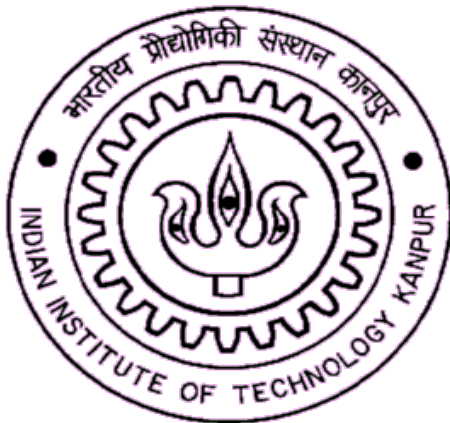


Introduction to Robotics



Amitabha Mukerjee

IIT Kanpur, India

What is a Robot? Mobile Robots

Robot properties:

- Flexibility in Motion
- Mobile robots

daksh ROV: de-mining robot
20 commissioned in Indian
army 2011.

100+ more on order
built by R&D Engineers, Pune

daksh platform derived
gun mounted robot (GMR)

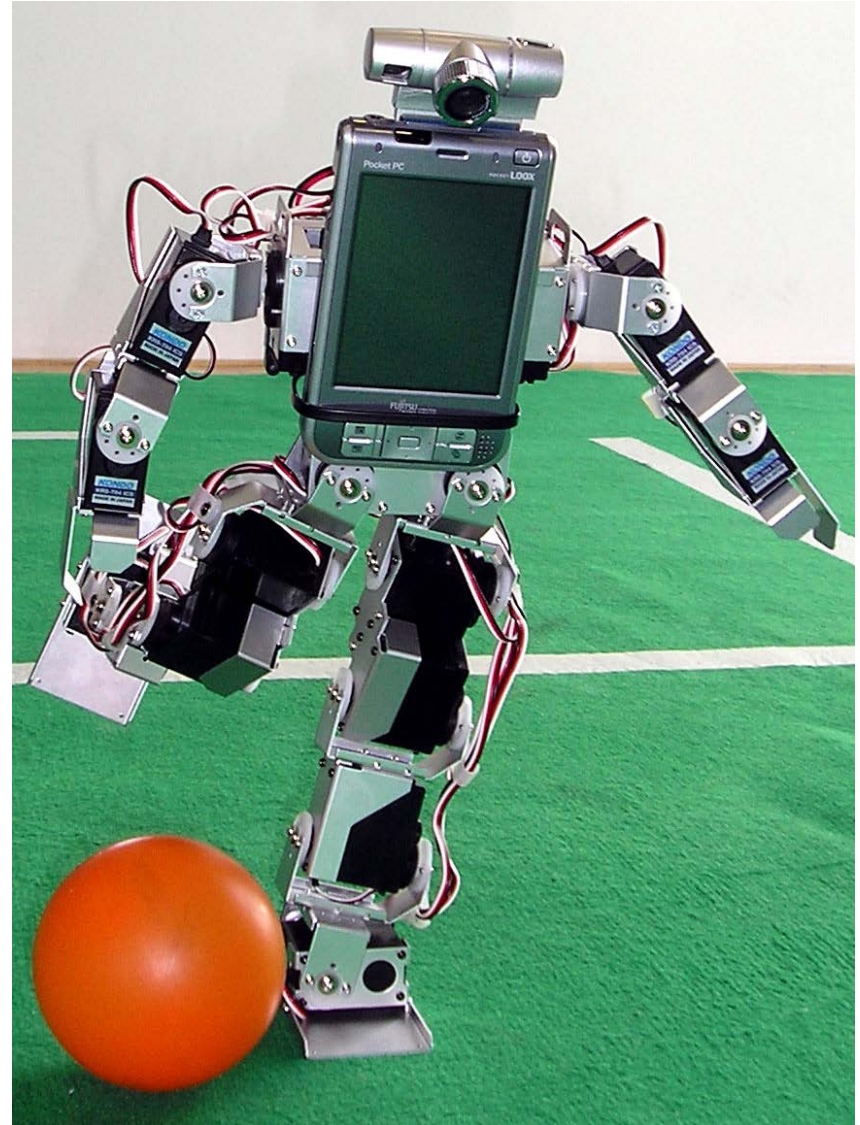


What is a Robot? Articulated Robots

Robot properties:

- Flexibility in Motion
 - Mobile robots
 - Articulated Robots

Soccer playing humanoid robot
[\[http://labintsis.com\]](http://labintsis.com)



Robot you can own



Roomba vacuum
Cleaning robot

By i-robot
Price: ~ rs. 15-30K

Algorithms for Robot motion



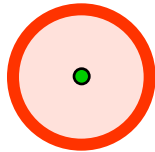
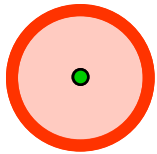
Roomba vacuum
Cleaning robot

By i-robot
Price: ~ rs. 30K

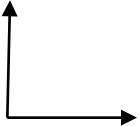
<https://www.youtube.com/watch?v=dweVBqei9L>
Λ

Models of Robot Motion

Circular robot

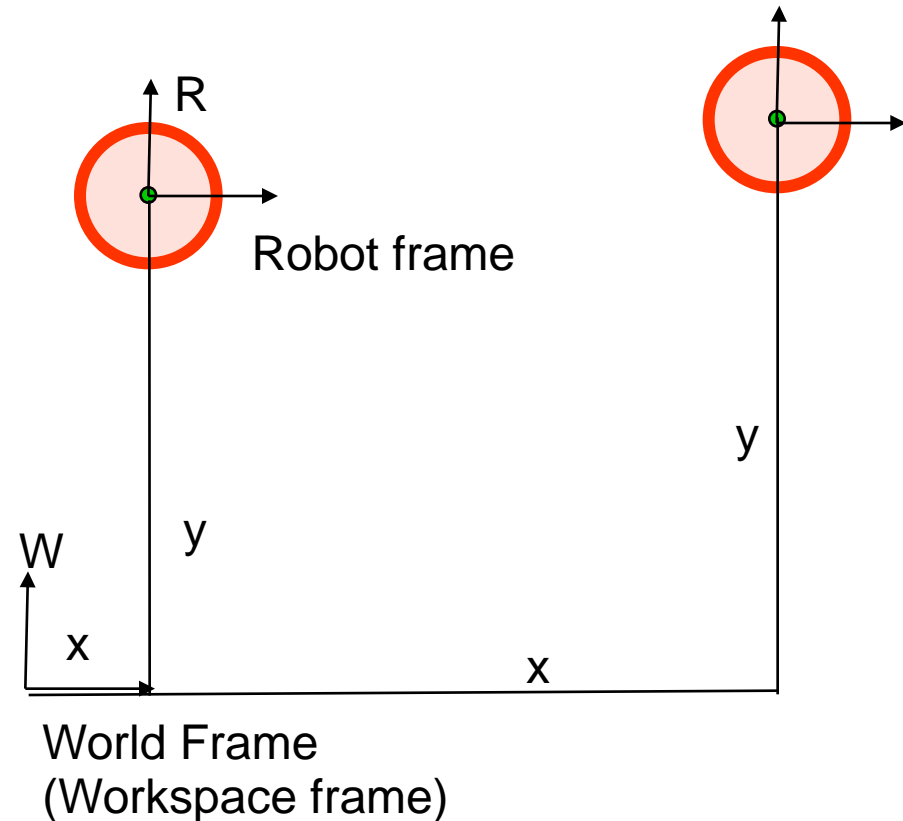


W



World Frame
(Workspace frame)

Models of Robot Motion



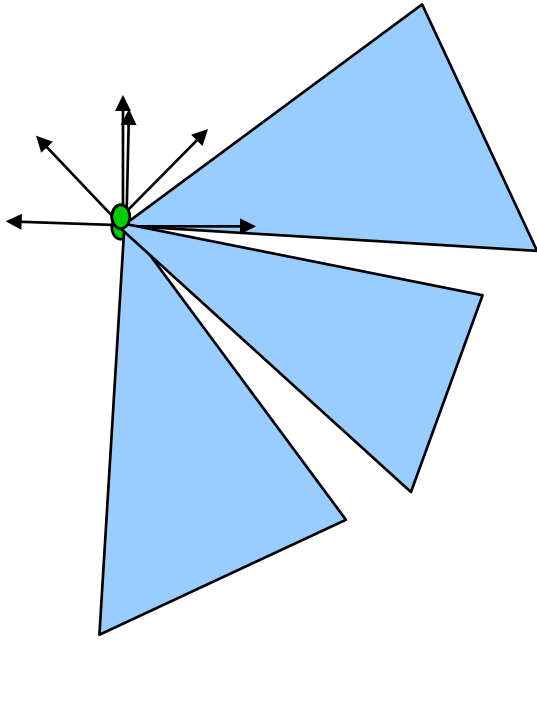
DEFINITION:

degrees of freedom:
NOTE: Given robot frame R, every point on the robot is known
number of parameters needed to fix the robot frame R in the world frame W

$(x, y) =$ **configuration**
(vector \mathbf{q})

given configuration \mathbf{q}
for a certain pose of the robot, the set of points on the robot is a function of the configuration: say $R(\mathbf{q})$

Non-Circular Robot



DEFINITION:

degrees of freedom:

number of parameters needed
to fix the robot frame R
in the world frame W

How many parameters needed to fix
the robot frame if it can only translate?

How many if it can rotate as well?

Motion in 3-D:

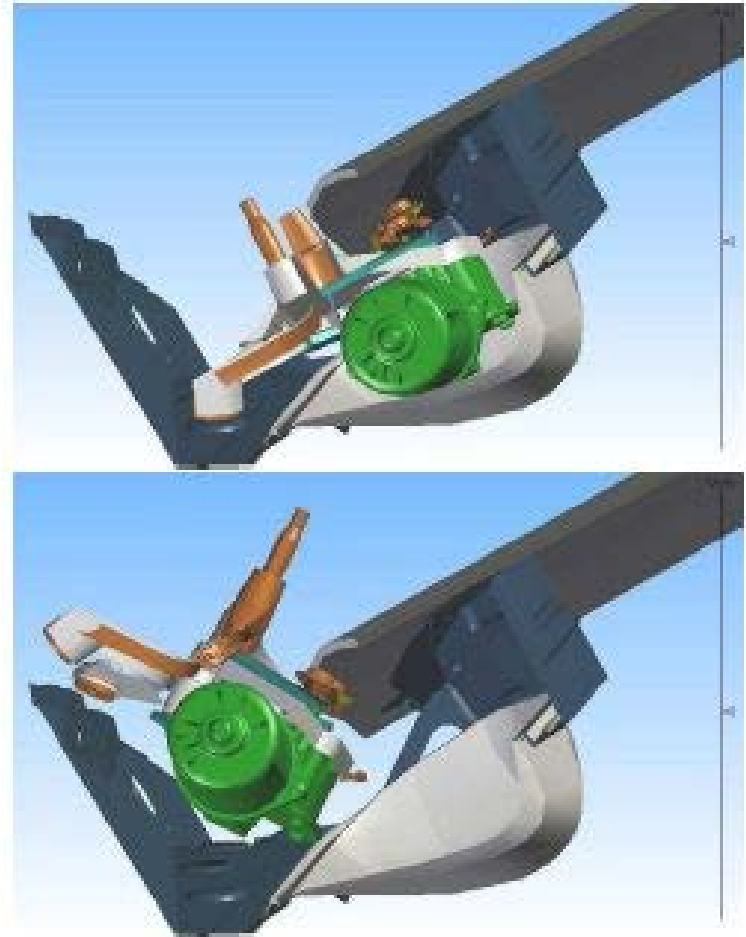
Piano movers problem

General 3D motion:

How many parameters
needed to fix the pose?

Can a design be
assembled?

Test based on CAD models



Research mobile robot



Turtlebot

Based on i-robot (roomba) platform
(with kinect RGB-D sensor)

ROS (open-source) software

Price: ~ 75K

Articulated robots

What is a Robot?

