

GEOMETRIC DATA STRUCTURES

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January 10-12, 2012 - NIT Surathkal
Introduction to Graph and Geometric Algorithms

SCOPE OF THE LECTURE

- ▶ **BINARY SEARCH TREES AND 2-D RANGE TREES**

We consider 1-d and 2-d range queries for point sets.

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2-d orthogonal range searching with Kd-trees.

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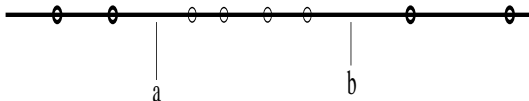
- ▶ **PLANAR POINT LOCATION**

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- ▶ **HIERARCHICAL REPRESENTATION OF A CONVEX POLYGON**

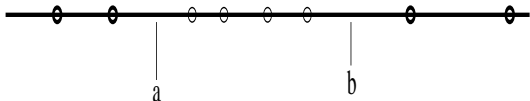
Detecting the intersection of a convex polygon with a query line..

1-DIMENSIONAL RANGE SEARCHING



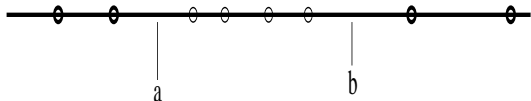
- ▶ Problem: Given a set P of n points $\{p_1, p_2, \dots, p_n\}$ on the real line, report points of P that lie in the range $[a, b]$, $a \leq b$.

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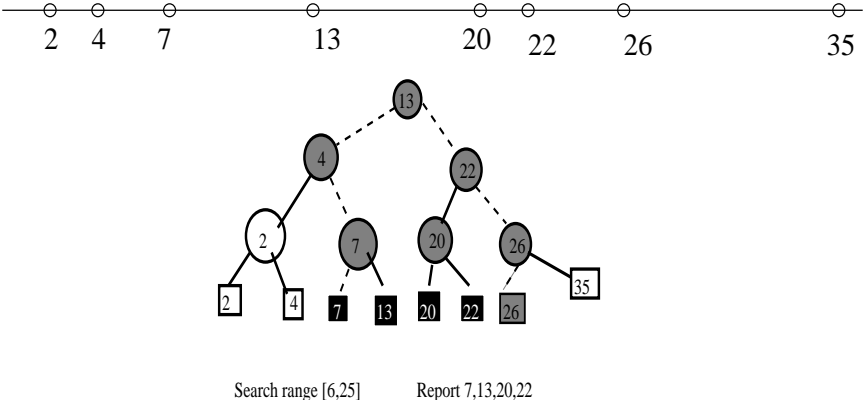
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- ▶ Using binary search on an array we can answer such a query in $O(\log n + k)$ time where k is the number of points of P in $[a, b]$.

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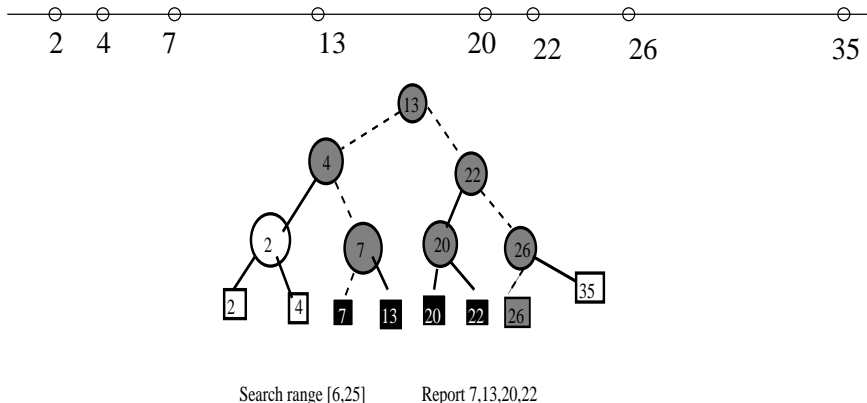
- ▶ Problem: Given a set P of n points $\{p_1, p_2, \dots, p_n\}$ on the real line, report points of P that lie in the range $[a, b]$, $a \leq b$.
- ▶ Using binary search on an array we can answer such a query in $O(\log n + k)$ time where k is the number of points of P in $[a, b]$.
- ▶ However, when we permit insertion or deletion of points, we cannot use an array answering queries so efficiently.

1-DIMENSIONAL RANGE SEARCHING



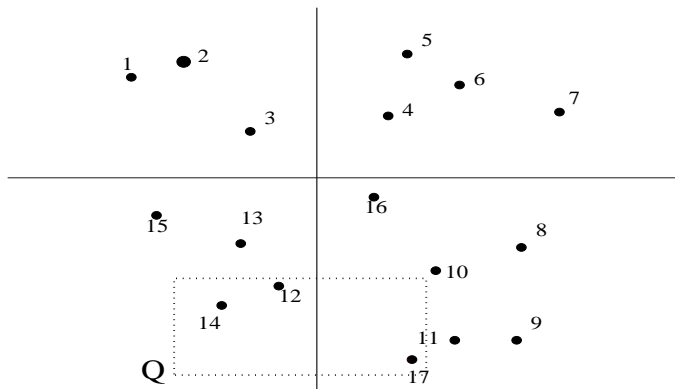
- ▶ We use a *binary leaf search tree* where leaf nodes store the points on the line, sorted by x-coordinates.

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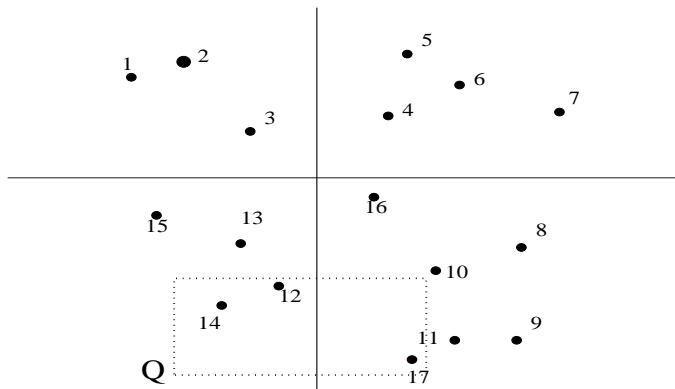
- ▶ We use a *binary leaf search tree* where leaf nodes store the points on the line, sorted by x-coordinates.
- ▶ Each internal node stores the x-coordinate of the rightmost point in its left subtree for guiding search.

2-DIMENSIONAL RANGE SEARCHING



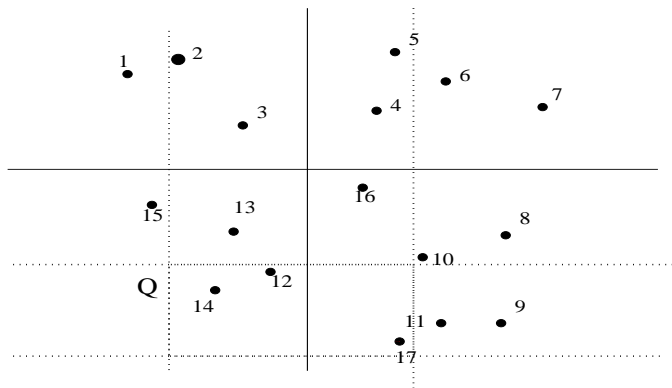
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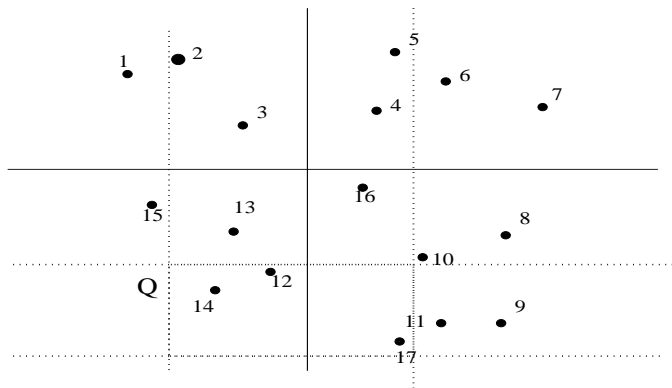
- ▶ Problem: Given a set P of n points in the plane, report points inside a query rectangle Q whose sides are parallel to the axes.
- ▶ Here, the points inside R are 14, 12 and 17.

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2-DIMENSIONAL RANGE SEARCHING



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- ▶ The cost incurred may exceed the actual output size of the 2-d range query.

RANGE SEARCHING WITH RANGE TREES AND KD-TREES

- ▶ Given a set S of n points in the plane, we can construct a *2d-range tree* in $O(n \log n)$ time and space, so that rectangle queries can be executed in $O(\log^2 n + k)$ time.

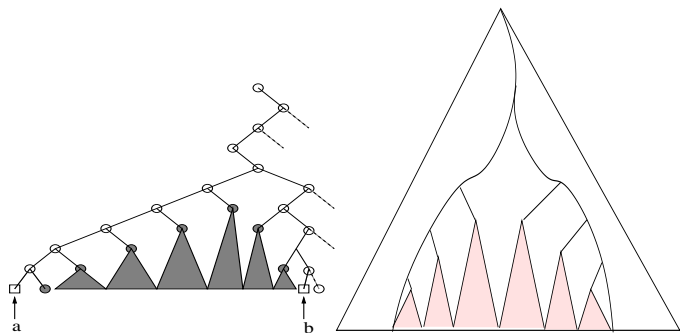
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- ▶ The query time can be improved to $O(\log n + k)$ using the technique of *fractional cascading*.
- ▶ Given a set S of n points in the plane, we can construct a Kd-tree in $O(n \log n)$ time and $O(n)$ space, so that *rectangle queries* can be executed in $O(\sqrt{n} + k)$ time. Here, the number of points in the query rectangle is k .

