Rainbow Coloring of Graphs

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Rainbow Connection Number and Graph Powers.

- If G is a connected graph, then $r(G^k) \leq rc(G) \leq 2.r(G^k) + 1$ for any $k \geq 2$.
- Upper bound is tight up to additive constant of 1.
- Note that $r(G^k) = \left\lceil \frac{r(G)}{k} \right\rceil$

Cartesian Product

- If G and H are two non-trivial connected graphs, then $r(G \times H) \leq rc(G \times H) \leq 2r(G \times H)$.
- Bounds are tight.
- Note that $r(G \times H) = r(G) + r(H)$.

Lexico Graphic Products

- If G and H are two non-trivial graphs such that G is connected, then we have the following:
- If $r(G \times H) \geq 2$ then $r(G \times H) \leq rc(G \times H) \leq 2r(G \times H)$.
- If $r(G \times H) = 1$, then $1 \leq rc(G \times H) \leq 3$.
- Both bounds are tight.

Strong Products

- If G and H are two connected, non-trivial graphs, the $r(G \times H) \leq rc(G \times H) \leq 2.r(G \times H) + 2$.
- The upper bound is tight up to an additive constant 2.
- Note that $r(G \times H) = \max(r(G), r(H))$.

Rainbow Connection Number and Connectivity

Let the vertex connectivity be κ and the edge connectivity be λ. Then

- $rc(G) \leq \frac{3n}{\lambda+1} + 3$
- $rc(G) \leq \frac{3n}{\kappa+1} + 3$

The above result follows from our earlier result: $rc(G) \leq \frac{3n}{\delta+1} + 3$, since $\kappa \leq \lambda \leq \delta$.

Now can we get a better bound in terms of κ and λ ?

In terms λ, we cannot improve much, but in terms of κ we can.

Improvement In terms of vertex connectivity

- We can improve it to $rc(G) \leq (2+\epsilon)(n/\kappa) + 23/\epsilon^2$, for any $\epsilon > 0$.
- We conjecture that $rc(G) \leq n/\kappa(G) + O(1)$

For some Special Cases, we prove that the conjecture is true:

- For $\kappa(G) = 2$:
- For chordal Graphs.
- Graphs of Girth at least 7.