Robots properties:

- Flexibility in Motion
 - Mobile robots
 - Articulated robots

SCARA 4-axis arm
(4 degrees-of-freedom)

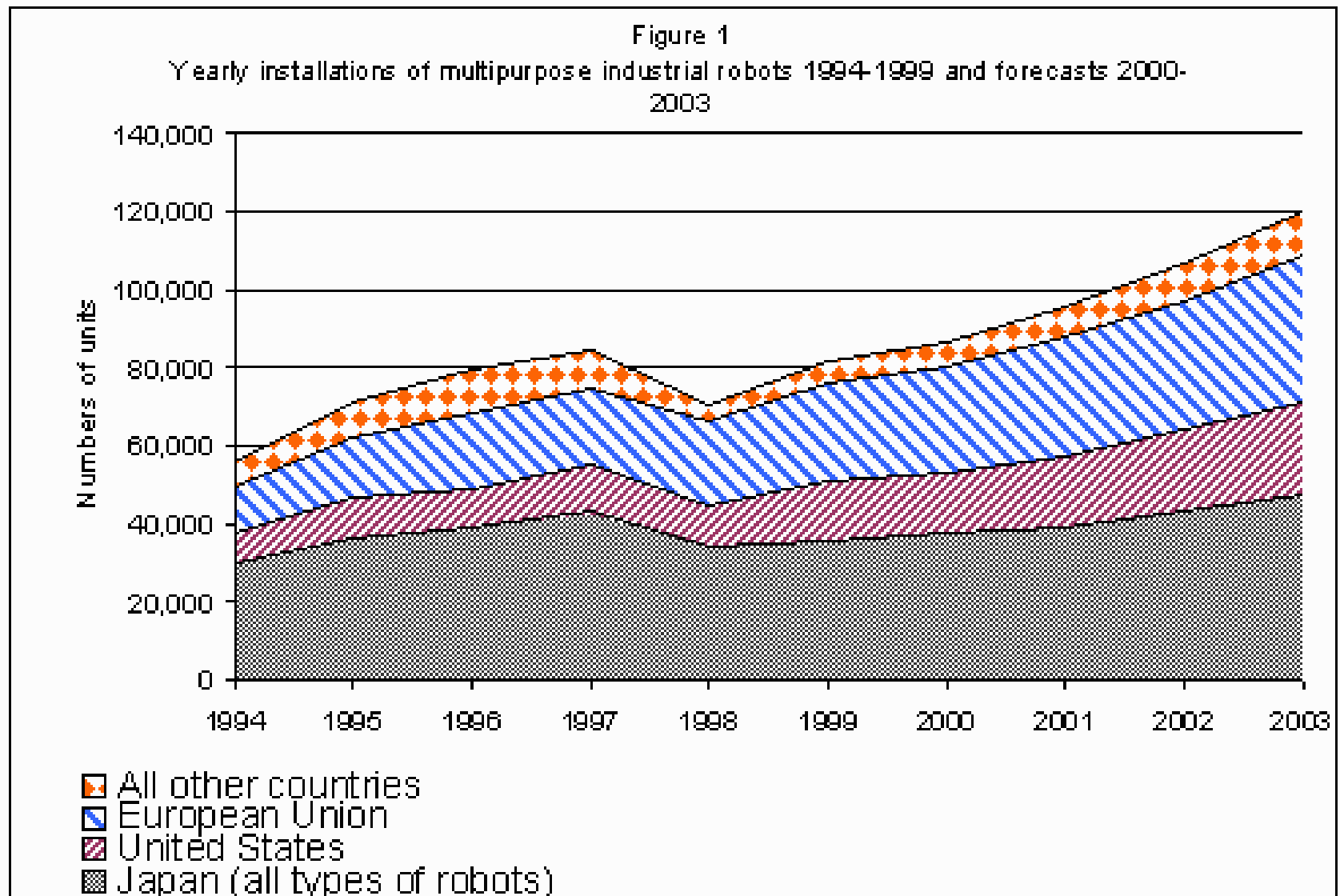
by Systemantics
Bangalore



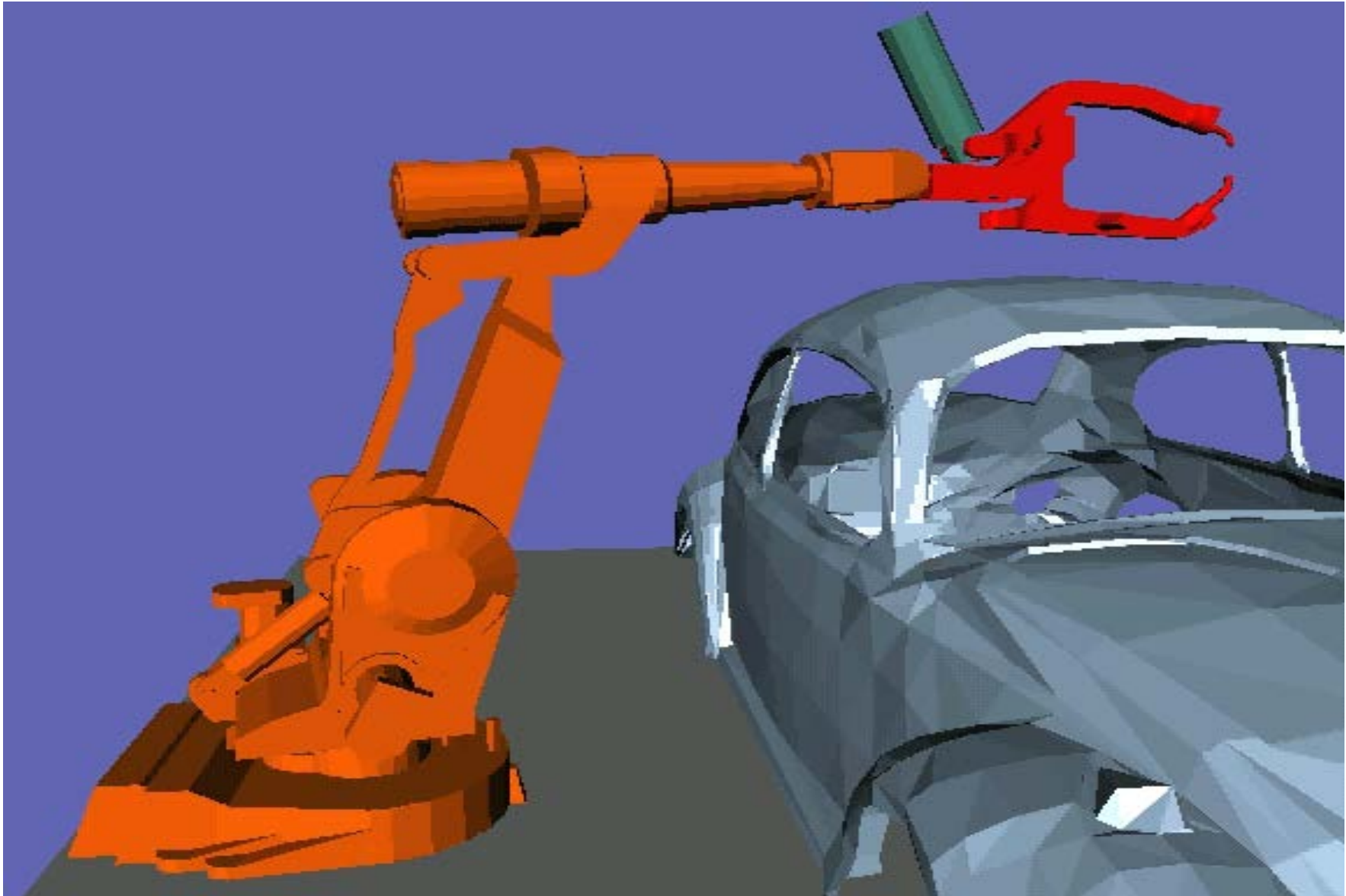
Industrial Robot



Industrial Robots



How to program a welding robot?



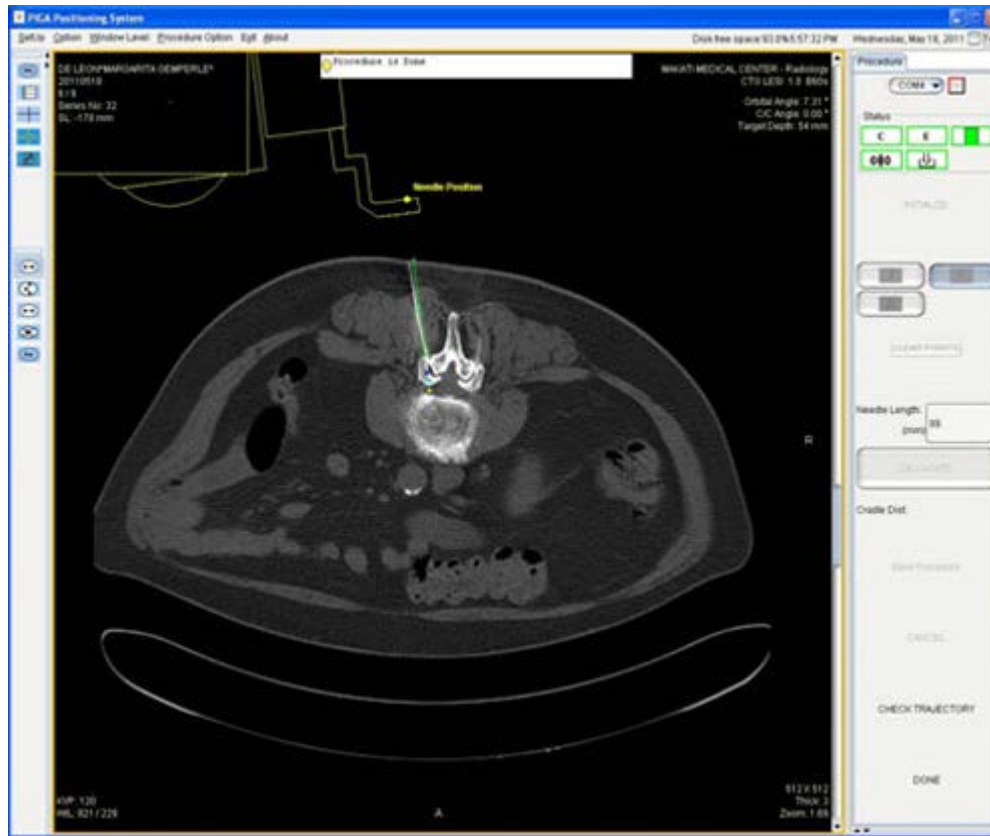
What is a Robot?

Robot properties

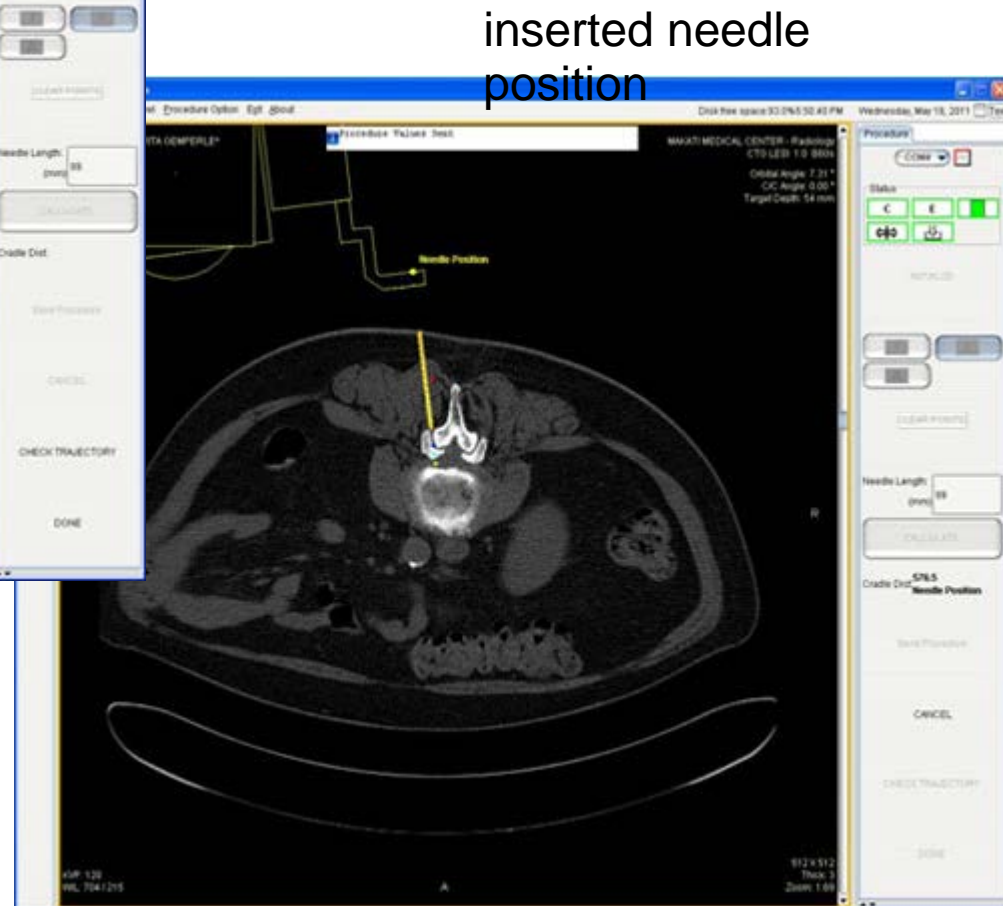
- Flexibility in M
- Mobile robot
- Articulated r
- Industrial r
- Surgical robots