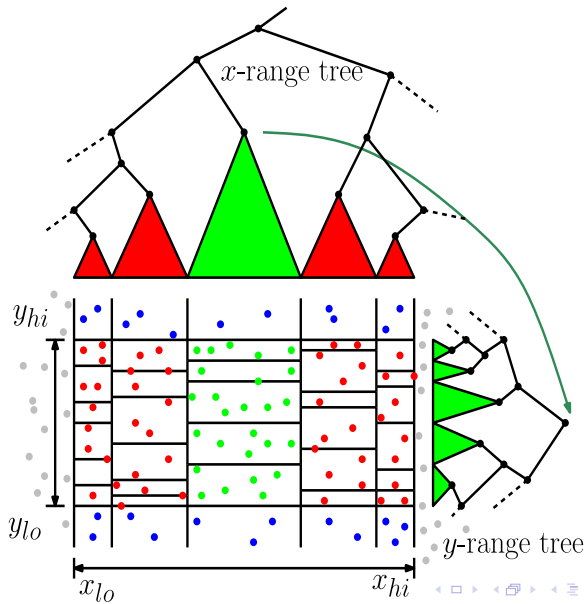
RANGE SEARCHING IN THE PLANE USING RANGE TREES



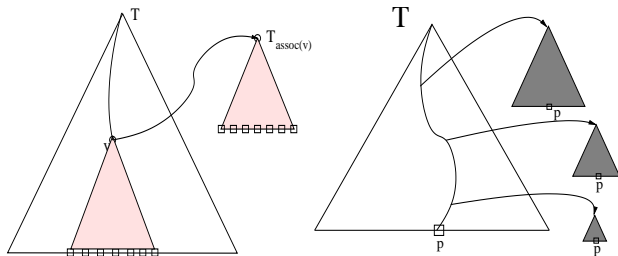
Given a 2-d rectangle query $[a, b] \times [c, d]$, we can identify subtrees whose leaf nodes are in the range $[a, b]$ along the X-direction.

Only a subset of these leaf nodes lie in the range $[c, d]$ along the Y-direction.

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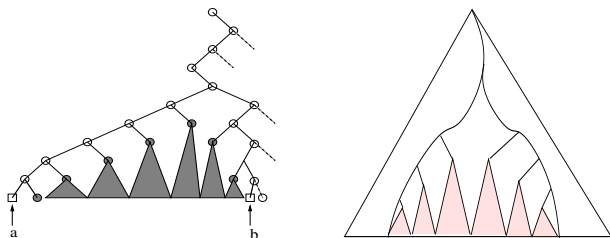


$T_{assoc(v)}$ is a binary search tree on y -coordinates for points in the leaf nodes of the subtree rooted at v in the tree T .

The point p is duplicated in $T_{assoc(v)}$ for each v on the search path for p in tree T .

The total space requirement is therefore $O(n \log n)$.

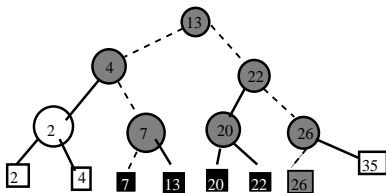
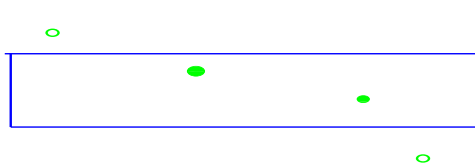
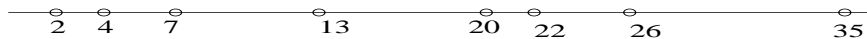
RANGE SEARCHING IN THE PLANE USING RANGE TREES



We perform 1-d range queries with the y -range $[c, d]$ in each of the subtrees adjacent to the left and right search paths within the x -range $[a, b]$ in the tree T .

Since the search path is $O(\log n)$ in size, and each y -range query requires $O(\log n)$ time, the total cost of searching is $O(\log^2 n)$. The reporting cost is $O(k)$ where k points lie in the query rectangle.

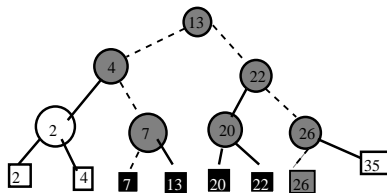
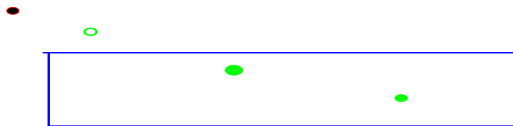
2-RANGE TREE SEARCHING



Search range [6,25]

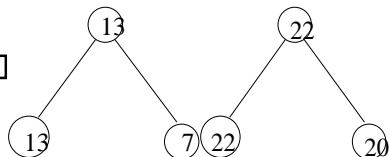
Report 7,13,20,22

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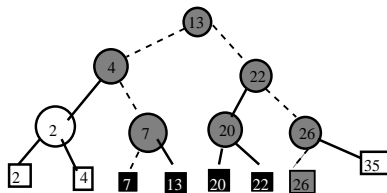
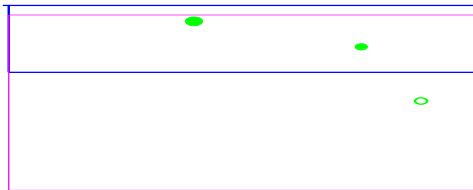
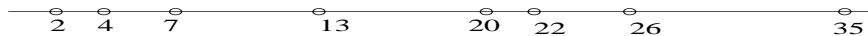


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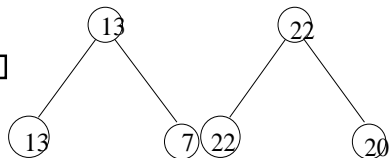


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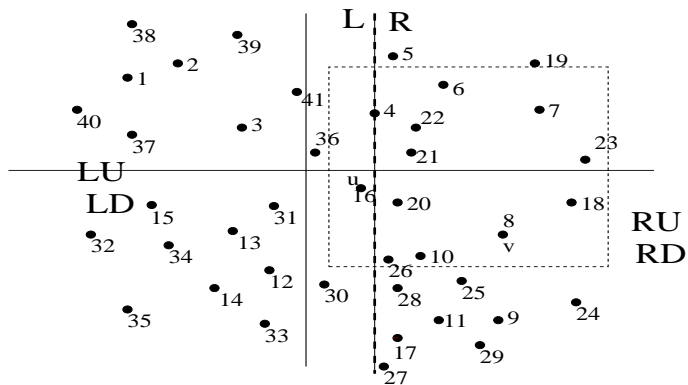


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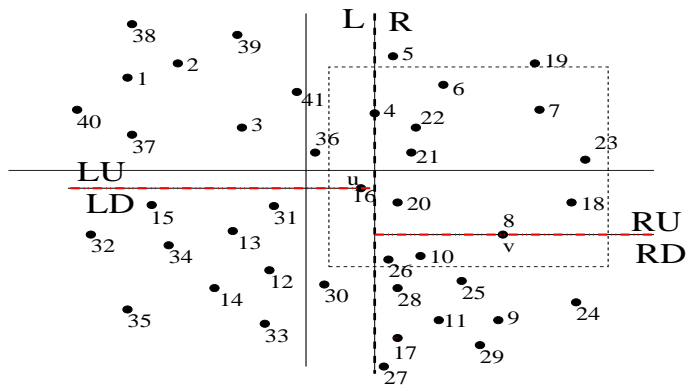
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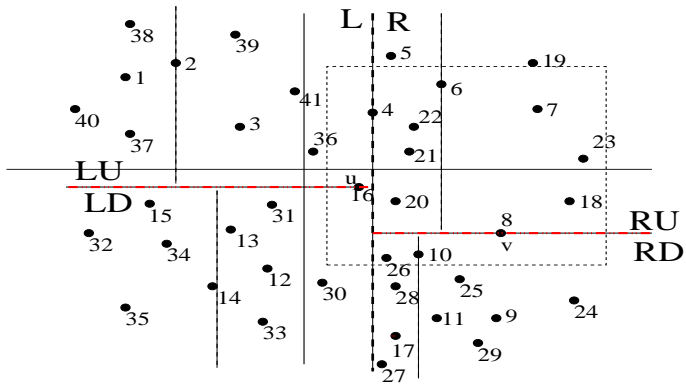
PARTITION BY THE MEDIAN OF X-COORDINATES



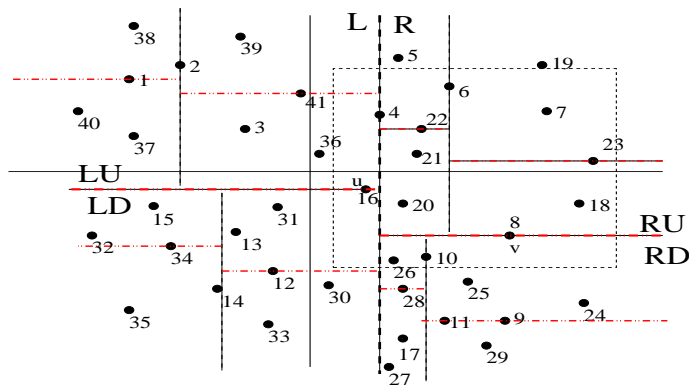
PARTITION BY THE MEDIAN OF Y-COORDINATES



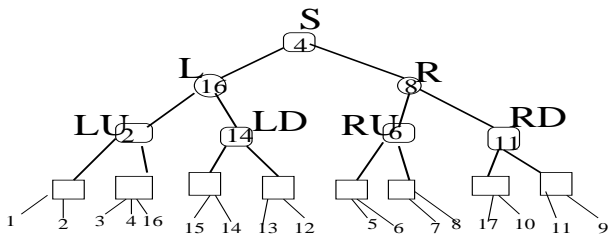
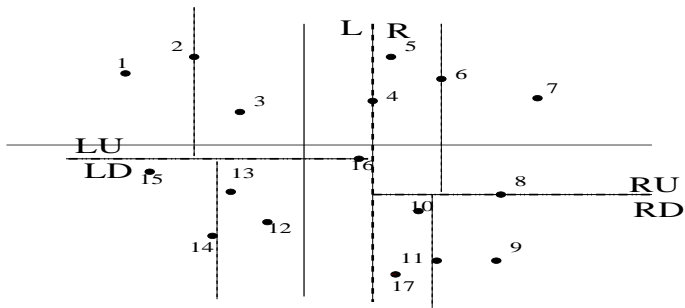
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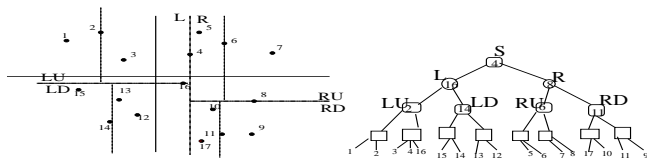
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2-DIMENSIONAL RANGE SEARCHING USING KD-TREES

