Introduction to Computational Geometry

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Outline

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- This branch of study is around thirty years old if one assumes Michael Ian Shamos's thesis [1] as the starting point.

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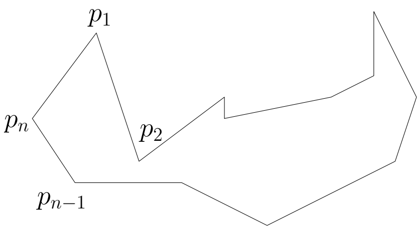
- Programming in CG is also a little difficult. Libraries like LEDA [5] and CGAL [6] are now available.
- CG algorithms suffer from the curse of degeneracies. So, we would make certain assumptions at times like no three points are collinear, no four points are cocircular, etc.

Outline

Area Computation

Problem

Given a simple polygon P of n points, compute its area.



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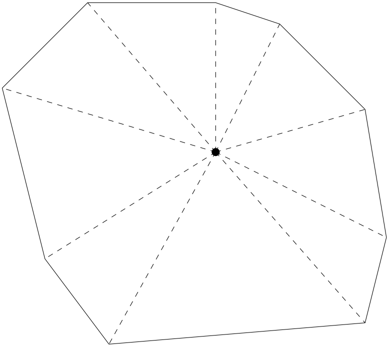
Area of a convex polygon

Find a point inside P, draw n triangles and compute the area.



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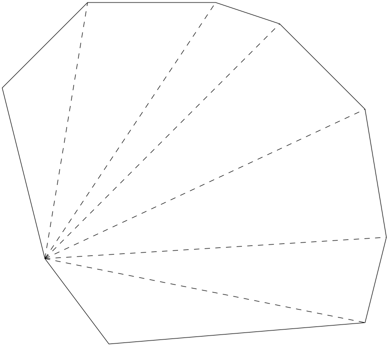
A better idea for convex polygon

We can triangulate P by non-crossing diagonals into $n - 2$ triangles and then find the area.



$(n-3)$ diagonals and $(n-2)$ triangles

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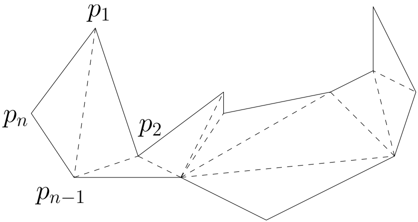
A better idea for simple polygon

We can do likewise.



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Area Computation and Polygon Triangulation

Moral of the story

A simple polygon can be triangulated into $(n-2)$ triangles by $(n-3)$ non-crossing diagonals.

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Time complexity

We can triangulate P by a very complicated $O(n)$ algorithm [7] OR by a *more or less simple* $O(n \log n)$ time algorithm [4].

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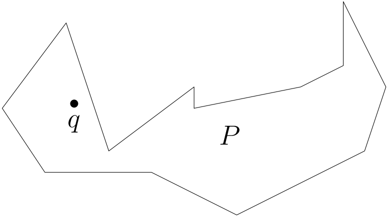

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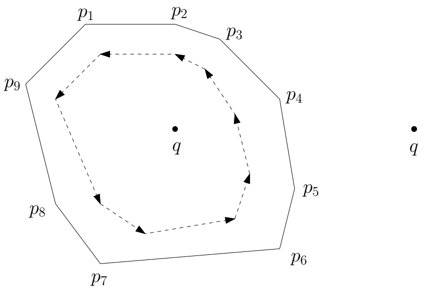

q is always to the left if $q \in P$, else, it varies.

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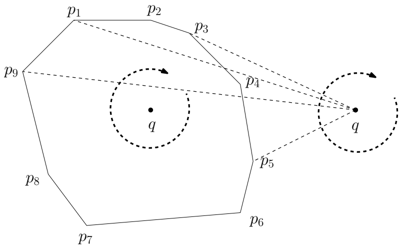
Another idea for convex polygon

- Stand at q and look around the polygon.
- We can show the same result for a simple polygon also.