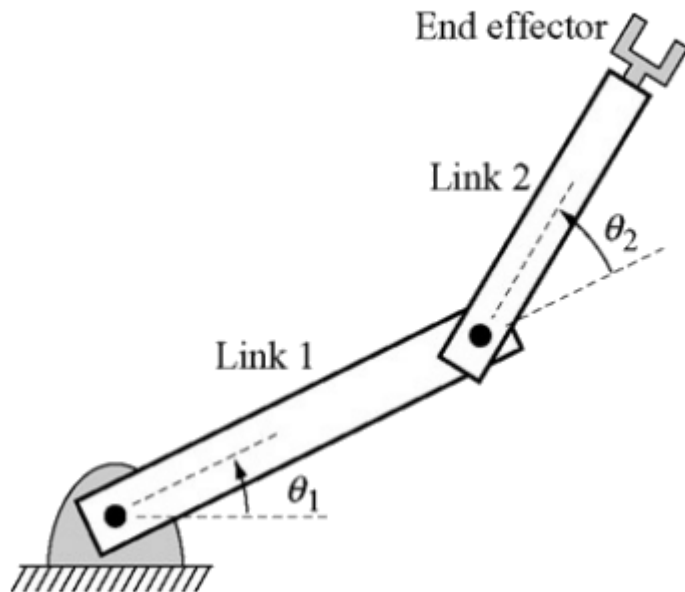
Surgical Robot : Lumbar biopsy



needle path as planned on CAT scan



Modeling Articulated Robots



Kinematic chain:

Pose of Link n depends on the poses of Links $1 \dots (n-1)$

Transformation between frame of link $(n-1)$ and link n , depends on a single motion parameter, say θ_n

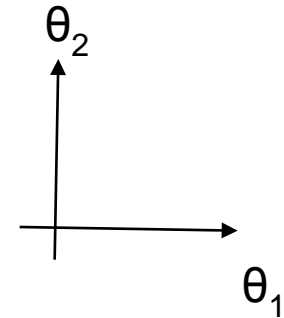
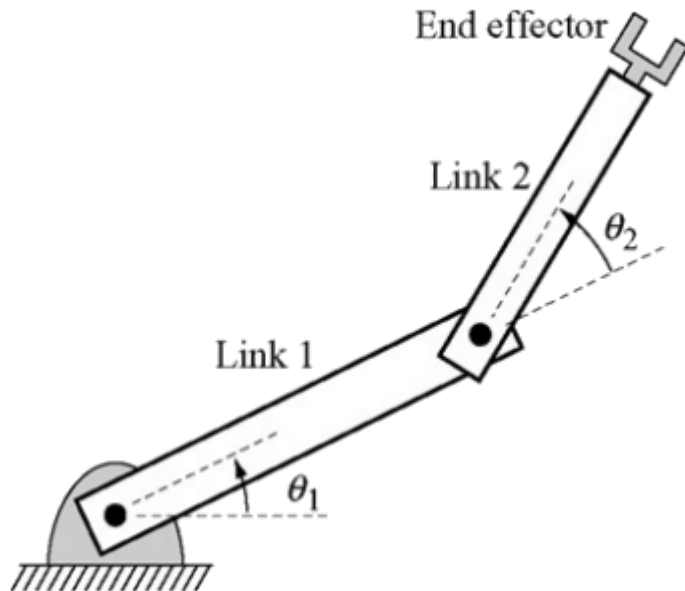
Exercise:

What are the coordinates of the origin of the end-effector center?

Modeling Articulated Robots

workspace

configuration
space



Exercise:

Sketch the robot pose for the configuration $[0, -90]$

Modeling Articulated Robots

Forward kinematics

Mapping from configuration \mathbf{q} to robot pose, i.e. $R(\mathbf{q})$

Usually, $R()$ is the product of a sequence of transformations from frame i to frame $i+1$.

Note: Must be very systematic in how frames are attached to each link

Inverse kinematics

a. Given robot pose, find \mathbf{q}

Or

b. Given end-effector pose, find \mathbf{q}

Q. Is the answer in (b) unique?

What is a Robot?

Robots properties

- Flexibility in Motion
 - Mobile robots
 - Articulated robots
 - Digital actors



Reality: limited functionality



Mobility isnt everything

What is a Robot?

Robots involve

- Flexibility in Motion
 - Dentists cradle?
 - Washing machine?
- Intentionality
 - Measure : not default probability distribution
 - e.g. Turn-taking (contingent behaviour)
 - Goal : intrinsic or extrinsic

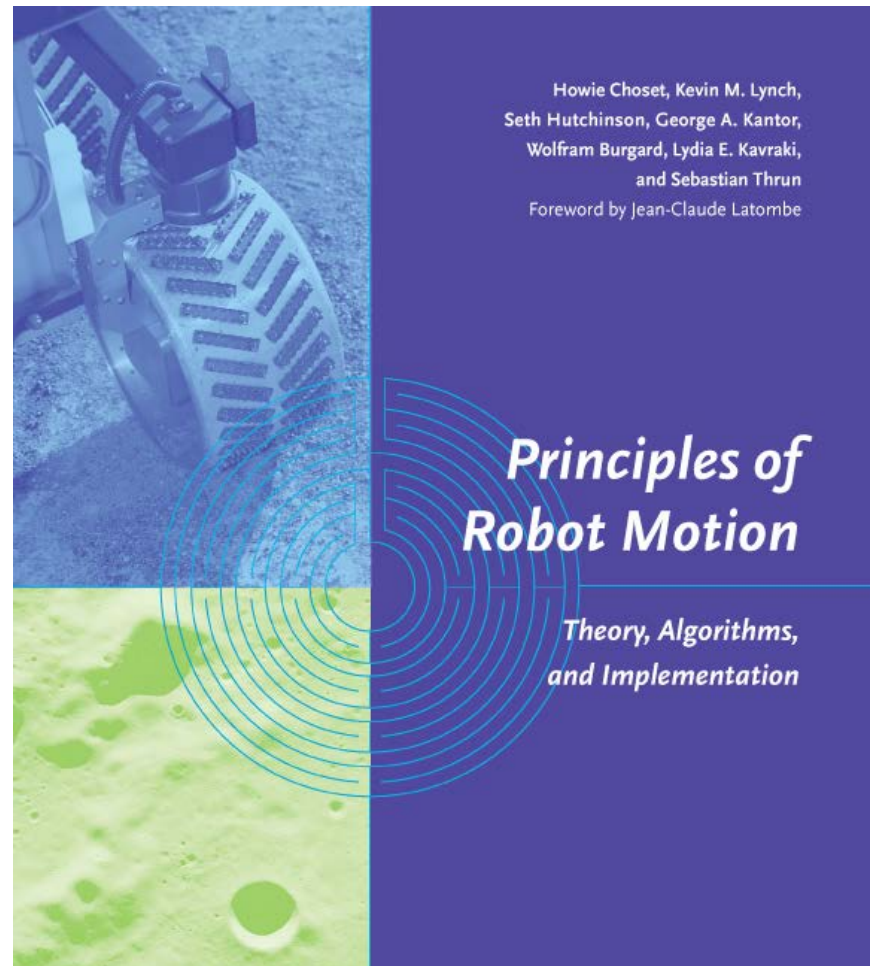
Robot Motion Planning



Amitabha Mukerjee

IIT Kanpur, India

indian edition
rs 425

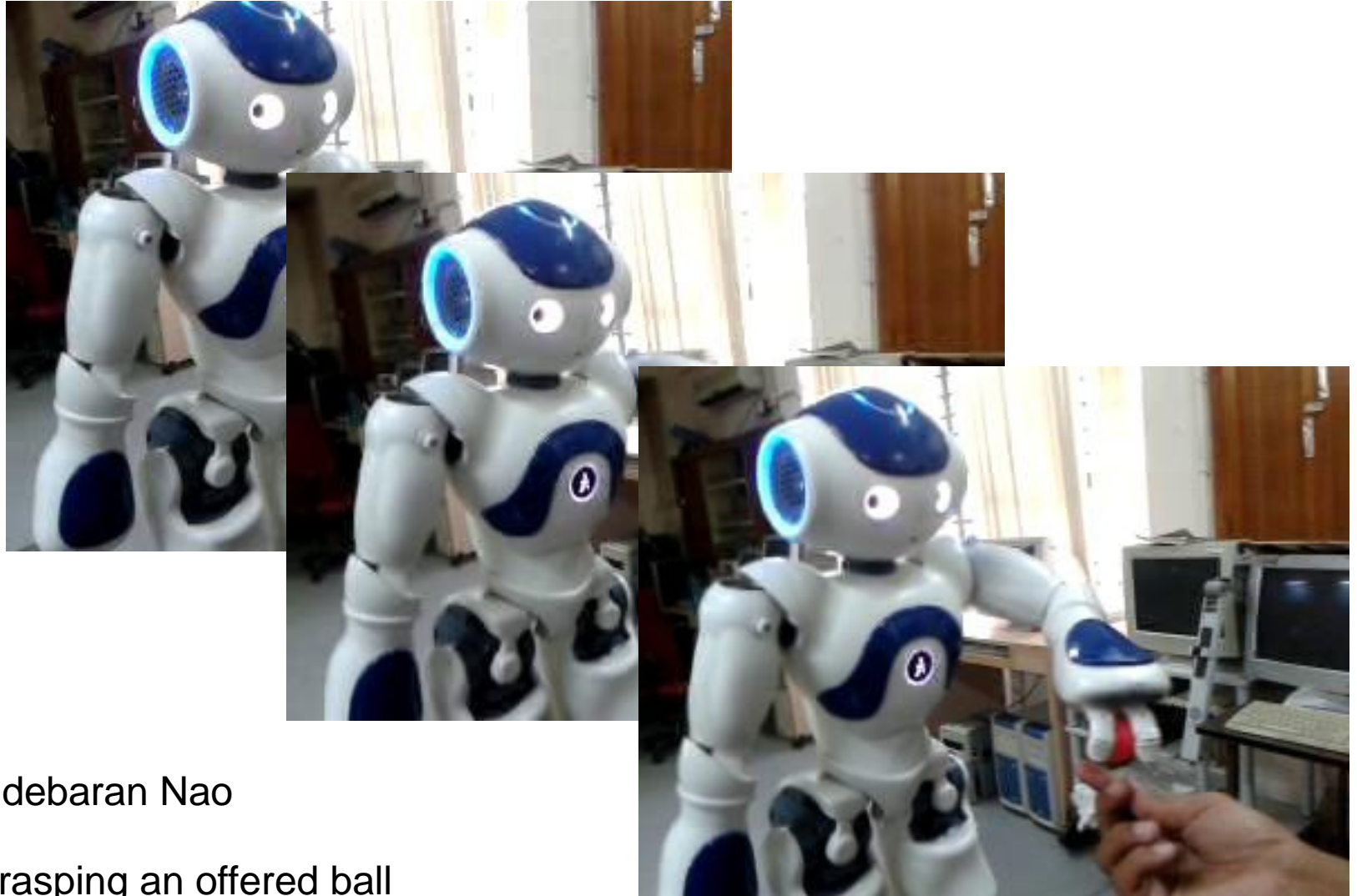


Sensing and Motion Planning



[bohori venkatesh singh mukerjee 05]
Bohori/Venkatesh/Singh/Mukerjee:2005

Programming a robot



Aldebaran Nao

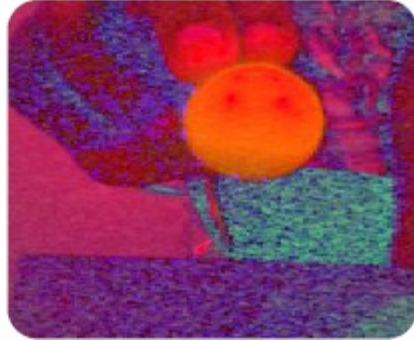
Grasping an offered ball

Programming a robot

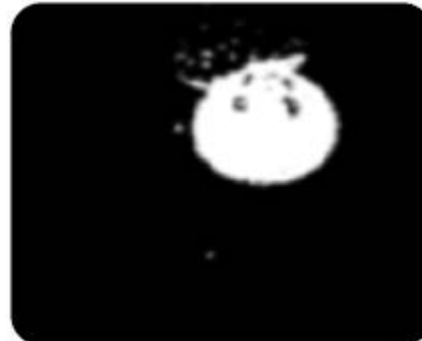
1. detect ball using colour:



image captured by nao



HSV



binarized



contour detected

2. estimate distance of ball (depth)
from image size

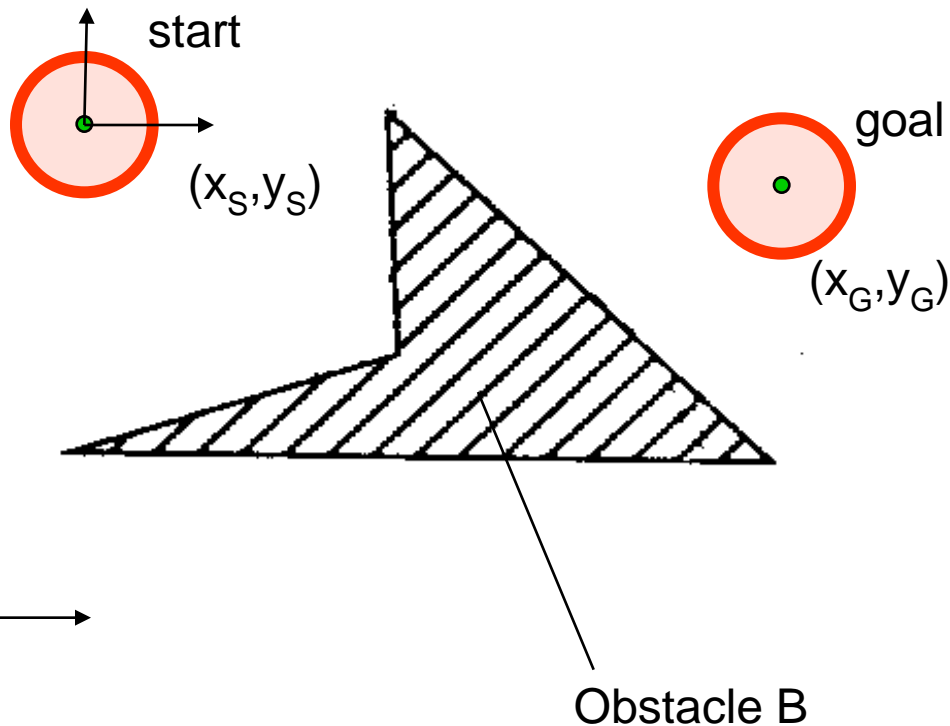
3. Inverse kinematics to grasp ball

Sensing in the workspace

Motion planning in C-space

Configuration Space

Robot Motion Planning



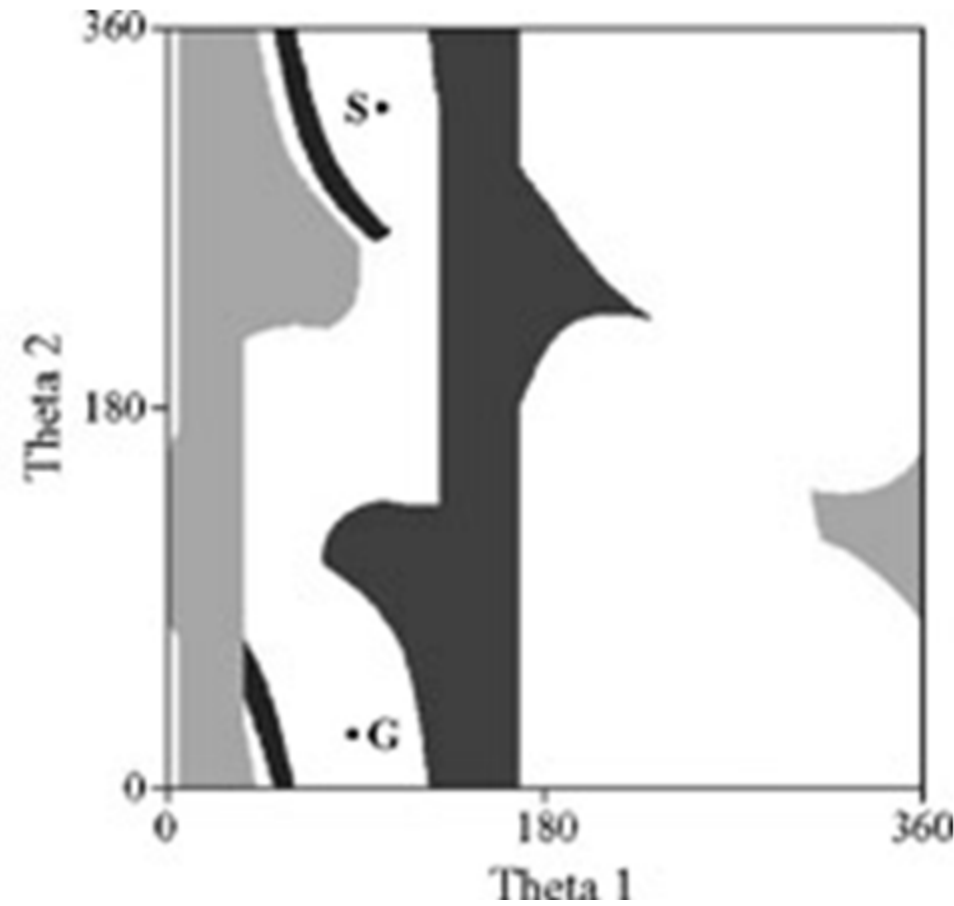
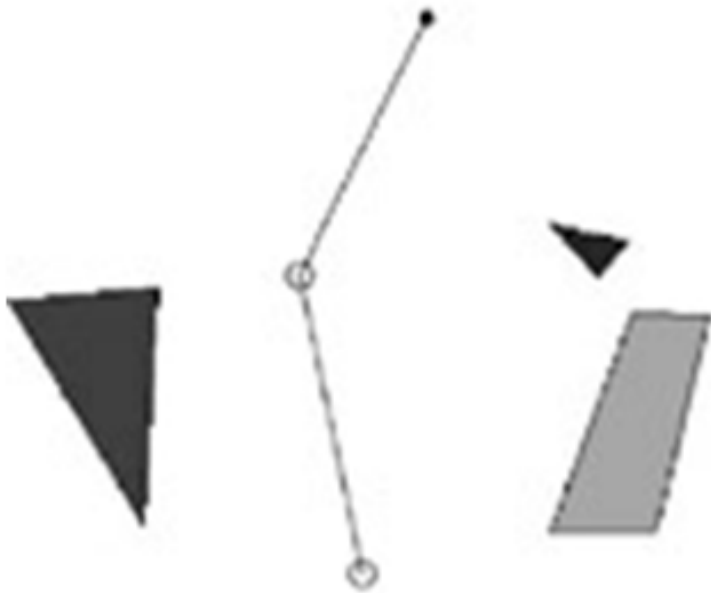
Valid paths will lie among those where the robot does not hit the obstacle

find path P from start to goal s.t.

$$\text{for all } t, R(t) \cap B = \emptyset$$

How to characterize the set of poses for which the robot does not hit the obstacle B ?

Robot Motion Planning

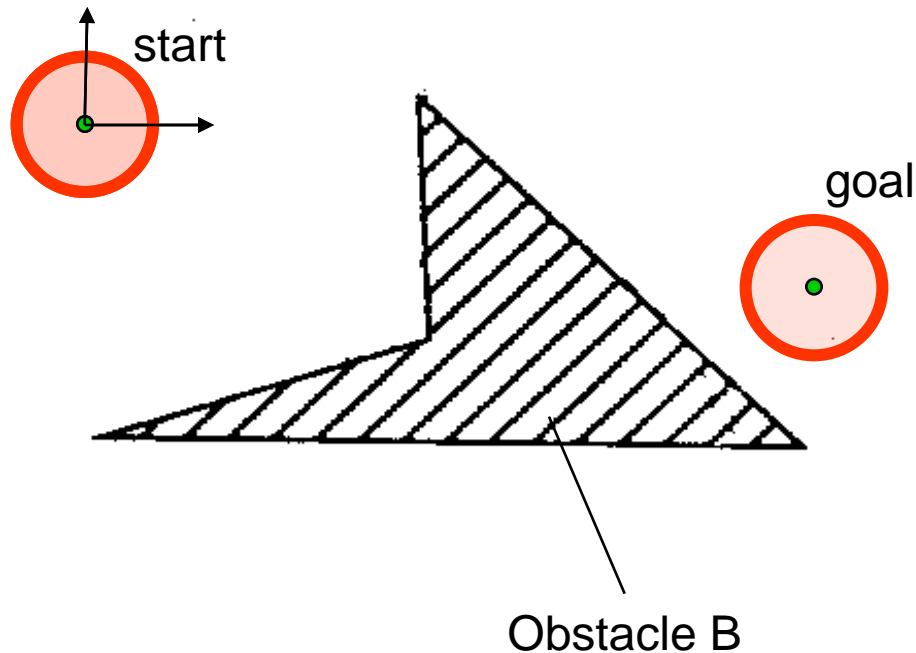


Continuum approaches vs Discretization

Two approaches to Robot motion planning:

- **continuum:**
treat motion space as single continuum
→ optimization
- **discretization:**
decompose motion space into regions / segments
→ graph-search

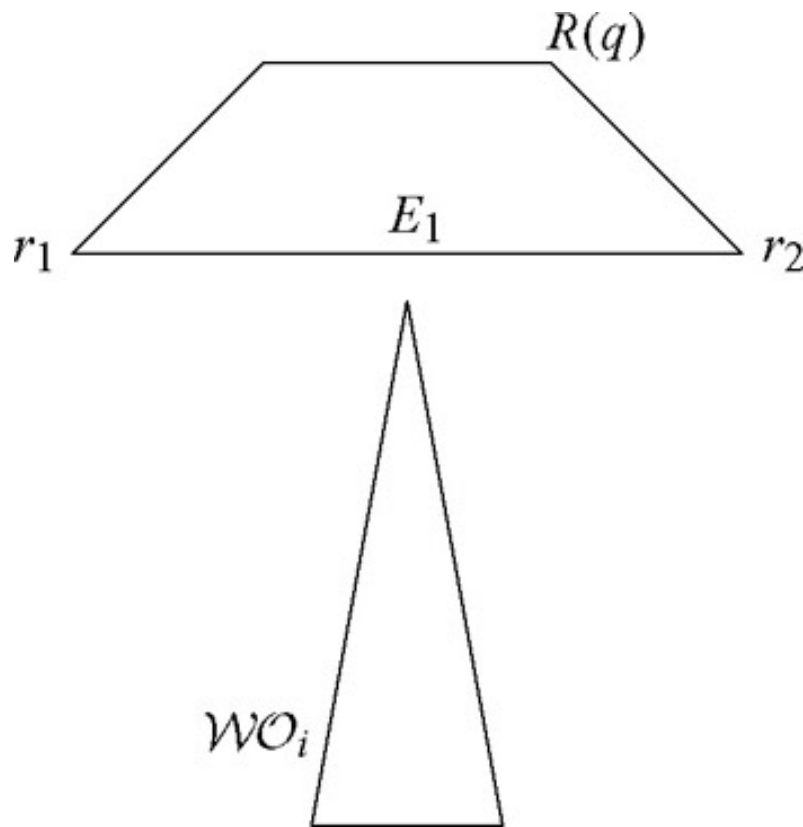
Potential fields



Potential fields

1. Goal: negative (attractive) potential
Obstacles: positive (repulsive) potential
2. Robot moves along gradient
3. Problems:
 - need to integrate the potential over the area of robot
 - problem of local minima

Finite area robots



Instead of integrating over robot area, restrict to a set of *control* points

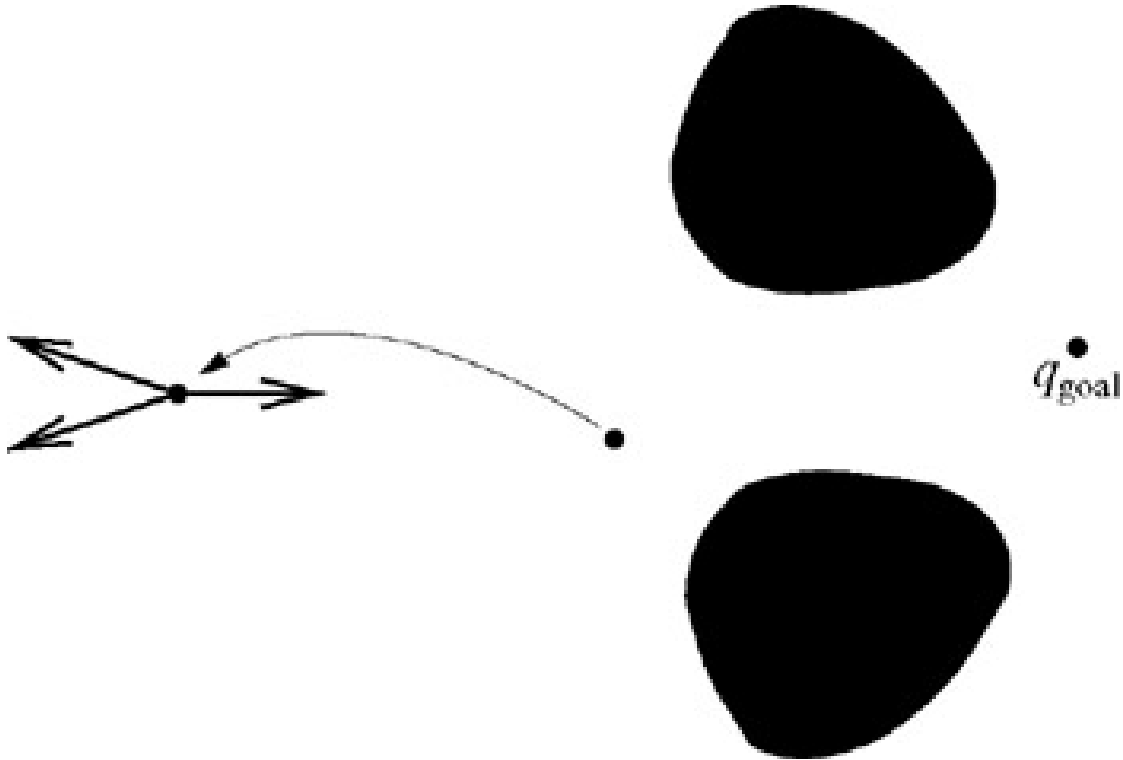
e.g. vertices

Problem:

With control points r_1 and r_2 on robot $R(q)$, edge E_1 may still hit Obstacle.