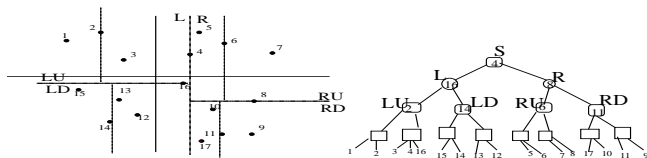


DESCRIPTION OF THE KD-TREE



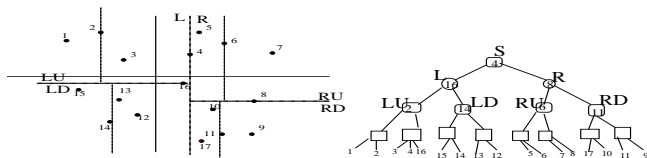
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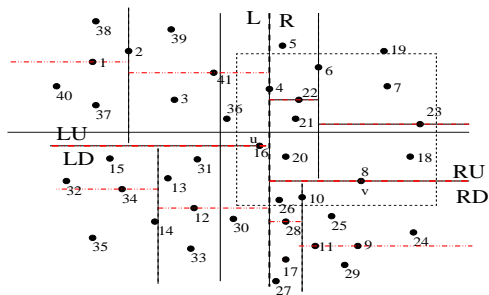
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- ▶ The point r stored in the root vertex T splits the set S into two roughly equal sized sets L and R using the median x -coordinate $x_{median}(S)$ of points in S , so that all points in L (R) have abscissae less than or equal to (strictly greater than) $x_{median}(S)$.

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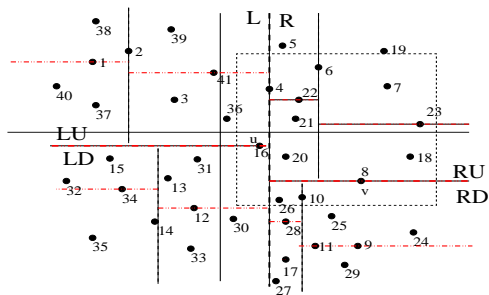
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- ▶ The entire plane is called the $region(r)$.

ANSWERING RECTANGLE QUERIES



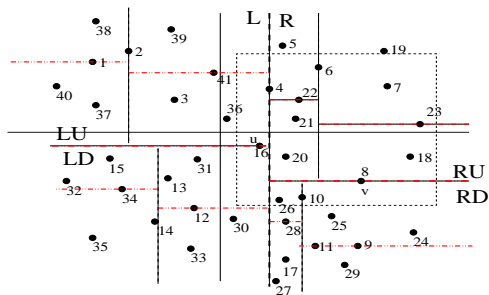
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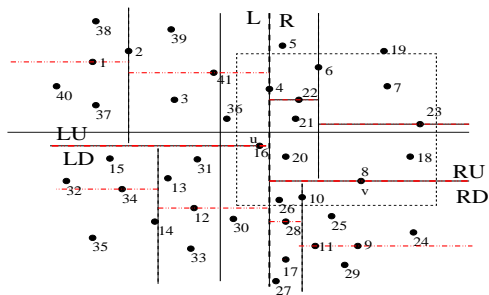
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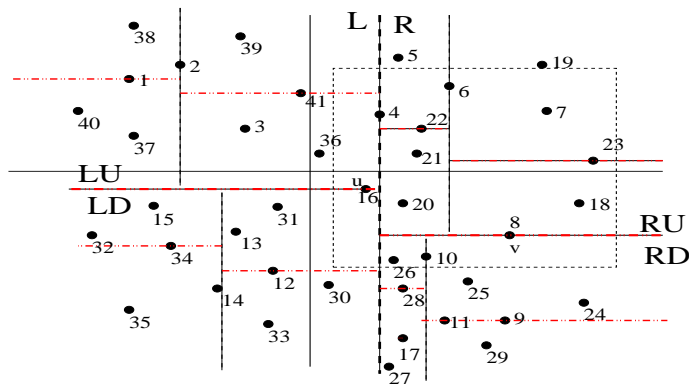
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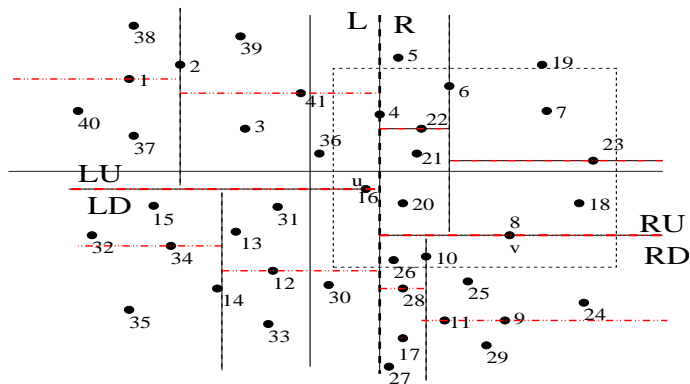
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- ▶ If R overlaps $region(p)$ then we check whether R also overlaps the two regions of the children of the node N .

2-DIMENSIONAL RANGE SEARCHING: KD-TREES



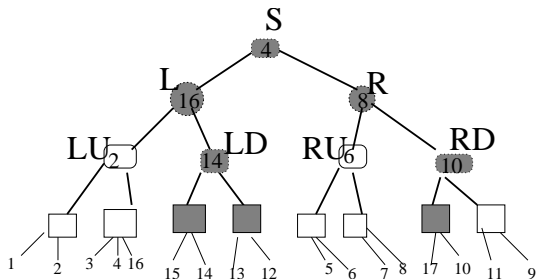
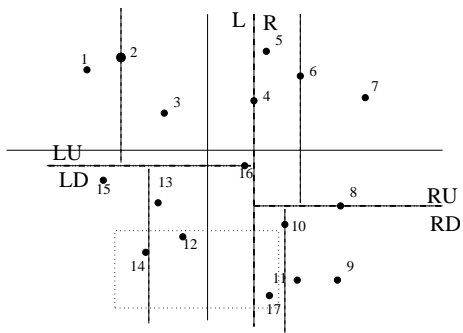
- ▶ The set L (R) is split into two roughly equal sized subsets LU and LD (RU and RD), using point u (v) that has the median y -coordinate in the set L (R), and including u in LU (RU).

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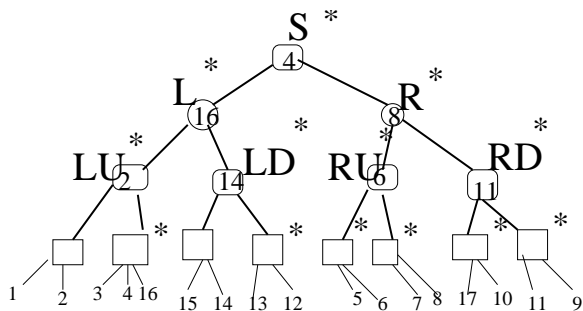
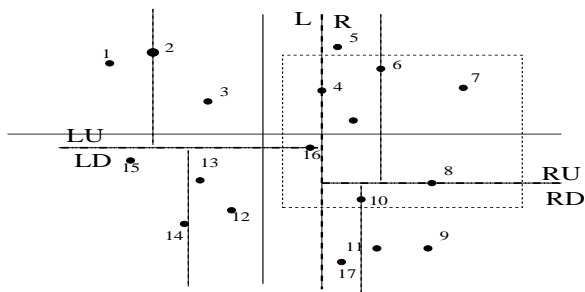


- ▶ The set L (R) is split into two roughly equal sized subsets LU and LD (RU and RD), using point u (v) that has the median y -coordinate in the set L (R), and including u in LU (RU).
- ▶ The entire halfplane containing set L (R) is called the *region*(u) (*region*(v)).

NODES TRAVERSED IN THE KD-TREE



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- ▶ This cost is borne for all leaf level regions intersected by R .

WORST-CASE COST OF TRAVERSAL

- ▶ It is sufficient to determine the upper bound on the number of (internal) nodes whose regions are intersected by a single vertical (horizontal) line.

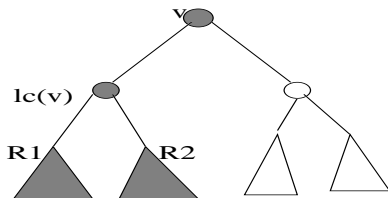
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- ▶ Any vertical line intersecting S can intersect either L or R but not both, but it can meet both RU and RD (LU and LD).
- ▶ Any horizontal line intersecting R can intersect either RU or RD but not both, but it can meet both children of RU (RD).

TIME COMPLEXITY OF RECTANGLE QUERIES FOR KD-TREES

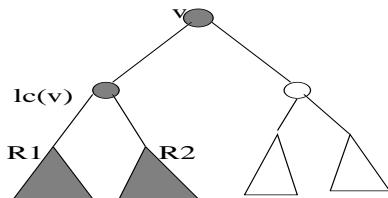


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$$T(n) = 2 + 2T\left(\frac{n}{4}\right)$$

$$T(1) = 1$$

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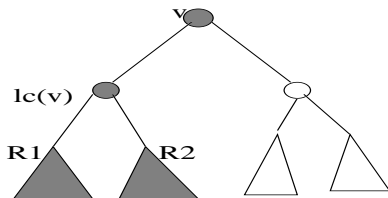
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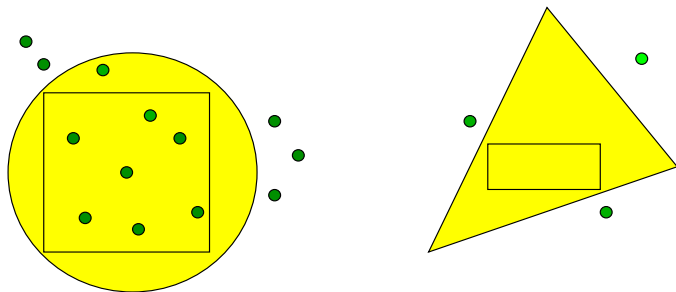
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- ▶ The total cost of reporting k points in R is therefore $O(\sqrt{(n)} + k)$.

