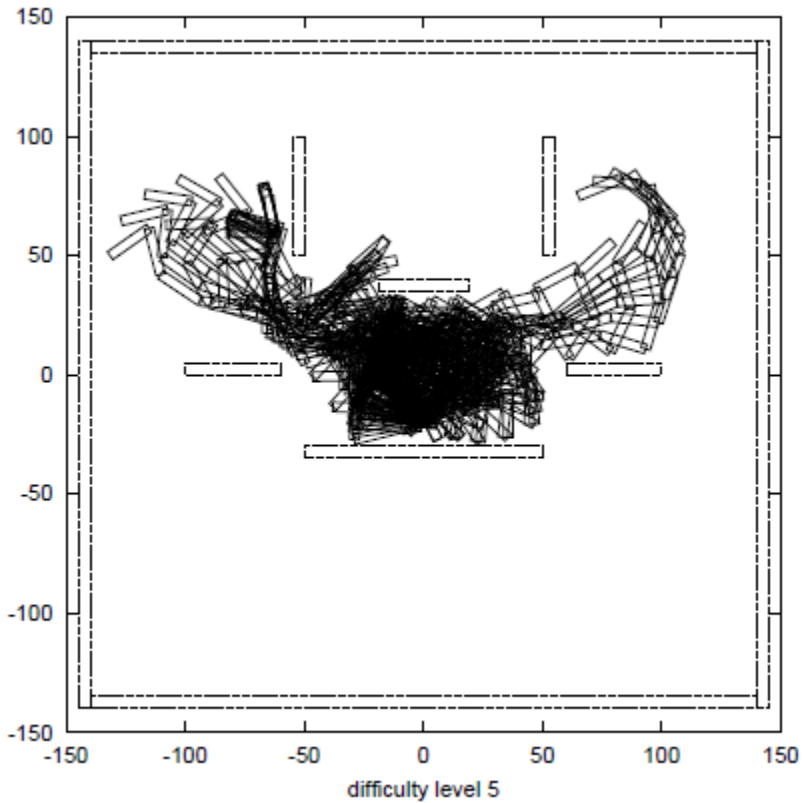
Use local planning to determine if a path exists between neighbours q_i and q_j .

Resulting graph = *Probabilistic Roadmap*

Probabilistically complete:

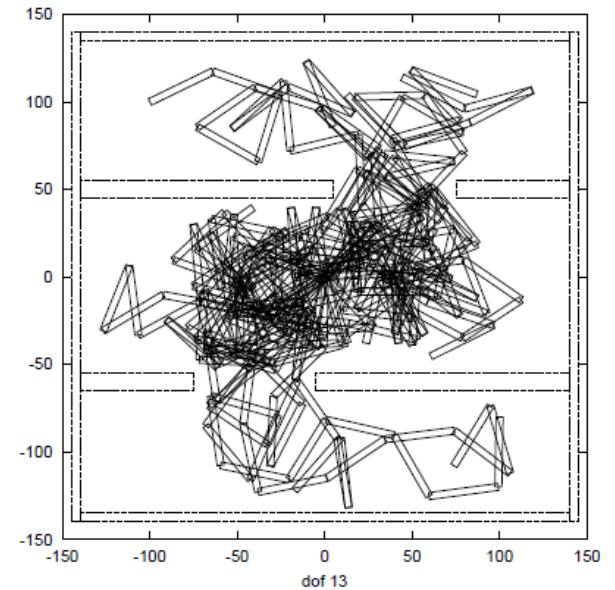
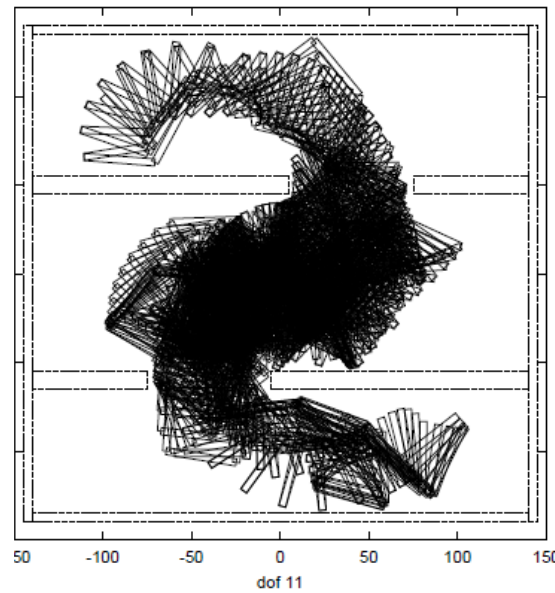
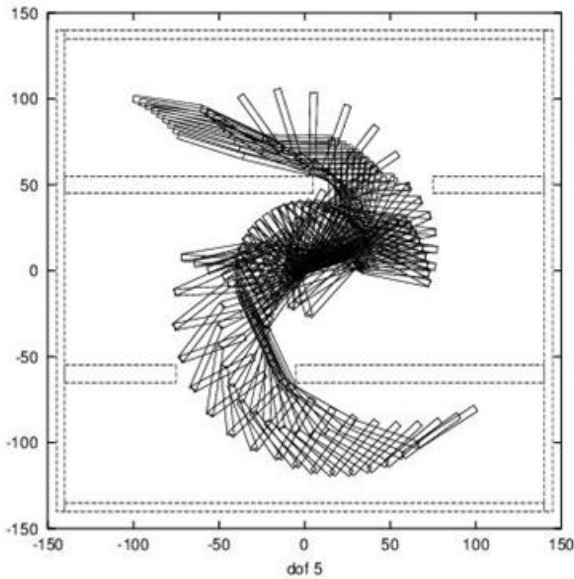
As #samples $n \rightarrow \infty$, Prob (success) $\rightarrow 1$