→ Attempt to reduce computation to points

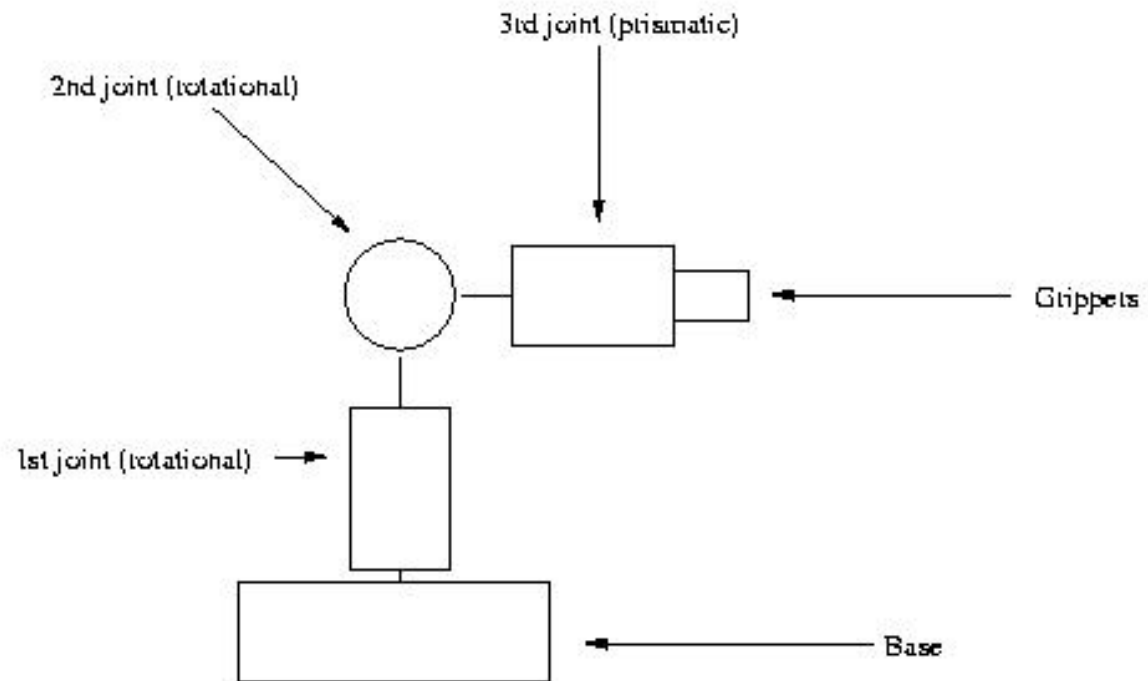
Local Minima



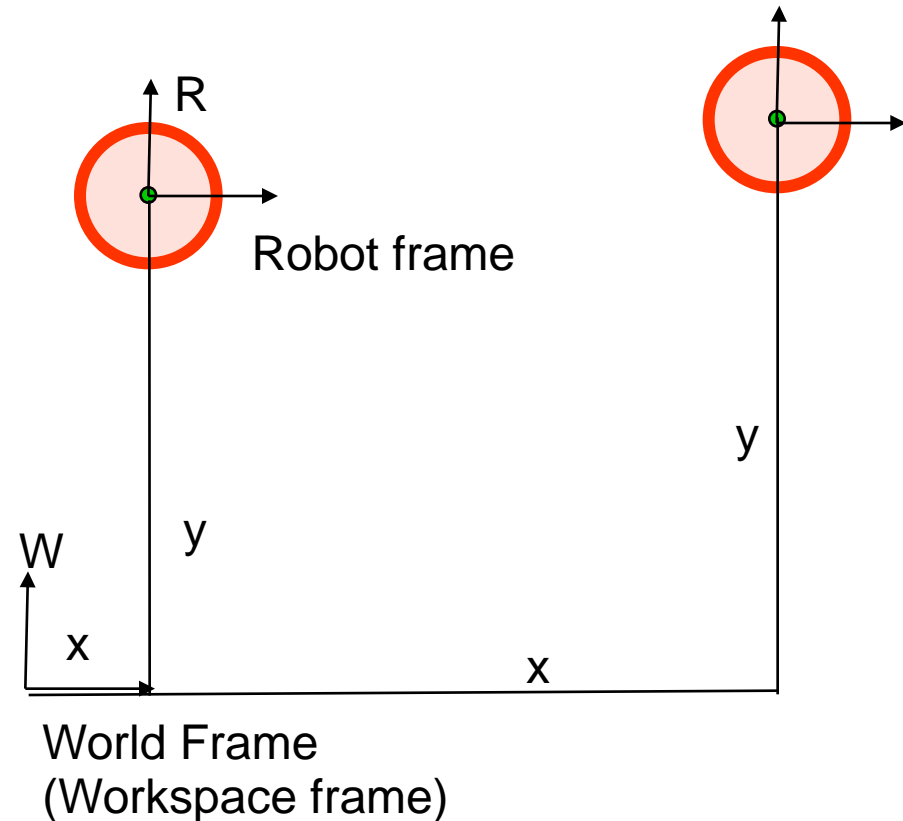
persists even for point robots

Nature of Configuration spaces

Robot Model



Models of Robot Motion



DEFINITION:

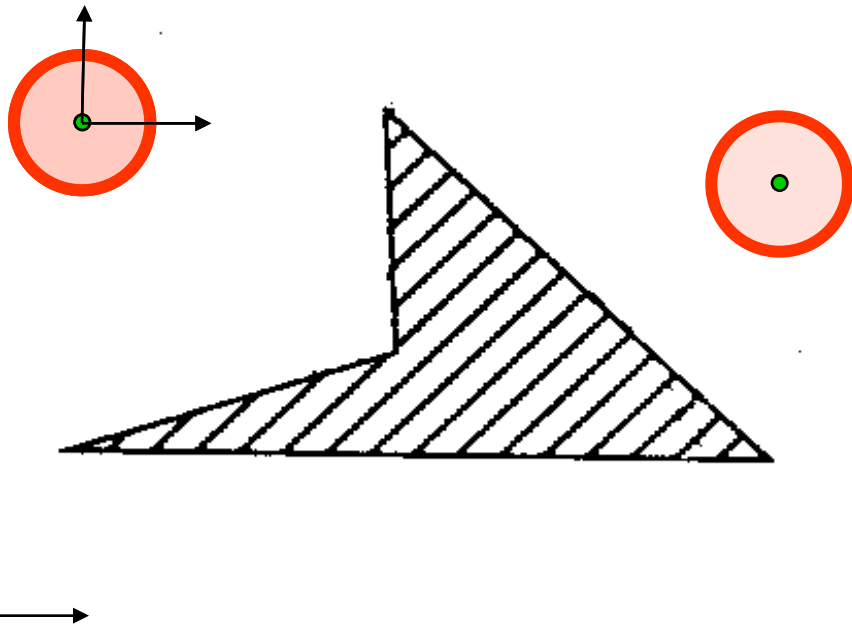
degrees of freedom:

number of parameters needed
to fix the robot frame R
in the world frame W

$(x, y) = \text{configuration}$
(vector \mathbf{q})

given configuration \mathbf{q}
for a certain pose of the
robot, the set of points on
the robot is a function of the
configuration: say $R(\mathbf{q})$

Robot Motion Planning

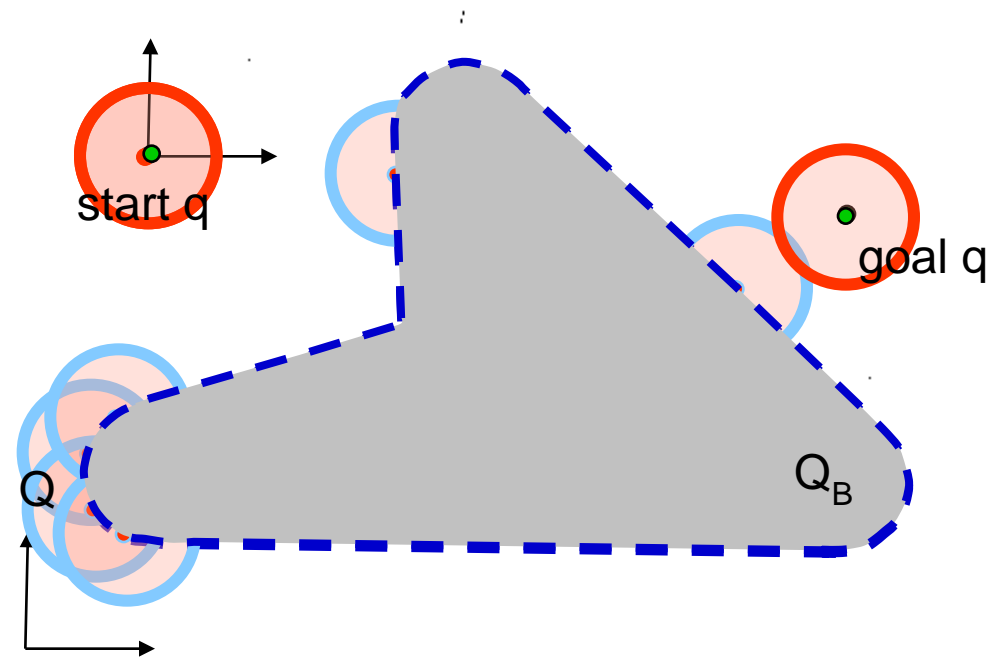


find path P from \mathbf{q}_S to \mathbf{q}_G s.t. for all $\mathbf{q} \in P$, $R(\mathbf{q}) \cap B = \emptyset$

? generate paths and check each point on every path?

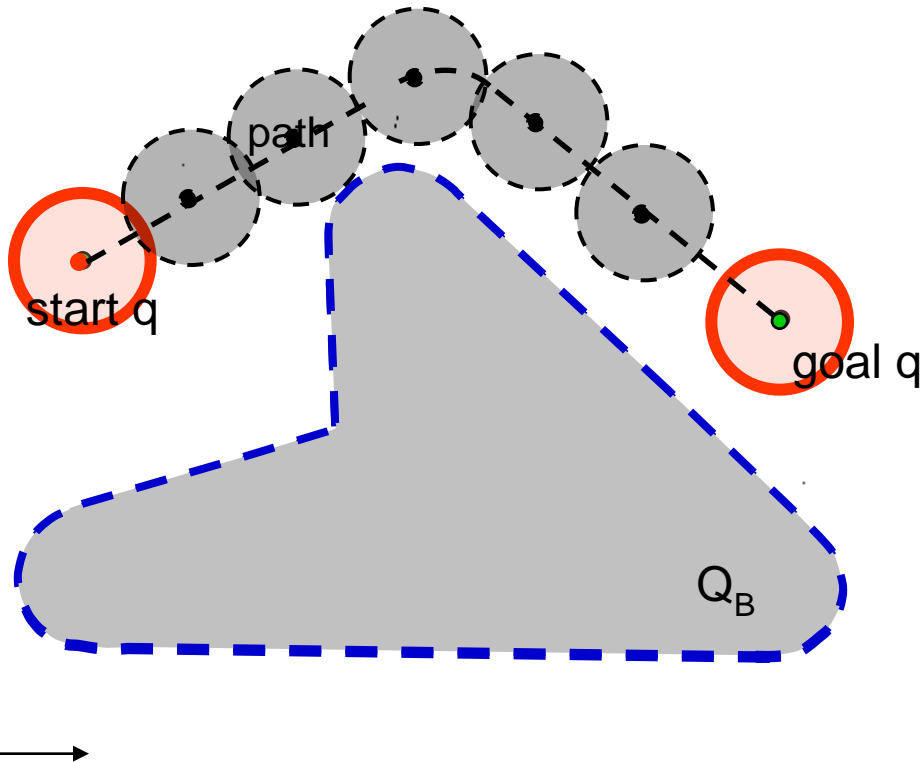
Would it be easier to identify Q_{free} first?