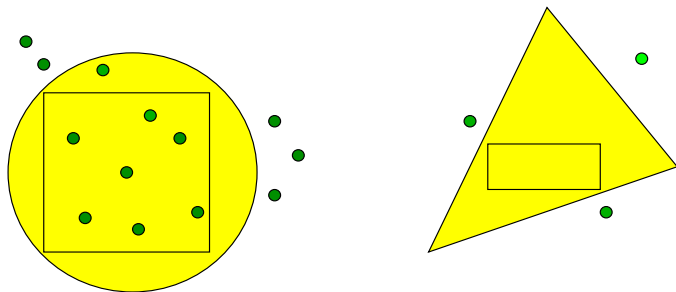
MORE GENERAL QUERIES



General Queries:

- ▶ Triangles can be used to simulate polygonal shapes with straight edges.

MORE GENERAL QUERIES



General Queries:

- ▶ Triangles can be used to simulate polygonal shapes with straight edges.
- ▶ Circles cannot be simulated by triangles either.

TRIANGLE QUERIES

- ▶ Using $O(n^2)$ space and time for preprocessing, triangle queries can be reported in $O(\log^2 n + k)$ time, where k is the number of points inside the query triangle.

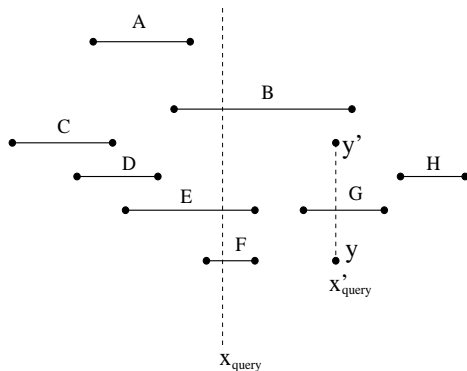
Goswami, Das and Nandy: Comput. Geom. Th. and Appl. 29 (2004) pp. 163-175.

TRIANGLE QUERIES

- ▶ Using $O(n^2)$ space and time for preprocessing, triangle queries can be reported in $O(\log^2 n + k)$ time, where k is the number of points inside the query triangle.
- ▶ For counting the number k of points inside a query triangle, worst-case optimal $O(\log n)$ time suffices.

Goswami, Das and Nandy: Comput. Geom. Th. and Appl. 29 (2004) pp. 163-175.

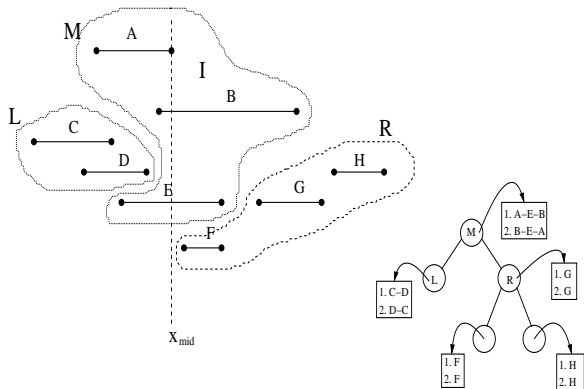
FINDING INTERVALS CONTAINING A VERTICAL QUERY LINE/SEGMENT



Simpler queries ask for reporting all intervals intersecting the vertical line $X = x_{query}$.

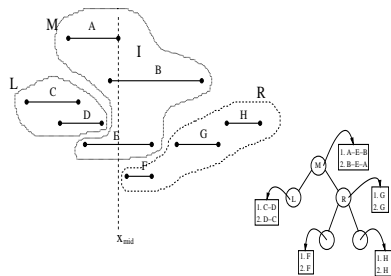
More difficult queries ask for reporting all intervals intersecting a vertical segment joining (x'_{query}, y) and (x'_{query}, y') .

CONSTRUCTING THE INTERVAL TREE



The set M has intervals intersecting the vertical line $X = x_{mid}$, where x_{mid} is the median of the x -coordinates of the $2n$ endpoints. The root node has intervals M sorted in two independent orders (i) by right end points ($B-E-A$), and (ii) left end points ($A-E-B$).

ANSWERING QUERIES USING AN INTERVAL TREE



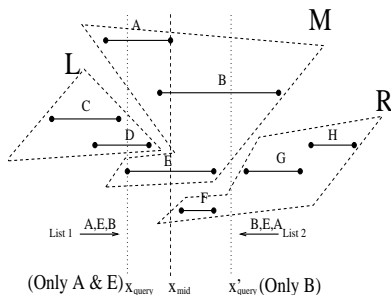
The set L and R have at most n endpoints each.

So they have at most $\frac{n}{2}$ intervals each.

Clearly, the cost of (recursively) building the interval tree is $O(n \log n)$.

The space required is linear.

ANSWERING QUERIES USING AN INTERVAL TREE

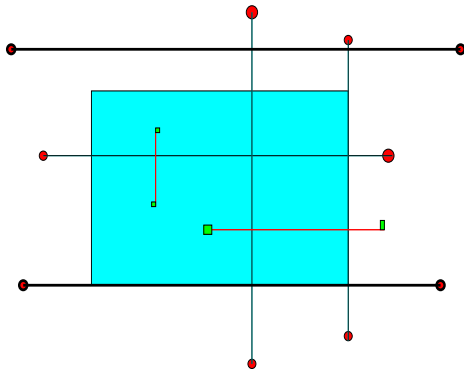


For $x_{query} < x_{mid}$, we do not traverse subtree for subset R .

For $x'_{query} > x_{mid}$, we do not traverse subtree for subset L .

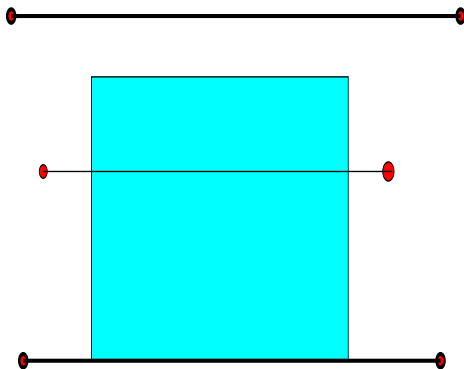
Clearly, the cost of reporting the k intervals is $O(\log n + k)$.

REPORTING (PORTIONS OF) ALL RECTILINEAR SEGMENTS INSIDE A QUERY RECTANGLE



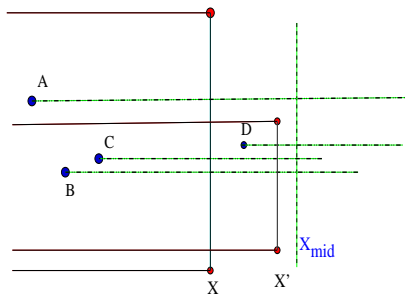
For detecting segments with one (or both) ends inside the rectangle, it is sufficient to maintain rectangular range query apparatus for output-sensitive query processing.

REPORTING SEGMENTS WITH NO ENDPOINTS INSIDE THE QUERY RECTANGLE



Report all (horizontal) segments that cut across the query rectangle or include an entire (top/bottom) bounding edge. Use either the right (or left) edge, and the top (or bottom) edge of the query rectangle.

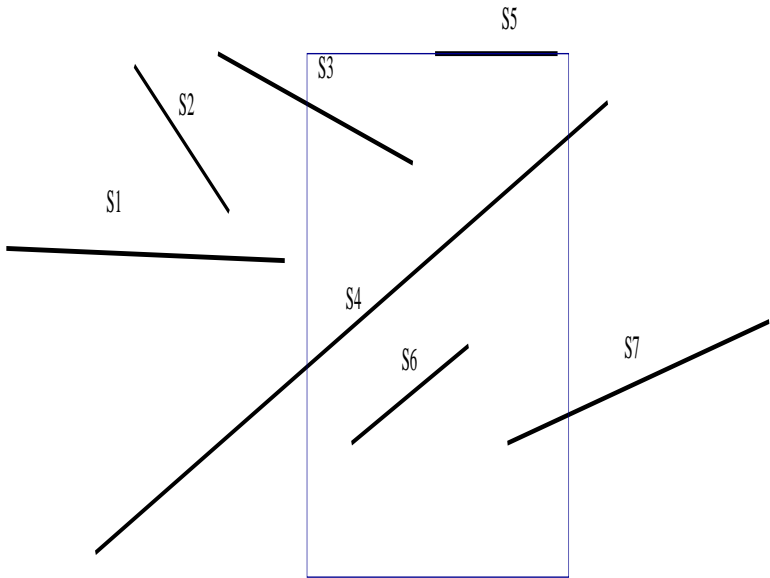
RIGHT EDGES X AND X' OF TWO QUERY RECTANGLES