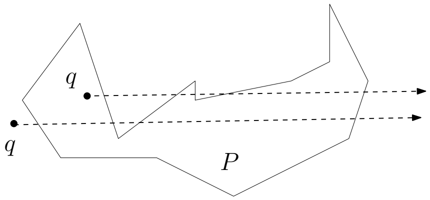


Total angular turn around$q$ is 2π if $q \in P$, else, 0

Point Inclusion

Another technique: Ray Shooting

Shoot a ray and count the number of crossings with edges of P. If it is odd, then $q \in P$. If it is even, then $q \notin P$. Some degenerate cases need to be handled. Time taken is $O(n)$.



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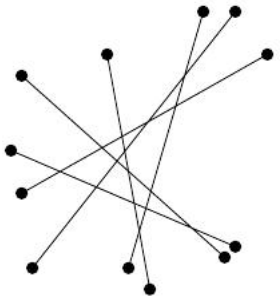
Line Segment Intersection

Input

A set of line segments $\mathcal{L}$ in general position in the plane. $|\mathcal{L}| = n$.

Output

Report the intersections.



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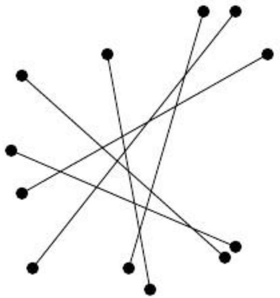

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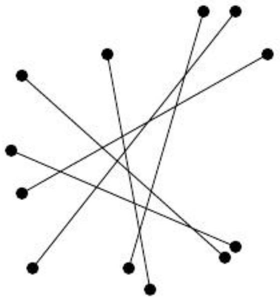
Output Sensitive Algorithm

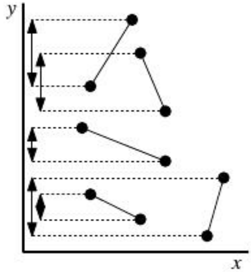
Number of intersections might vary from 0 to $\binom{n}{2} = O(n^2)$. So, the lower bound of the problem is $\Omega(n^2)$. The idea is now to look for an output sensitive algorithm.



An Output Sensitive Algorithm

The idea

- Avoid testing pairs of segments that are far apart.



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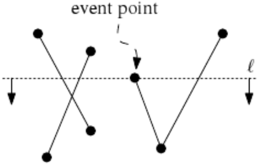

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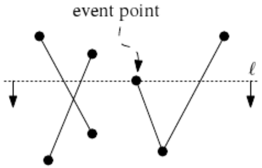

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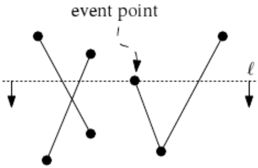

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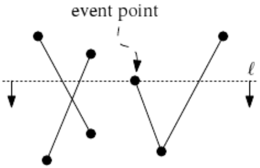

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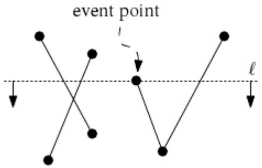
- Only at particular points known as event points, the status needs to be updated.



Event Points and Sweep Line Status

Event Points and the Event Queue

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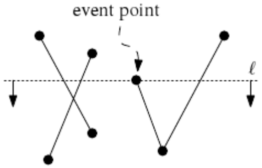

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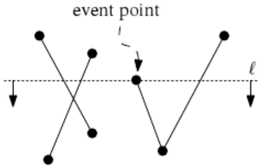

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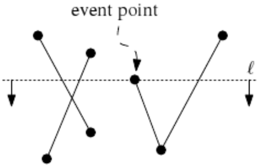

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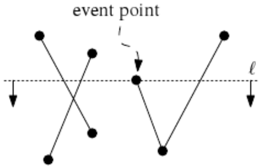

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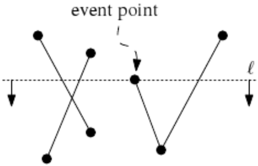