Robot Motion Planning



$$Q_B = [\mathbf{q} \mid R(\mathbf{q}) \cap B \neq \emptyset]$$

Motion Planning in C-space

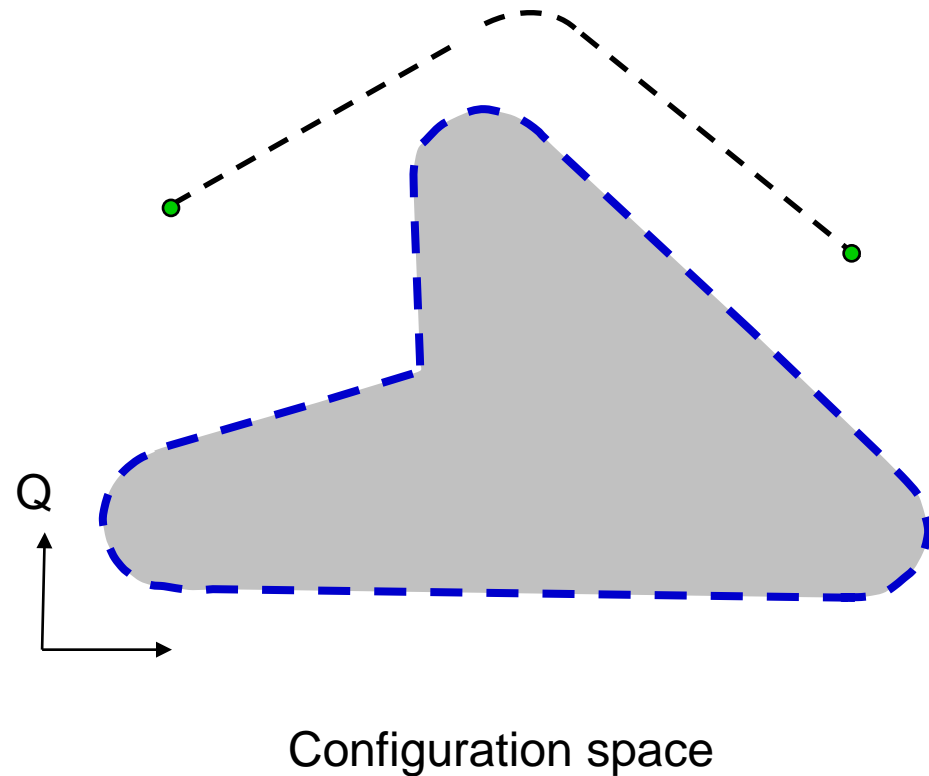
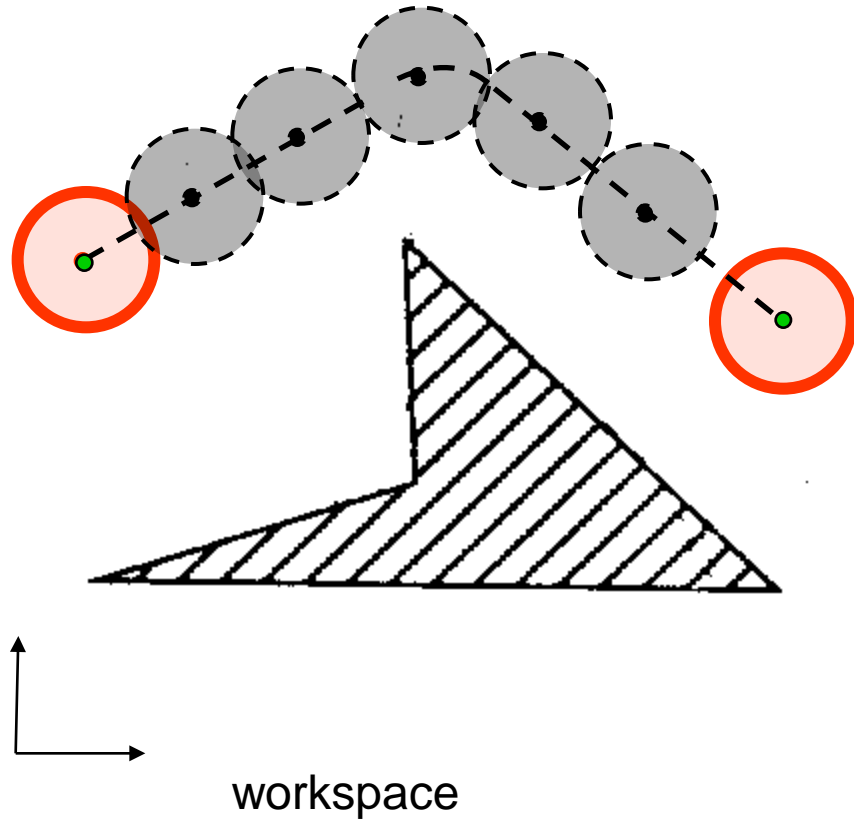