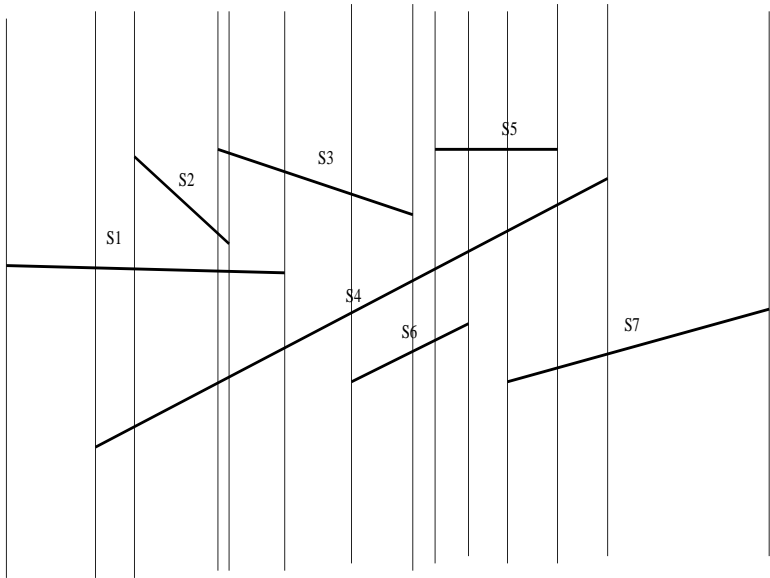


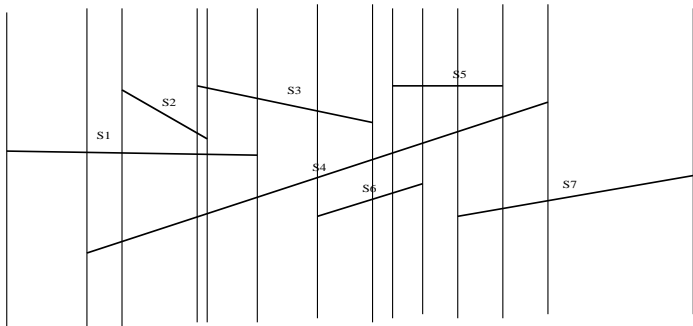
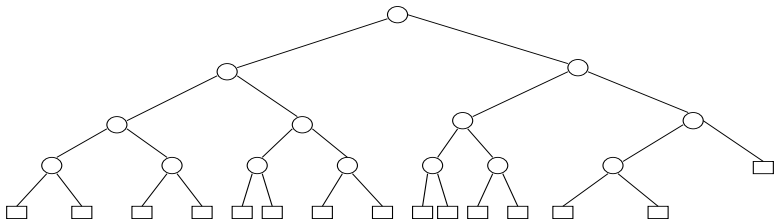
Use an interval tree of all the horizontal segments and the right bounding edge of the query rectangle like X or X' .

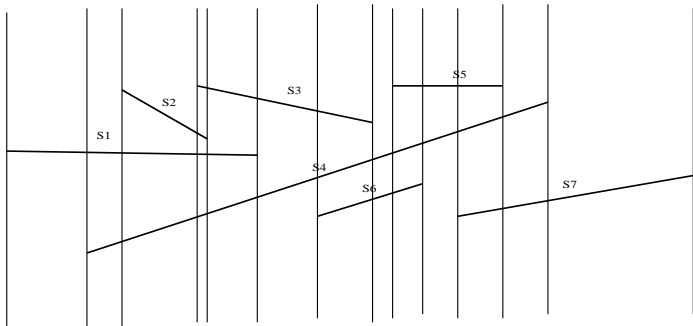
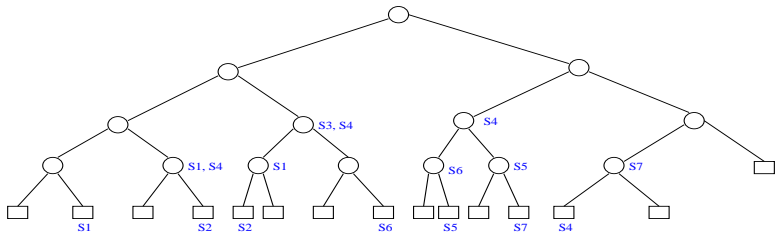
This helps reporting all segments cutting the right edge of the query rectangle.

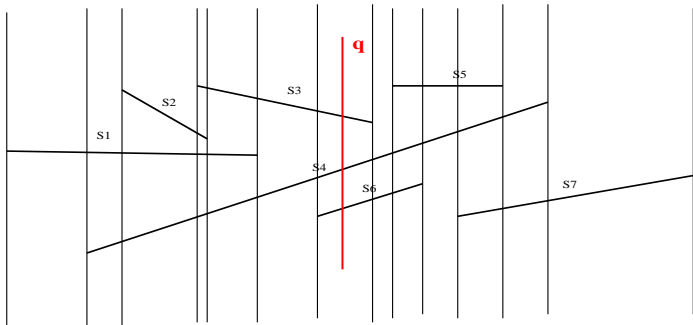
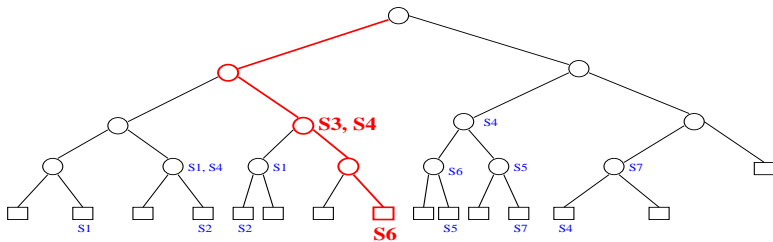
Use the rectangle query for vertical segment X and find points A , B and C in the rectangle with left edge at minus infinity. For X' , report B , C and D , similarly.

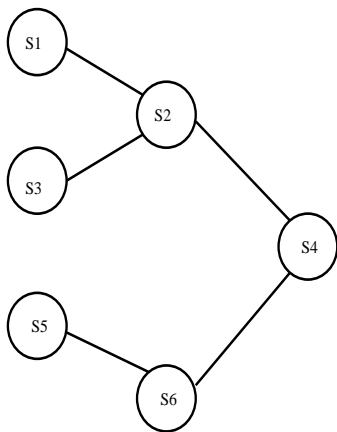
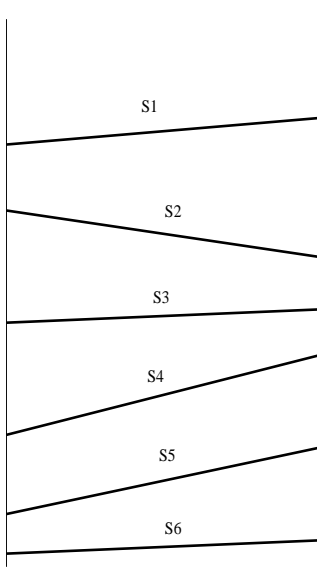






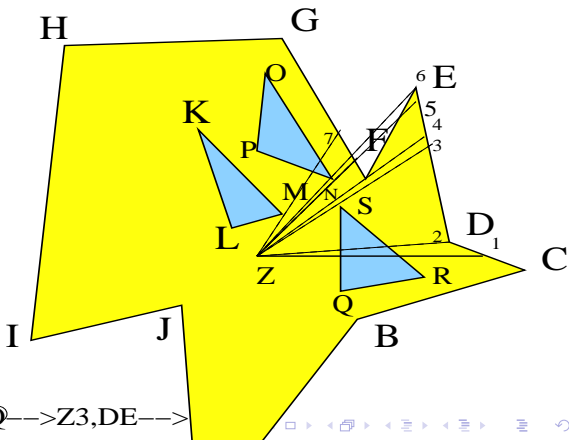






COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

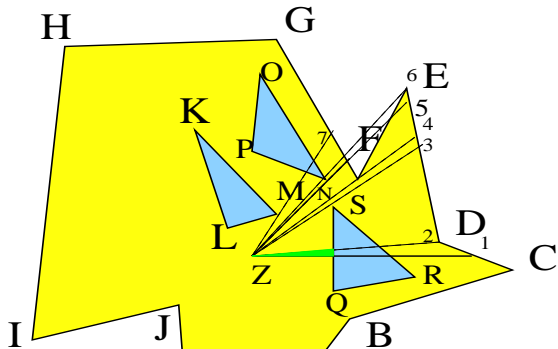
SQ,SR,DC,1-->SQ,SR,DE,2-->DE,3-->
FG,FE,DE,4-->NP,NO,FG,FE,DE,5-->
NP,NO,FG,FE,DE,6-->LM,MK,NP,NO,FG,7



Z1 ,SQ-->Z2,SQ-->Z3,DE-->

COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

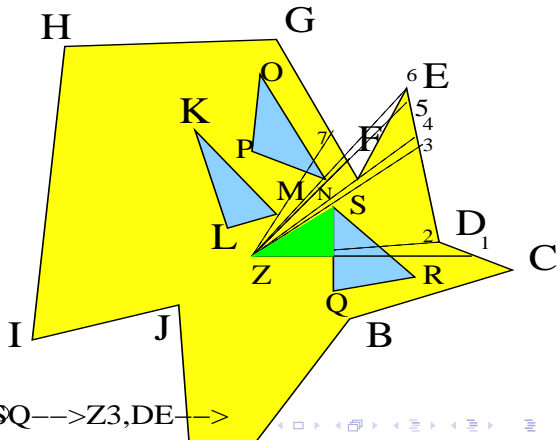
$SQ, SR, DC, 1 \rightarrow SQ, SR, DE, 2 \rightarrow DE, 3$
 $FG, FE, DE, 4 \rightarrow NP, NO, FG, FE, DE, 5 \rightarrow$
 $NP, NO, FG, FE, DE, 6 \rightarrow LM, MK, NP, NO, FG, 7$



$Z1, SQ \rightarrow Z2, SQ \rightarrow Z3, DE \rightarrow$

COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

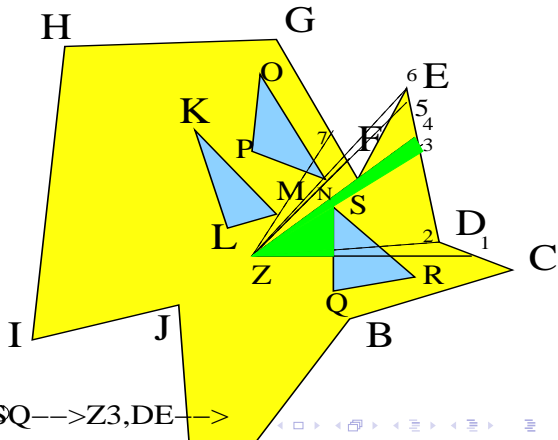
$SQ, SR, DC, 1 \rightarrow SQ, SR, DE, 2 \rightarrow DE, 3$
 $FG, FE, DE, 4 \rightarrow NP, NO, FG, FE, DE, 5 \rightarrow$
 $NP, NO, FG, FE, DE, 6 \rightarrow LM, MK, NP, NO, FG, 7$