Hyper-redundant robot motion planning



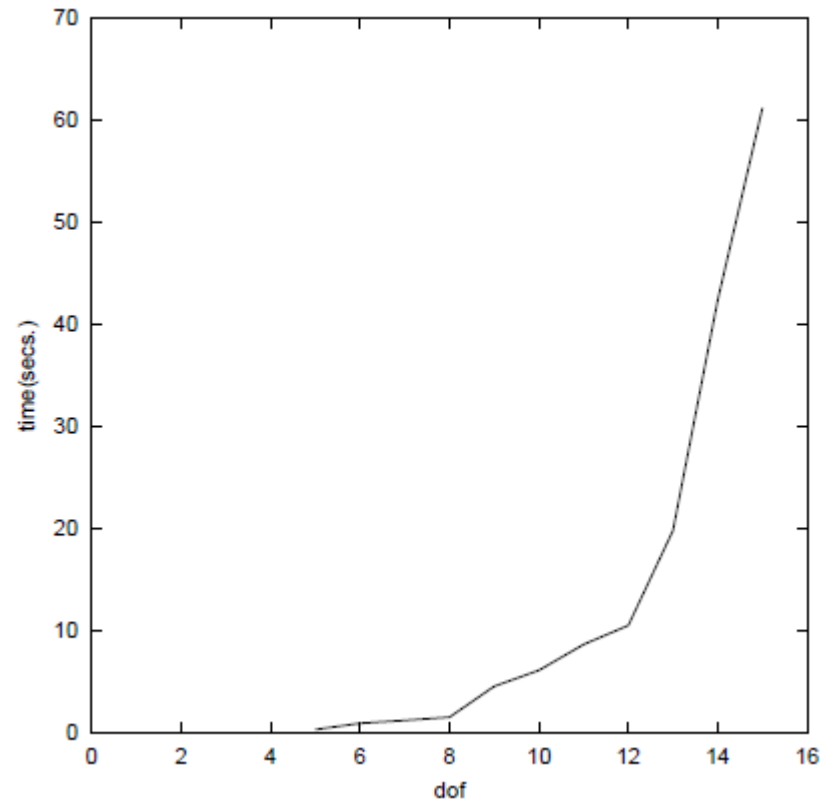
[sinha mukerjee dasgupta 02]

Hyper-redundant robot motion planning



[sinha mukerjee dasgupta 02]