configurations are points in C-space

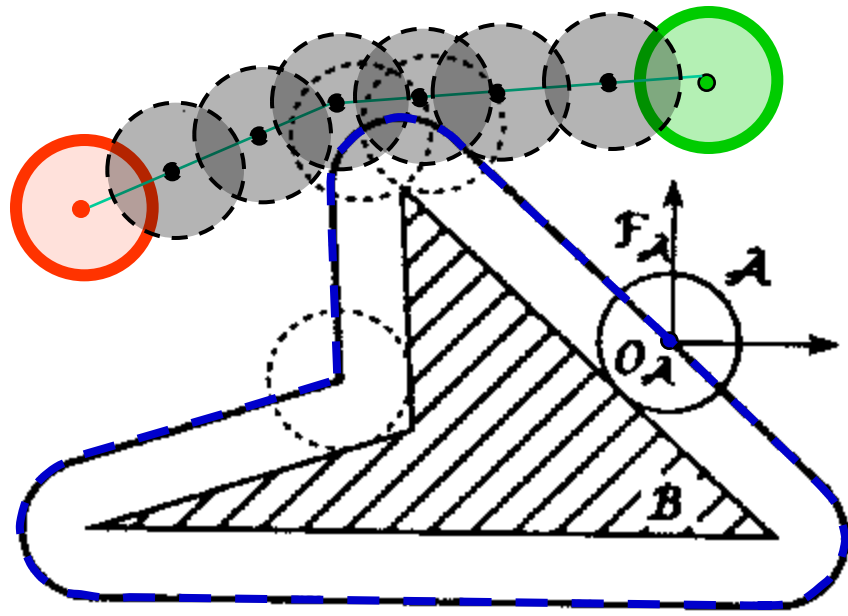
path P is a line

if $P \cap Q_B = \emptyset$, then path is in Q_{free}

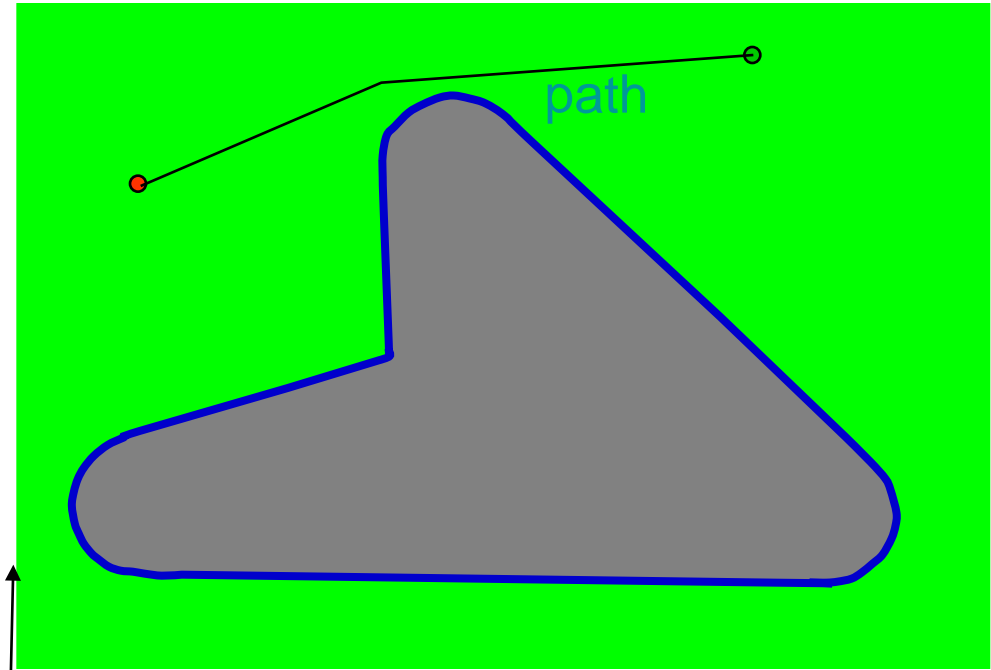
Motion Planning in C-space



Robot Motion Planning



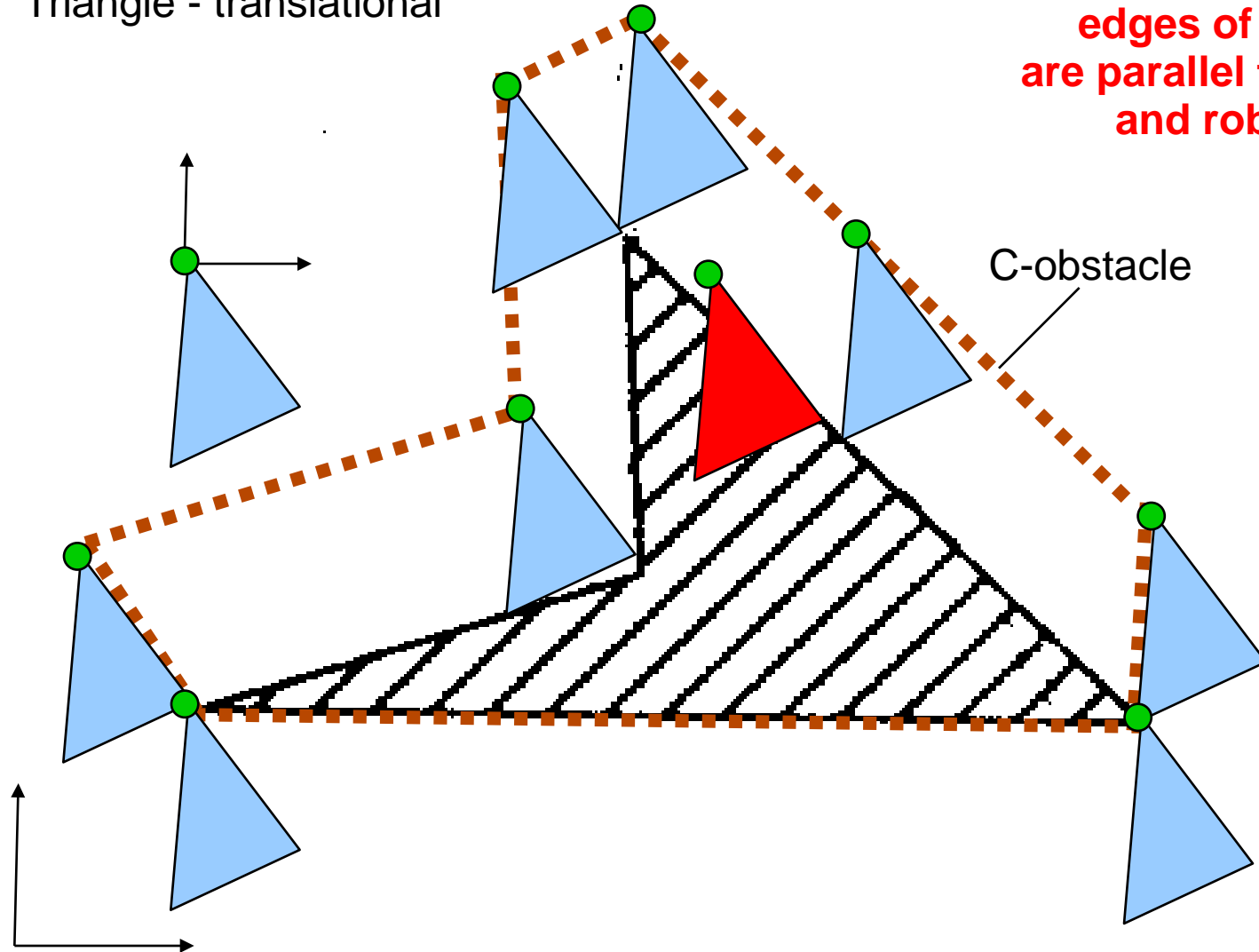
workspace
 W



configuration space
 C

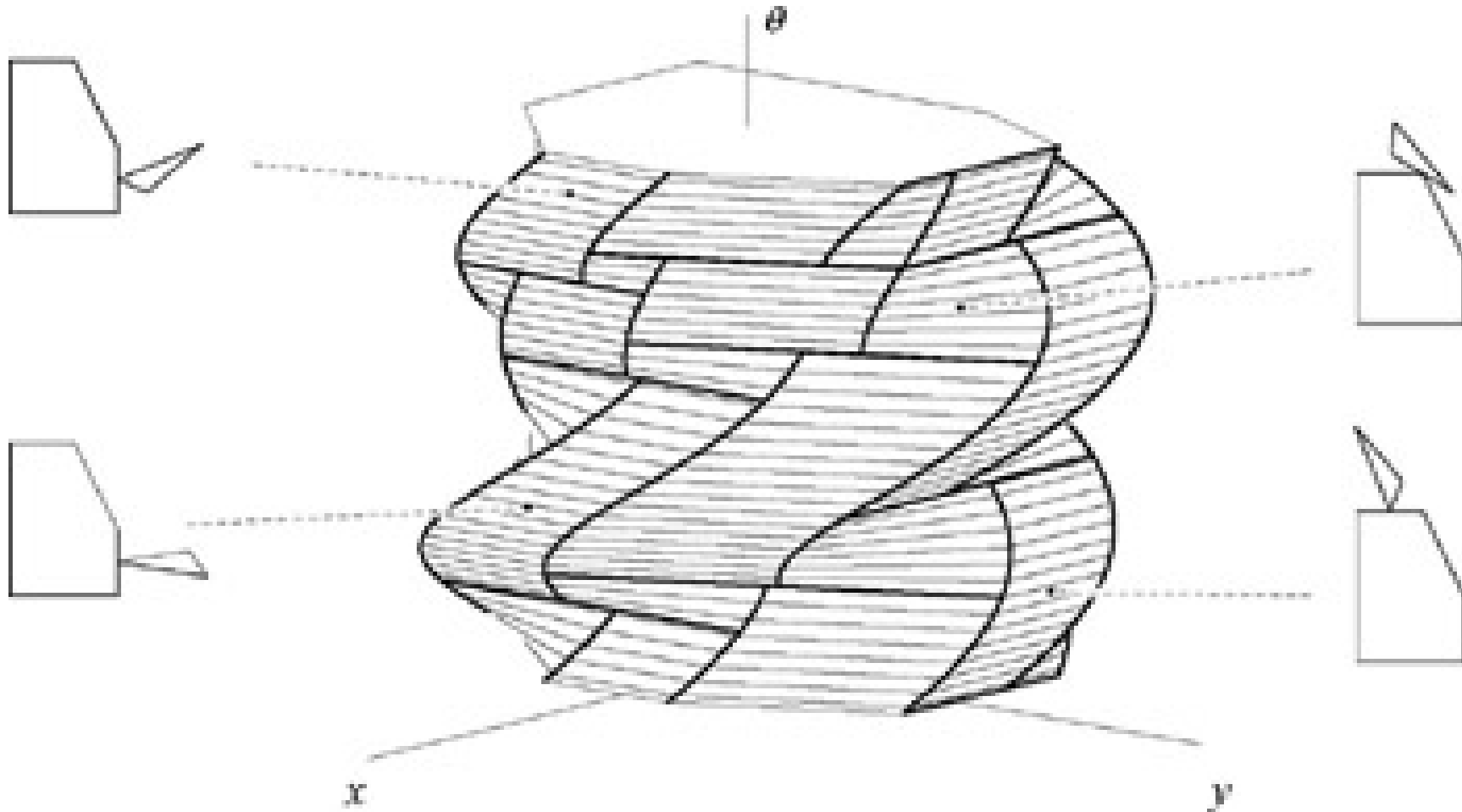
Non-circular mobile robots

Triangle - translational

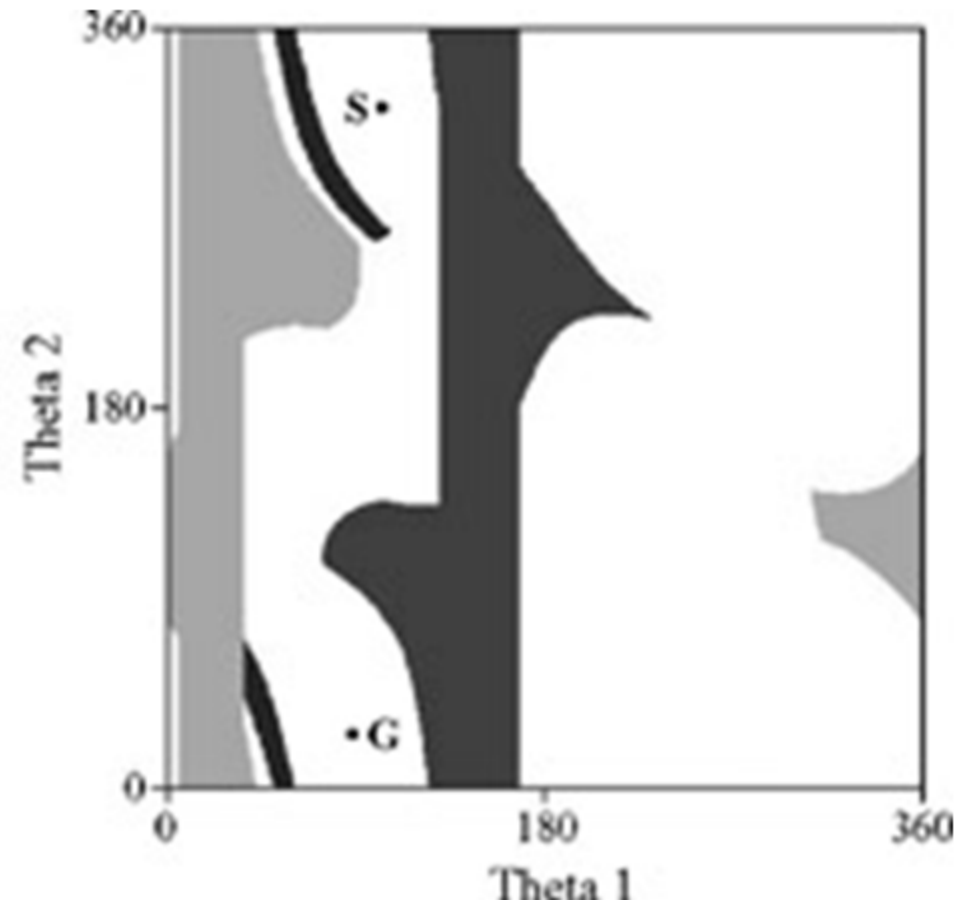
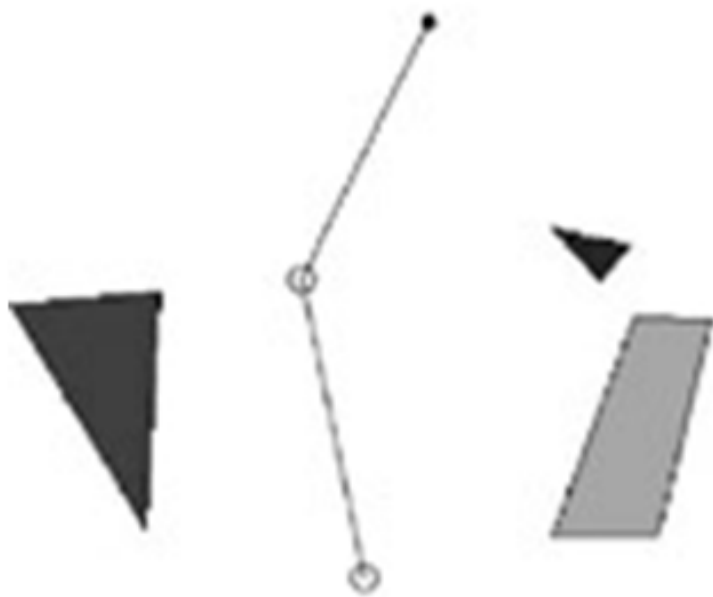


Non-circular mobile robots

C-space with rotation θ (polygonal obstacle)



Configuration Space for Articulated Robots

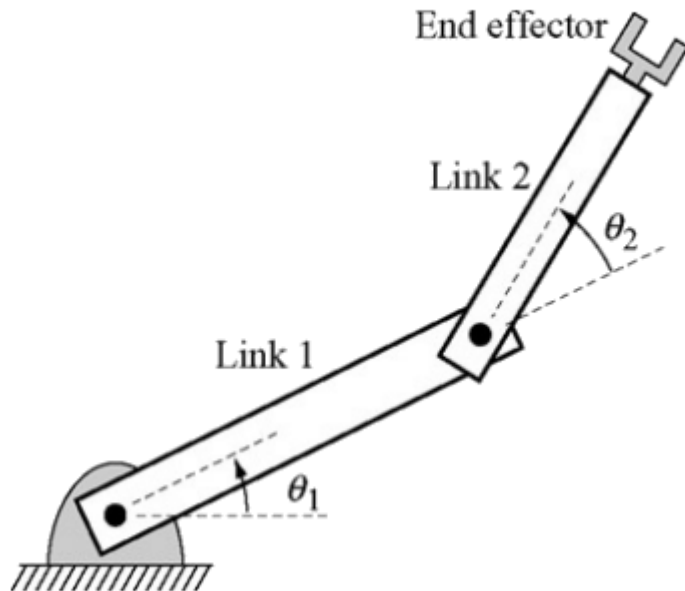


Configuration Space Analysis

Basic steps (for ANY constrained motion system):

1. determine degrees of freedom (DOF)
2. assign a set of configuration parameters \mathbf{q}
e.g. for mobile robots, fix a frame on the robot
3. identify the mapping $R : Q \rightarrow W$, i.e. $R(\mathbf{q})$ is the set of points occupied by the robot in configuration \mathbf{q}
4. For any \mathbf{q} and given obstacle B , can determine if $R(\mathbf{q}) \cap B = \emptyset$. \rightarrow can identify Q_{free}
Main benefit: The search can be done for a point
5. However, computation of C-spaces is not needed in practice; primarily a conceptual tool.

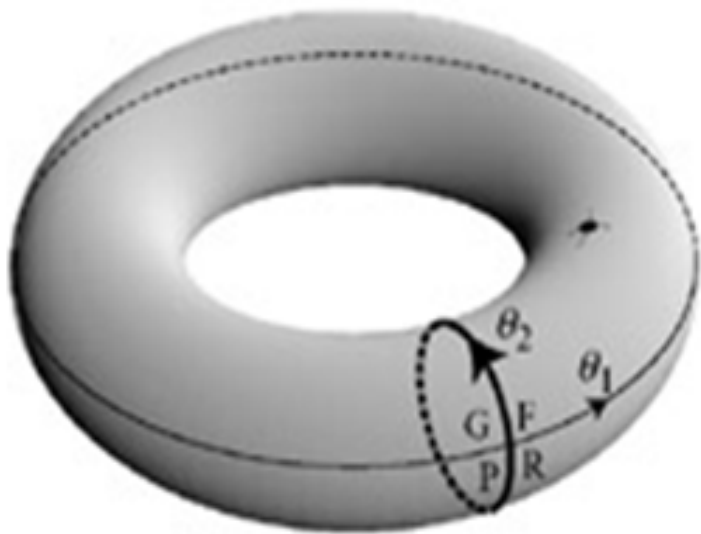
Articulated Robot C-space



How many parameters needed to fix the robot pose ?

What may be one assignment for the configuration parameters?

C-space as manifolds



Topology of C-space: Torus ($S^1 \times S^1$)

Choset, H et al 2007, Principles of robot motion: Theory, algorithms, and implementations, chapter 3

C-space as manifolds

- **manifold:** generalization of curves / surfaces

every point on manifold has a neighbourhood homeomorphic to an open set in \mathbb{R}^n

- Mapping $\Phi : S \leftrightarrow T$ is bijective (covers all of T and has unique inverse)

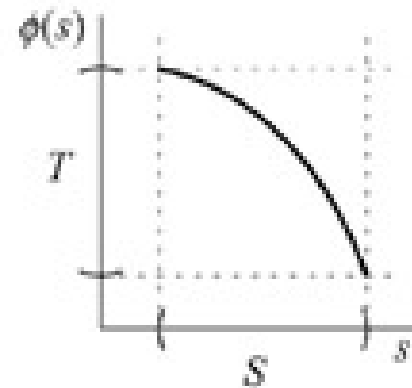
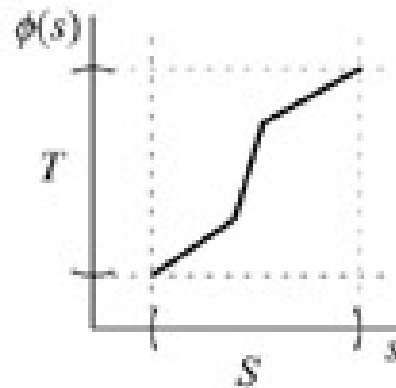
Φ is

homeomorphic:

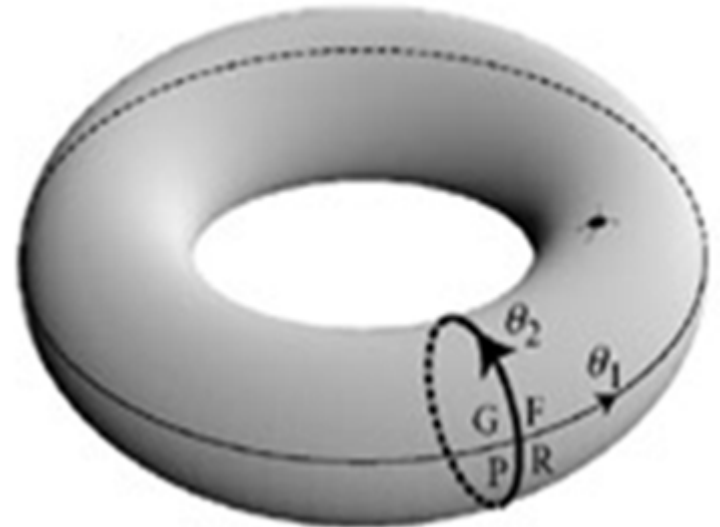
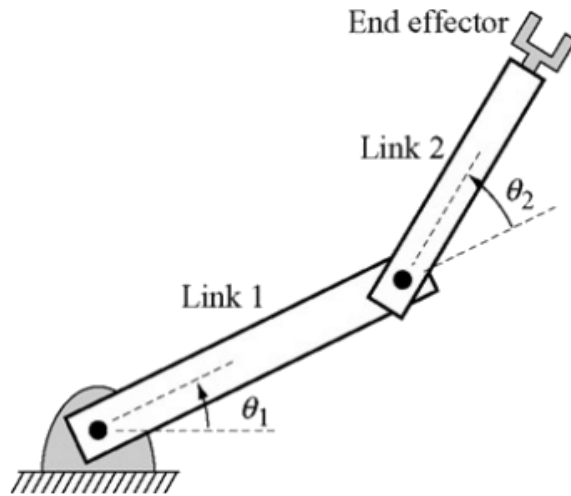
(f / f^{-1} are continuous)

diffeomorphic :

(f / f^{-1} are C^∞ smooth)



C-space as manifolds



Neighbourhood of q is mappable to \mathbb{R}^2

global topology is not \mathbb{R}^2 but $S^1 \times S^1$ (torus)

Map from C-space to W

Given configuration \mathbf{q} , determine volume occupied by $R(\mathbf{q})$ in workspace

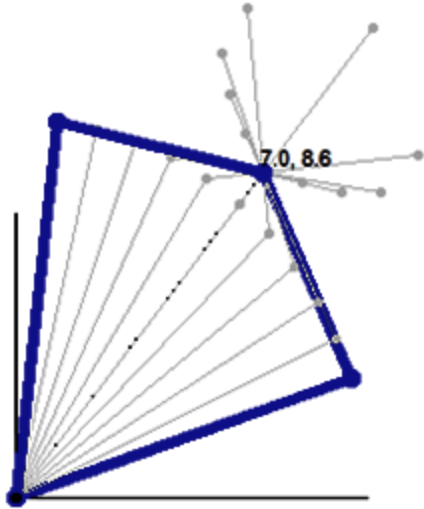
For multi-link manipulators, spatial pose of link $(n+1)$ depends on joint configuration \mathbf{q} for joints 1, 2, ..., n .