$Z1, SQ \rightarrow Z2, SQ \rightarrow Z3, DE \rightarrow$

COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

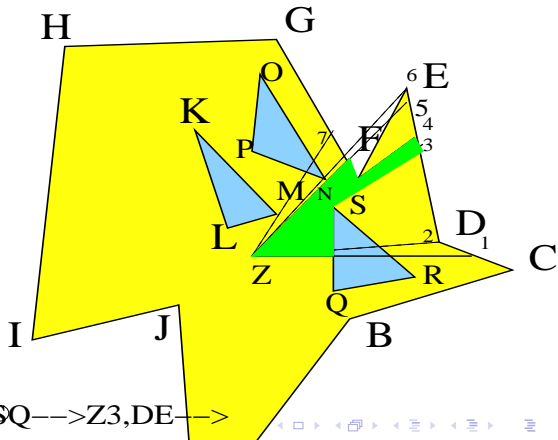
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 $NP, NO, FG, FE, DE, 6 \rightarrow LM, MK, NP, NO, FG, 7$



$Z1, SQ \rightarrow Z2, SQ \rightarrow Z3, DE \rightarrow$

COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

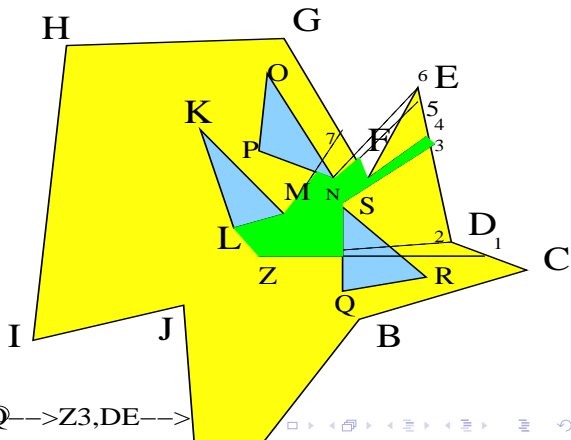
$SQ, SR, DC, 1 \rightarrow SQ, SR, DE, 2 \rightarrow DE, 3$
 $FG, FE, DE, 4 \rightarrow NP, NO, FG, FE, DE, 5 \rightarrow$
 $NP, NO, FG, FE, DE, 6 \rightarrow LM, MK, NP, NO, FG, 7$



$Z1, SQ \rightarrow Z2, SQ \rightarrow Z3, DE \rightarrow$

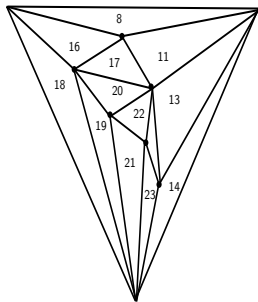
COMPUTING THE VISIBLE REGION IN A POLYGON WITH OPAQUE OBSTACLES

SQ,SR,DC,1-->SQ,SR,DE,2-->DE,3-->
 FG,FE,DE,4-->NP,NO,FG,FE,DE,5-->
 NP,NO,FG,FE,DE,6-->LM,MK,NP,NO,FG,7

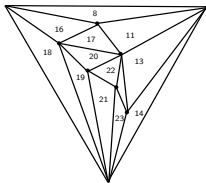


Z1 ,SQ-->Z2,SQ-->Z3,DE-->

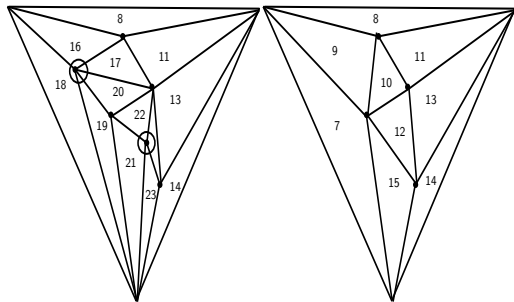
PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT



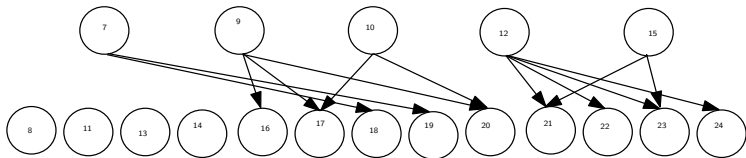
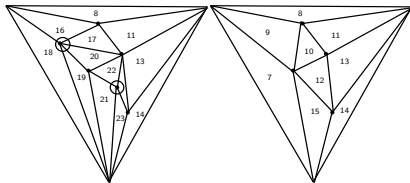
PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT

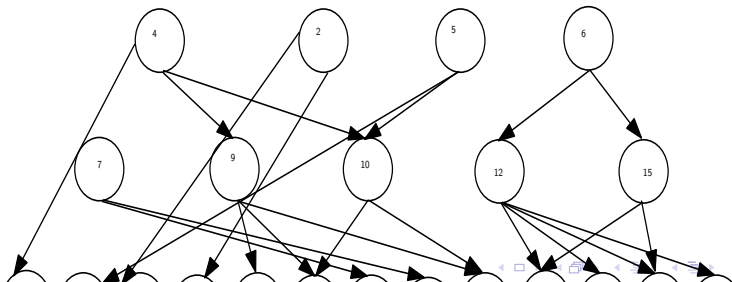
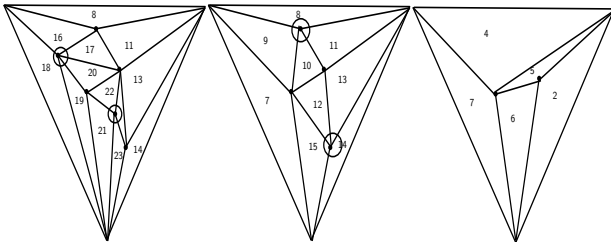


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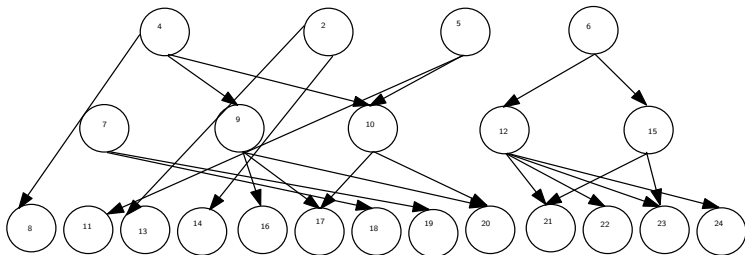
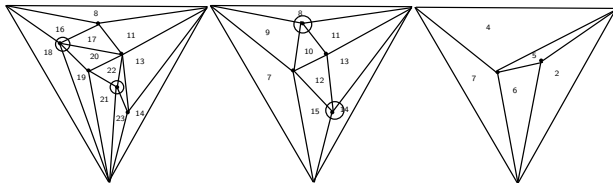


PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT

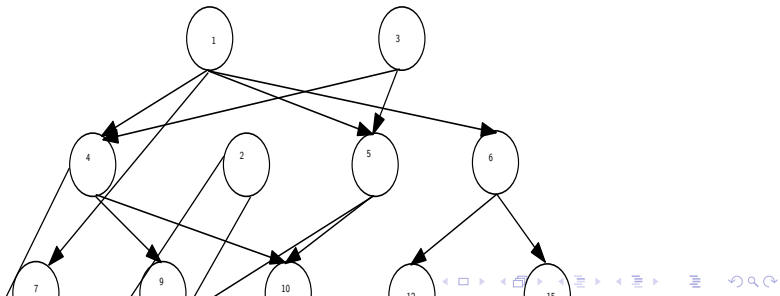
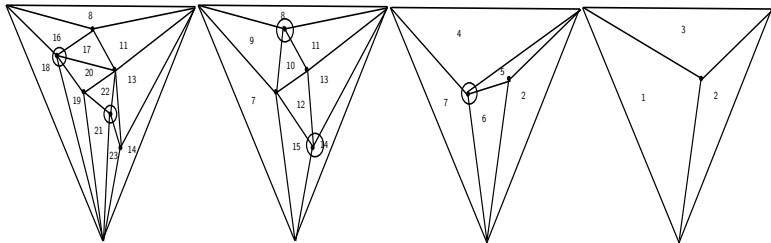




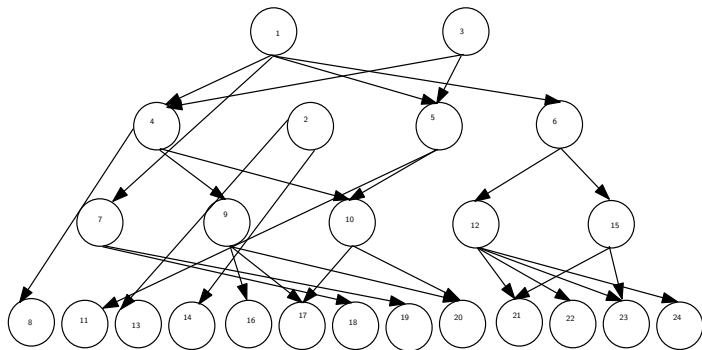
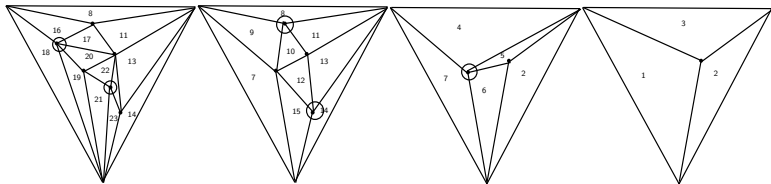
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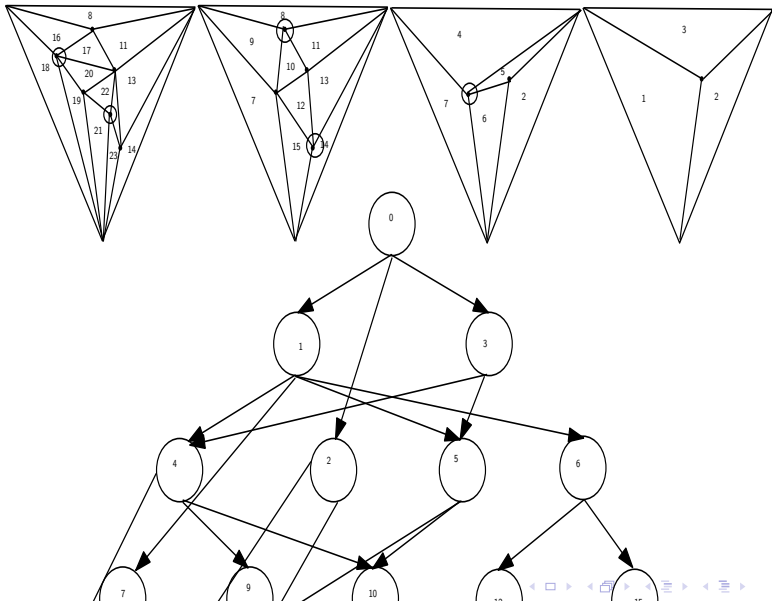
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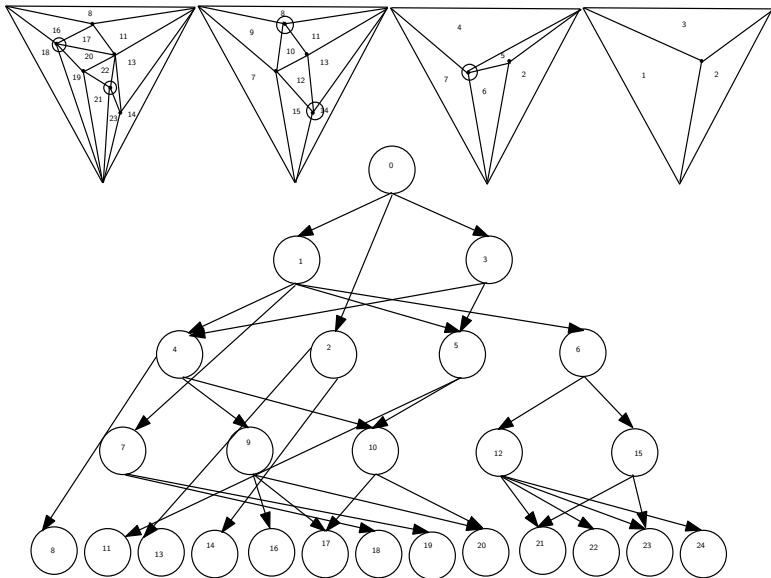
PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT



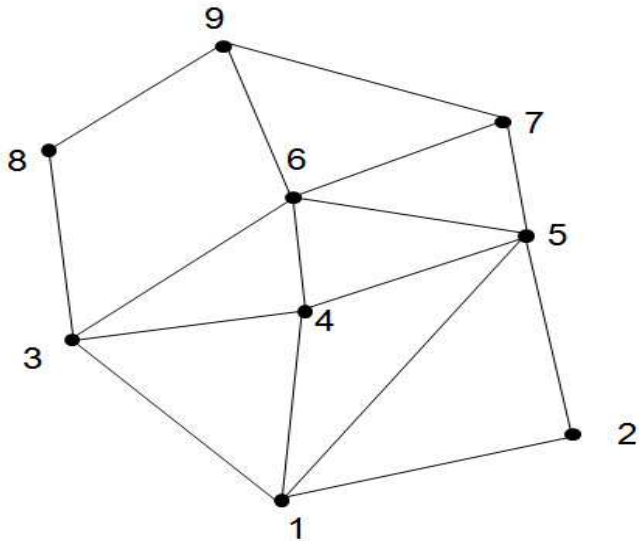
PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT



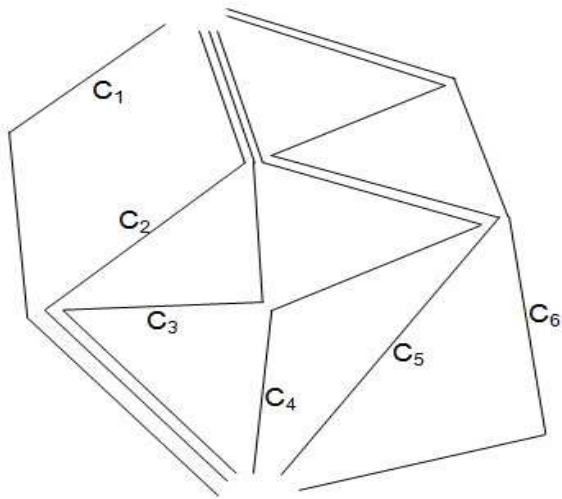
PLANAR POINT LOCATION BY TRIANGULATION REFINEMENT



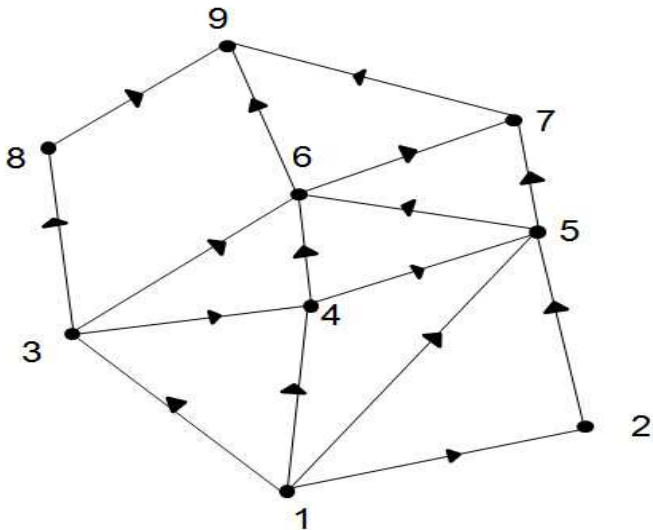
PLANAR POINT LOCATION USING MONOTONE CHAINS



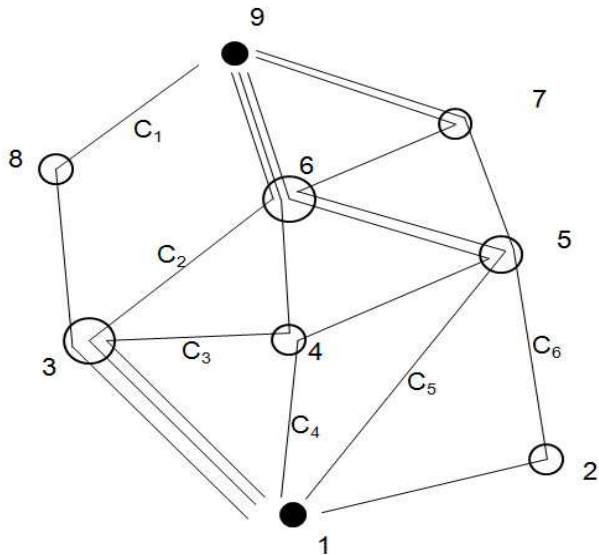
PLANAR POINT LOCATION USING MONOTONE CHAINS



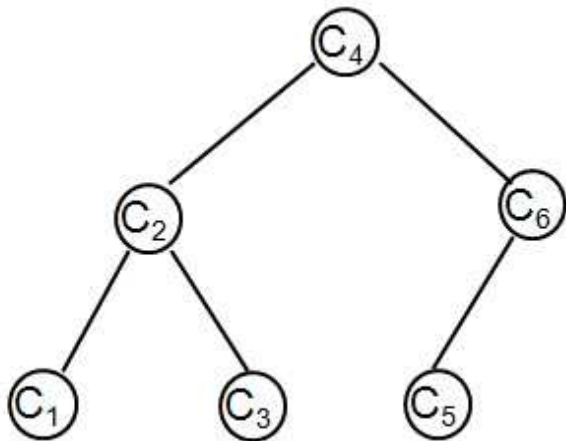
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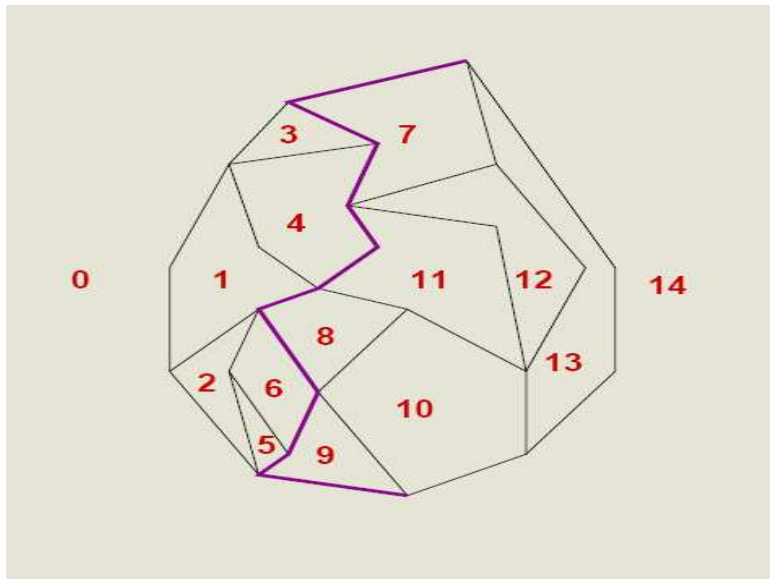
PLANAR POINT LOCATION USING MONOTONE CHAINS



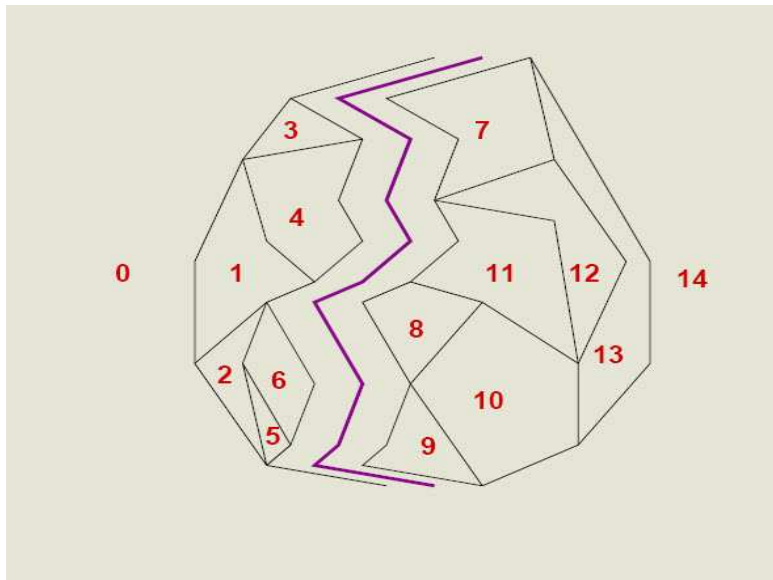
PLANAR POINT LOCATION USING MONOTONE CHAINS