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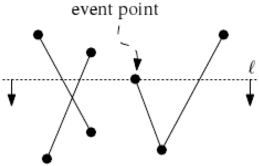

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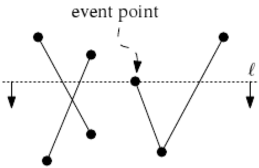

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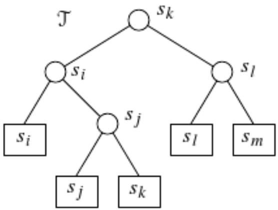

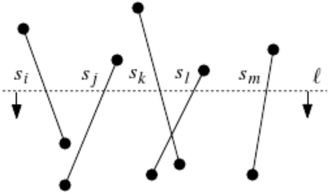

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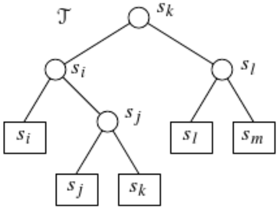

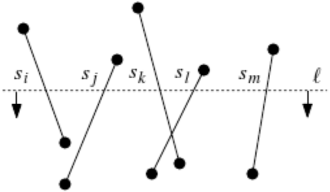

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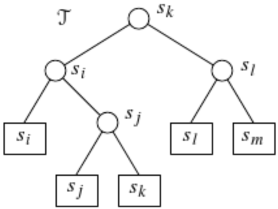

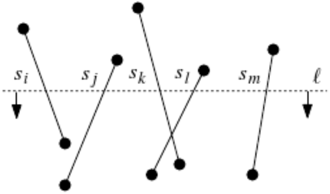

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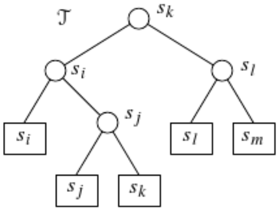
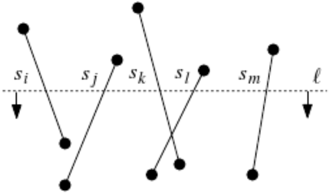
- There is an event point above the intersecting point where s_k and s_l are adjacent and are tested for intersection. So, no intersection point is ever missed.





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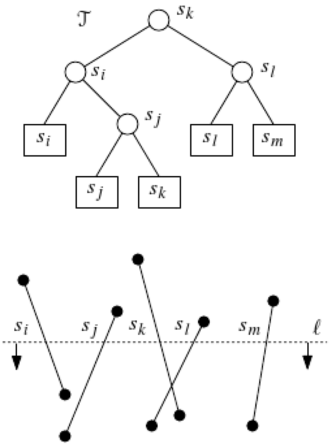

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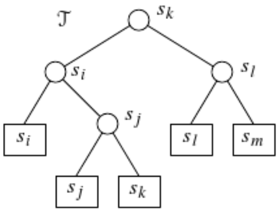

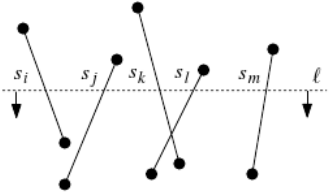

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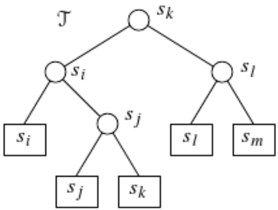
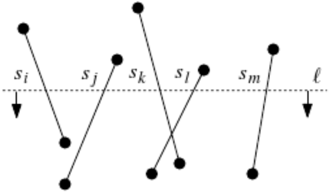
- Based on the three cases: segment top end point, segment bottom end point and intersection point, take necessary actions on $\mathcal{T}$.