Hyper-redundant motion planning



Time:
Exponential in DOFs

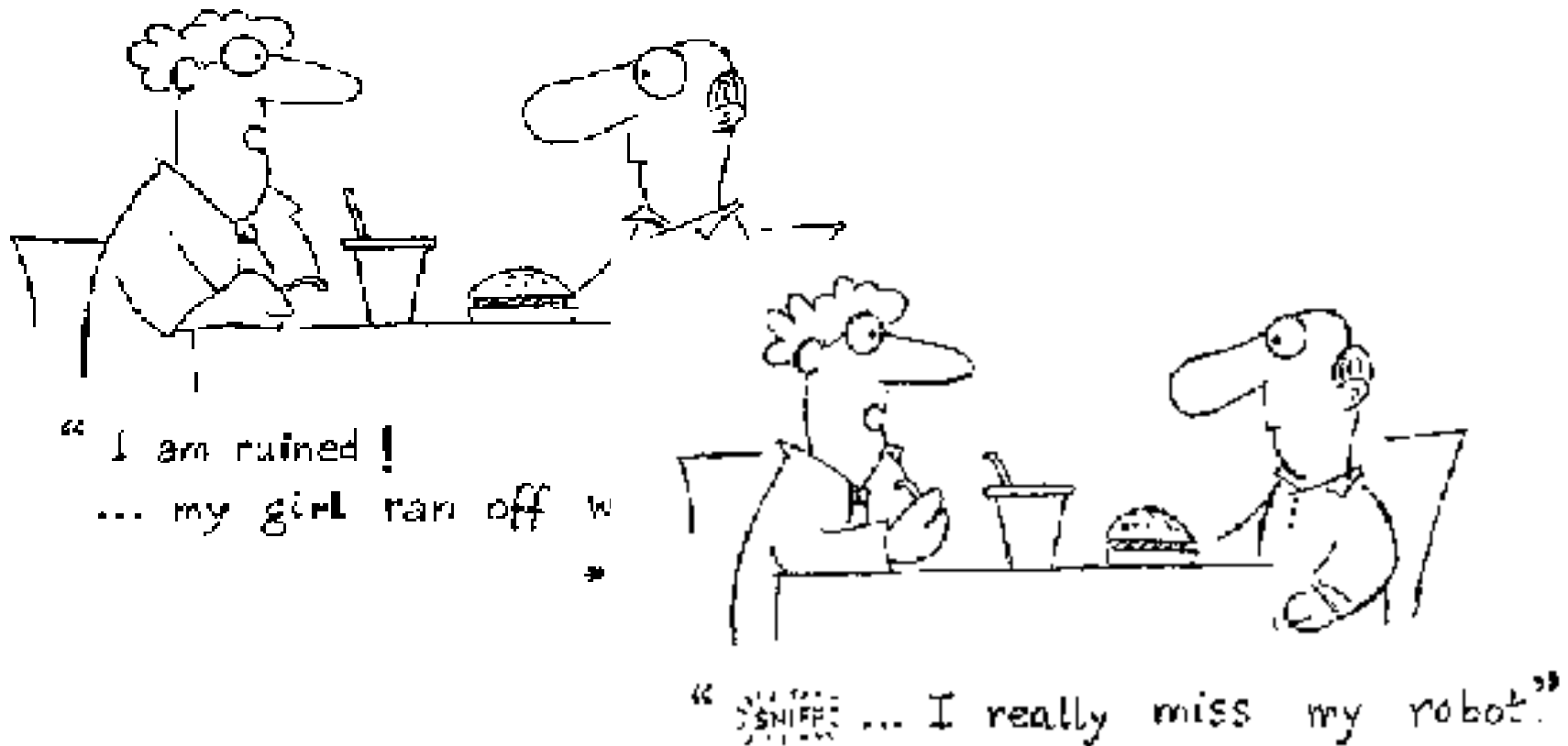
[sinha mukerjee dasgupta 02]

Conclusion

Beyond Geometry

- Geometry is not everything!!
- Real robots have limitations on acceleration owing to torque / inertia → **Dynamics**
- **Learning** to plan motions?
 - Babies learn to move arms
 - Learn low-dimensional representations of motion
- **Grasping** / Assembly : Motions along obstacle boundary

Humans and Robots



Madhur
24 Jan 02