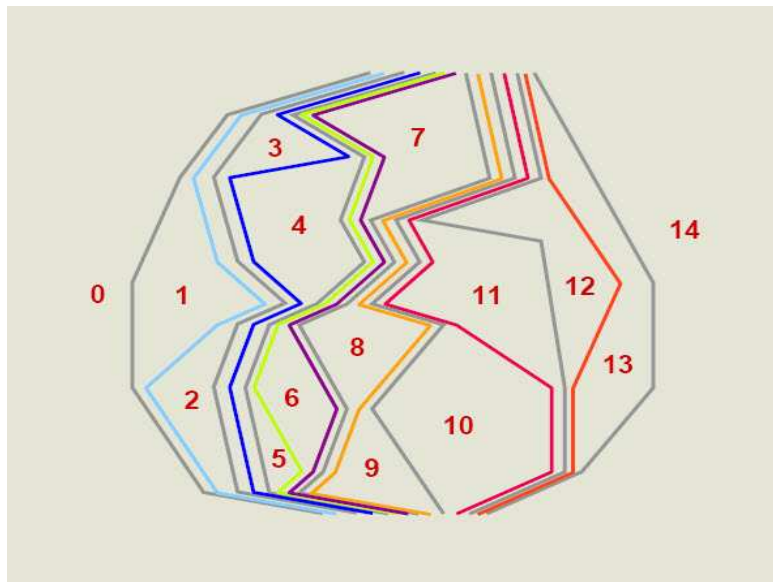
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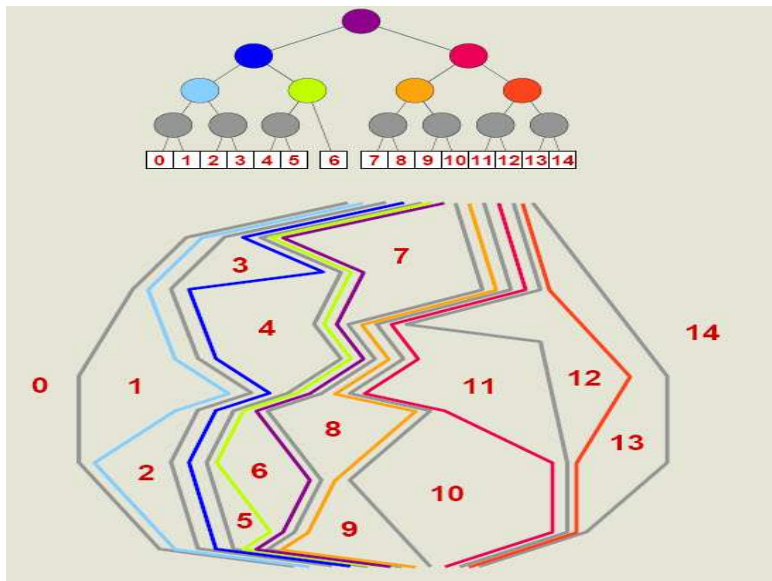
PLANAR POINT LOCATION USING MONOTONE CHAINS



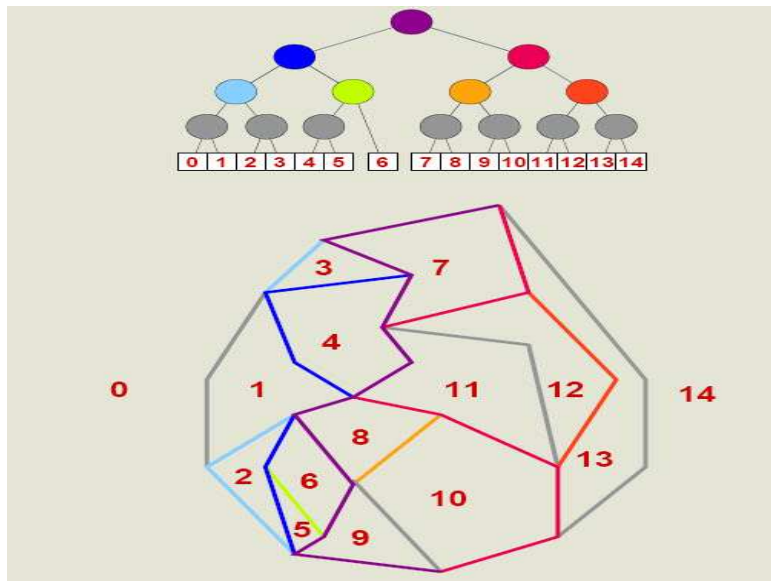
PLANAR POINT LOCATION USING MONOTONE CHAINS



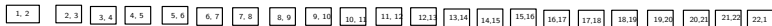
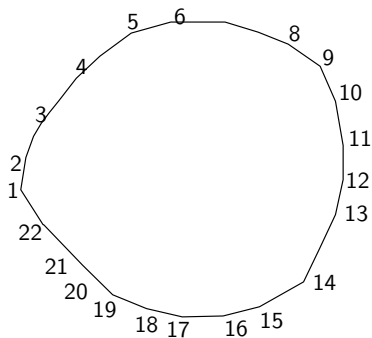
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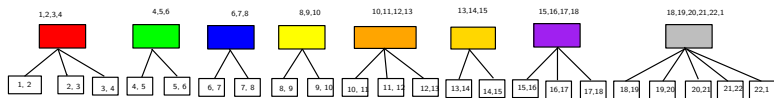
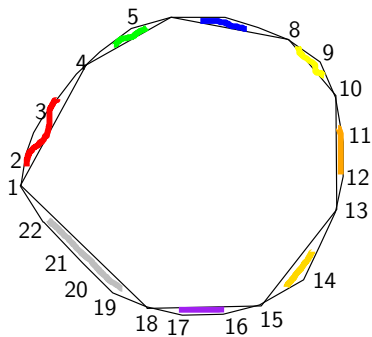
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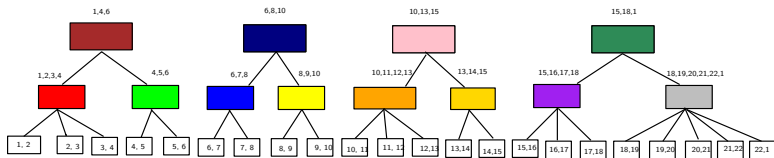
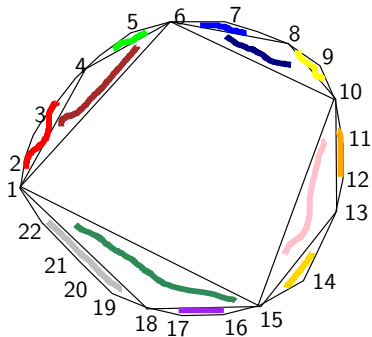
REPRESENTING A CONVEX OBJECT LAYER BY LAYER

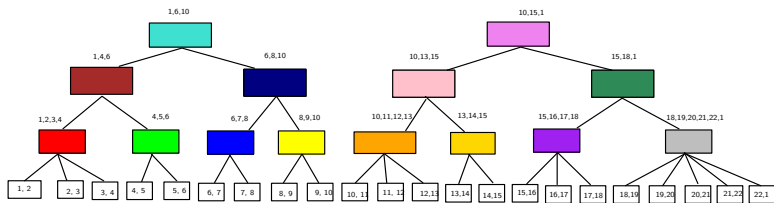
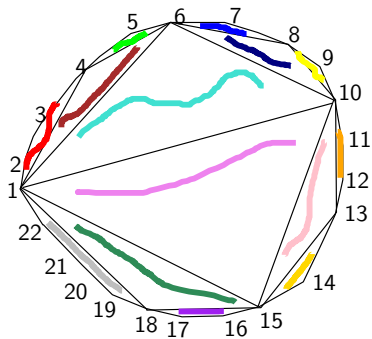


SECOND LAYER

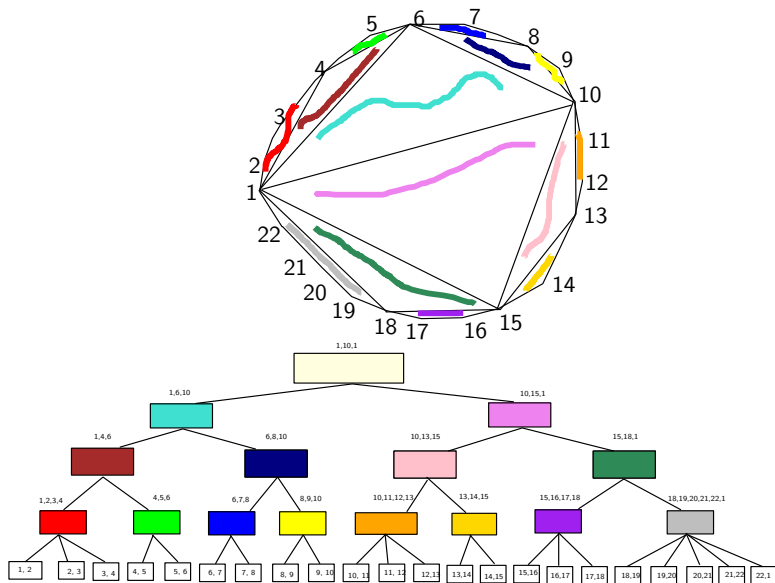






THIRD LAYER





POINT INCLUSION AND LINE INTERSECTION QUERIES



-  Mark de Berg, Otfried Schwarzkopf, Marc van Kreveld and Mark Overmars, Computational Geometry: Algorithms and Applications, Springer.
-  S. K. Ghosh, Visibility Algorithms in the Plane, Cambridge University Press, Cambridge, UK, 2007.
-  Kurt Mehlhorn, Data Structures and Algorithms, Vol. 3, Springer.
-  F. P. Preparata and M. I. Shamos, Computational Geometry: An Introduction, New York, NY, Springer-Verlag, 1985.