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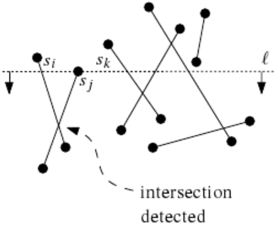

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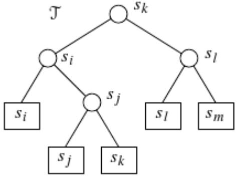

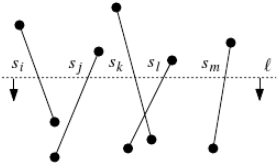

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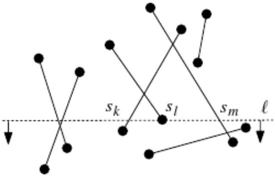

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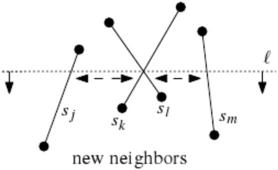
- **[Intersection point]** Swap the line segments' status in $\mathcal{T}$. Check for intersections of preceding and succeeding entries.



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- The balanced binary search tree $\mathcal{T}$ grows also to a size at most $2n + I$. So, each operation takes $O(\log n)$.

The Analysis

Analysis

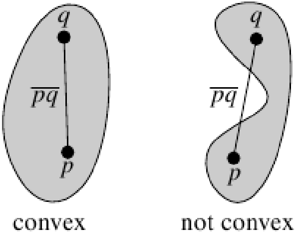
- Total number of event points is $2n + I$, where I is the number of intersections.
- The heap $\mathcal{H}$ grows to a size at most $2n + I$. Each operation takes $O(\log(2n + I))$. As $I < n^2$, so $O(\log(2n + I)) = O(\log n)$.
- The balanced binary search tree $\mathcal{T}$ grows also to a size at most $2n + I$. So, each operation takes $O(\log n)$.
- So, the total time taken is $O((2n + I) \log n) = O(n \log n + I \log n)$.

Outline

Convex Hull

Definition

A set $S \subset \mathcal{R}^2$ is convex if for any two points $p, q \in S$, $\overline{pq} \in S$.



Convex Hull

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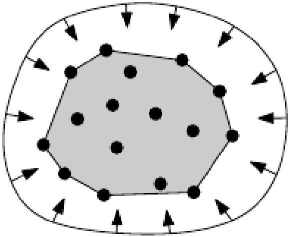
Definition

Let $\mathcal{P}$ be a set of points in $\mathcal{R}^2$. Convex hull of $\mathcal{P}$, denoted by $CH(\mathcal{P})$, is the smallest convex set containing $\mathcal{P}$.



Convex Hull Problem

Problem

Given a set of points $\mathcal{P}$ in the plane, compute the convex hull $CH(\mathcal{P})$ of the set $\mathcal{P}$.

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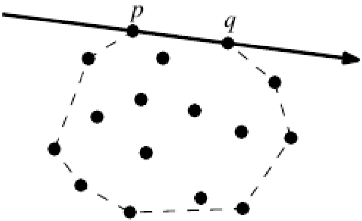
A Naive Algorithm

- Consider all line segments determined by $\binom{n}{2} = O(n^2)$ pairs of points.
- If a line segment has all the other $n - 2$ points on one side of it, then it is a hull edge.



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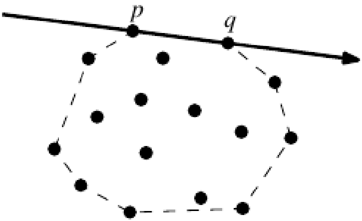
- We need $\binom{n}{2}(n-2) = O(n^3)$ time.



Towards a Better Algorithm

Way forward, but how much?

- **Better characterizations lead to better algorithms.**

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Towards a Better Algorithm

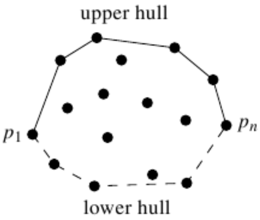
Way forward, but how much?

- **Better characterizations lead to better algorithms.**
- How much better can we make?
- Leads to the notion of lower bound of a problem.
- The problem of Convex Hull has a lower bound of $\Omega(n \log n)$. This can be shown by a reduction from the problem of sorting which also has a lower bound of $\Omega(n \log n)$.

Graham's Scan: An optimal algorithm for Convex Hull

A better characterization

- Consider a walk in clockwise direction on the vertices of a closed polygon.



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- Consider a walk in clockwise direction on the vertices of a closed polygon.
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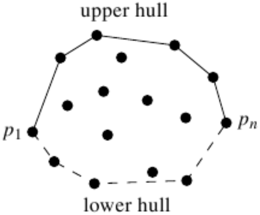

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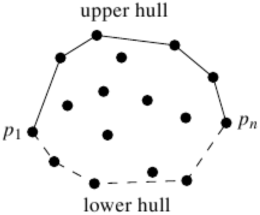

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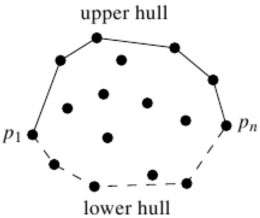

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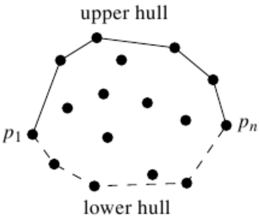
- We compute the upper hull first. The upper hull contains the convex hull edges that bound the convex hull from above.



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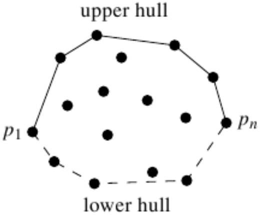
- Sort the points in P from left to right.



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             Delete the middle of the last
             three points from L_U;
    }
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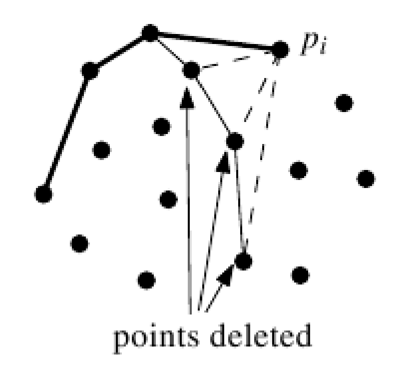
```

The Algorithm in Action

The algorithm in action



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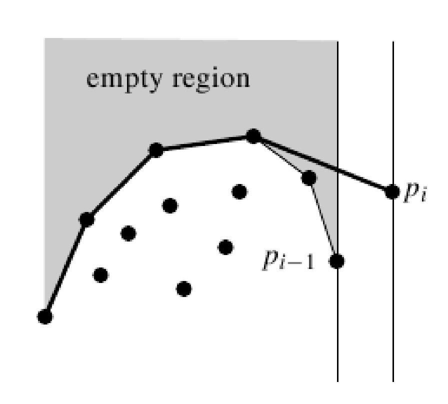
- Hence, the total time complexity is $O(n \log n)$.

Proof of Correctness

The Proof of Correctness

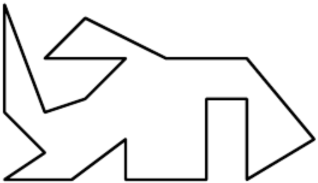


Outline

Art Gallery Problem

Problem

Given a simple polygon P of n vertices, find the minimum number of cameras that can guard P.



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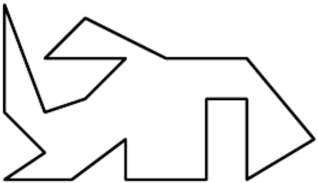
Hardness

The above problem is NP-Hard.



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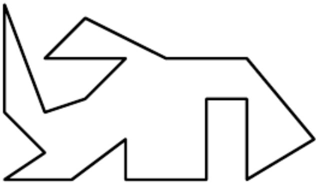

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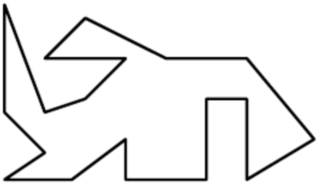

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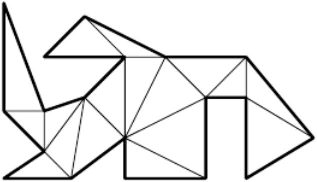
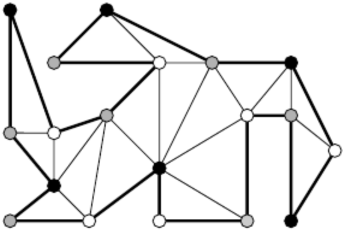
- Can we find as a function of n the number of cameras that suffices to guard P?
- Recall P can be triangulated into $n-2$ triangles. Place a guard in each triangle.



Art Gallery Problem

Can we bring the bound down?

- Place guards at vertices of the triangulation $\mathcal{T}$ of P.



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Can we bring the bound down?

- Place guards at vertices of the triangulation $\mathcal{T}$ of P.
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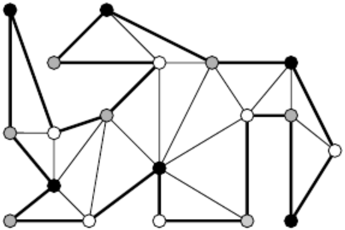

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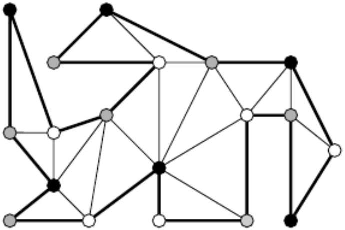
- Place guards at vertices of the triangulation $\mathcal{T}$ of P.
- We do a 3-coloring of the vertices of $\mathcal{T}$. Each triangle of $\mathcal{T}$ has a black, gray and white vertex.
- Choose the smallest color class to guard P.



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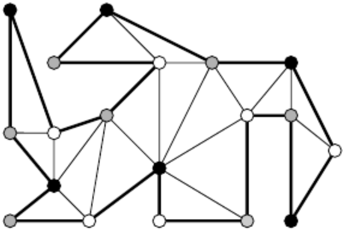

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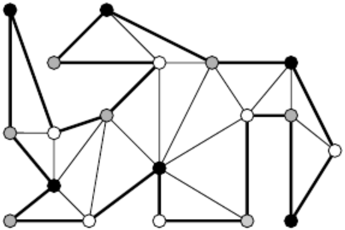
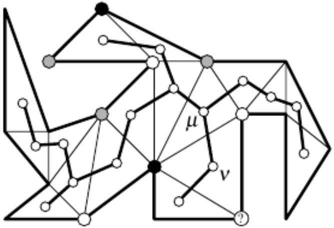
- Hence, $\lfloor \frac{n}{3} \rfloor$ guards suffice.
- But, does a 3-coloring always exist?



Art Gallery Problem

A 3-coloring always exists

- Consider the dual graph $\mathcal{G}_{\mathcal{T}}$ of $\mathcal{T}$ of P.



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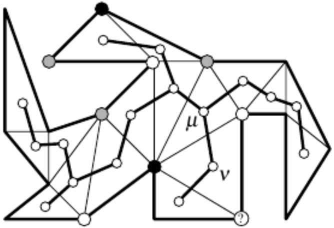

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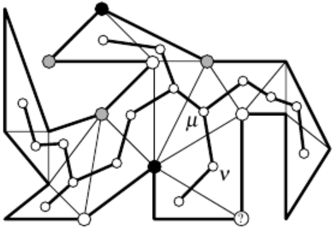

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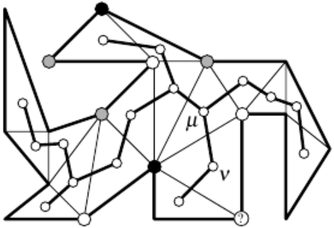

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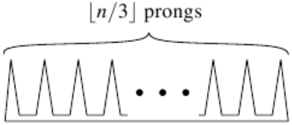

Necessity?

Are $\lfloor \frac{n}{3} \rfloor$ guards sometimes necessary?

Art Gallery Theorem

The Final Theorem

For a simple polygon with n vertices, $\lfloor \frac{n}{3} \rfloor$ cameras are always sufficient and occasionally necessary to have every point in the polygon visible from at least one of the cameras.

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