→ Forward Kinematics

Map from W to C-space: given pose in workspace, find \mathbf{q}

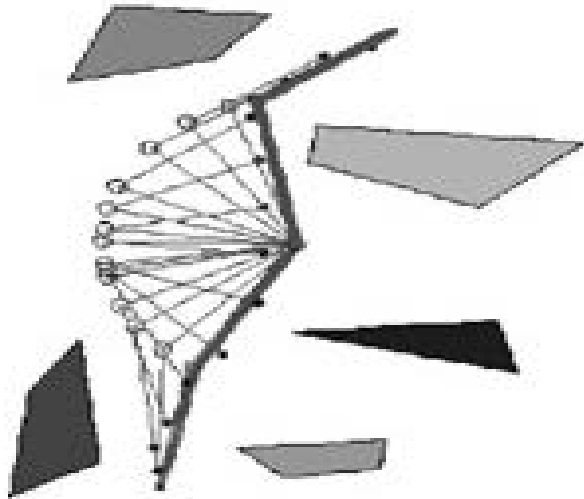
→ Inverse Kinematics

Mapping obstacles

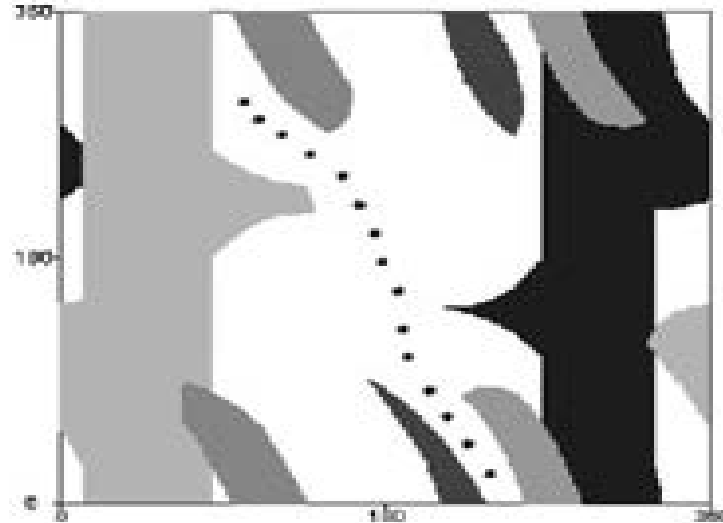


Point obstacle in
workspace

Articulated Robot C-space



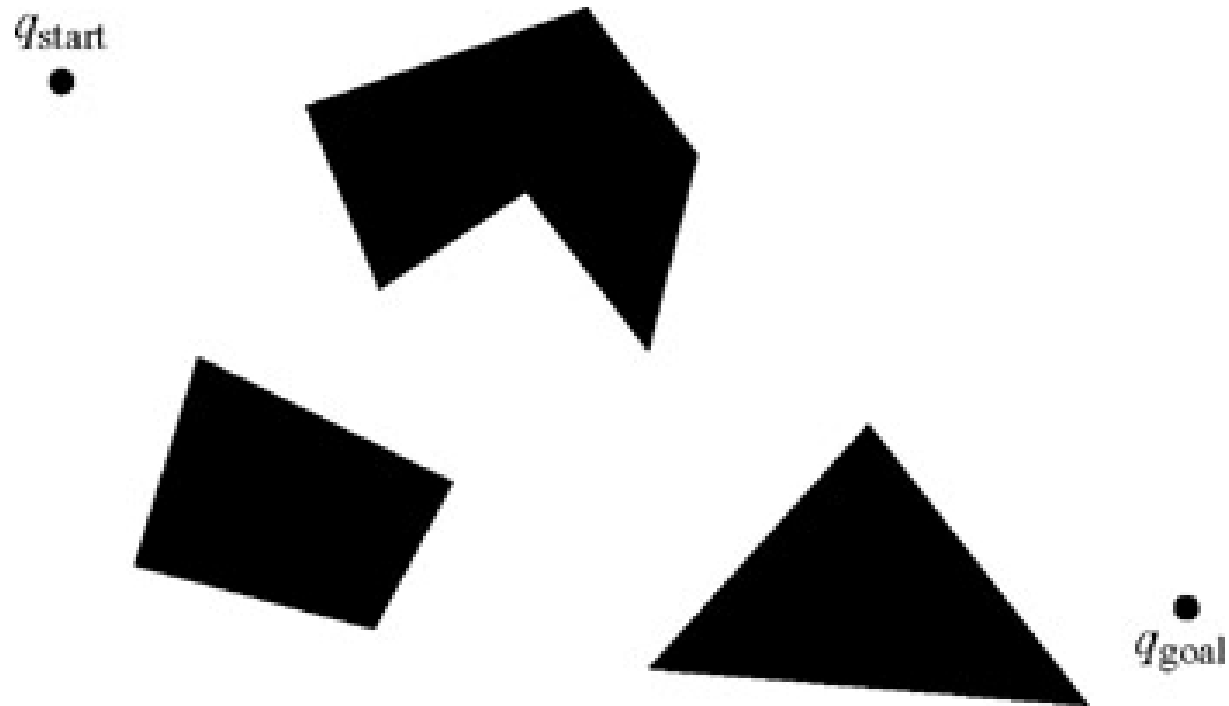
Path in workspace



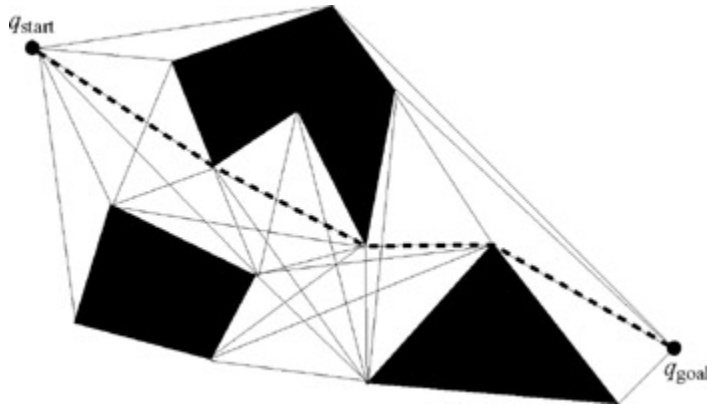
Path in Configuration Space

Graph-based approaches

Visibility Graph methods



Visibility Graph methods



Construct edges between visible vertices

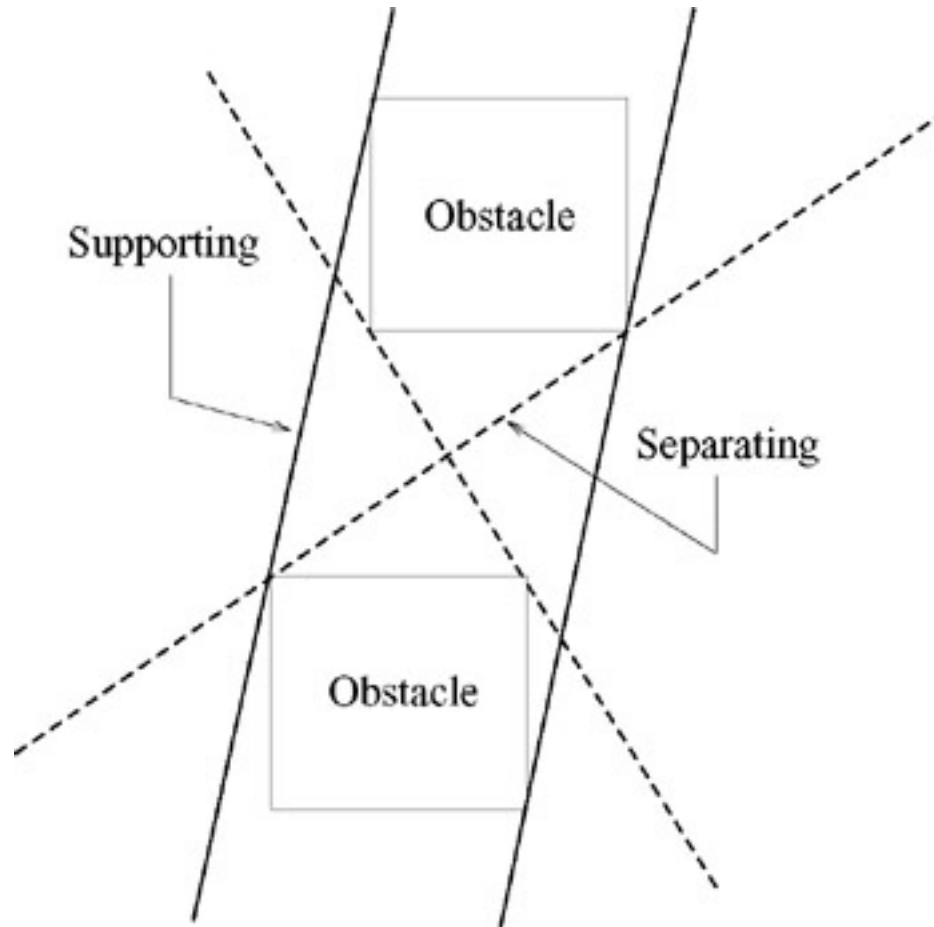
Sufficient to use only **supporting** and **separating** tangents

Complexity:

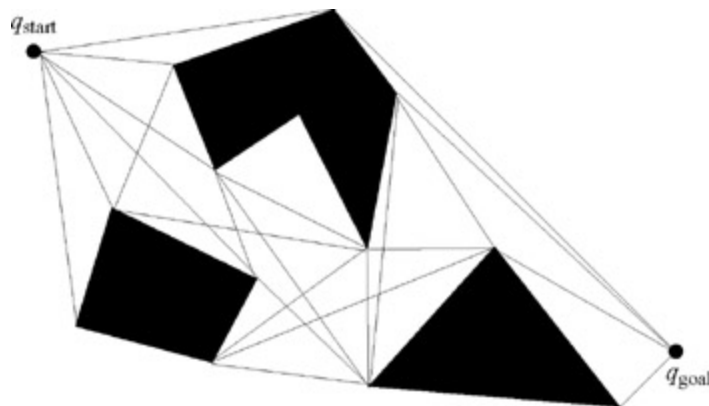
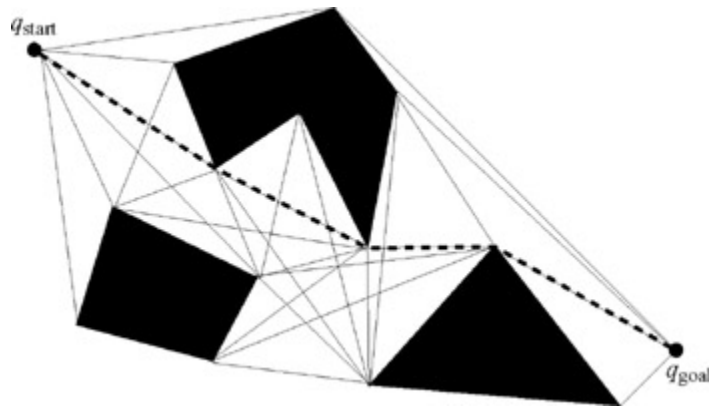
Direct visibility test: $O(n^3)$
(tests for each vtx: $O(n)$ emanations
x $O(n)$ obst edges)

Plane sweep algorithm: $O(n^2 \log n)$

Visibility Graph methods



Visibility Graph methods



Sufficient to use only **supporting** and **separating** tangents

Finds “shortest” path – but too close to obstacles

Cell decomposition methods

Trapezoidal decomposition:
Each cell is convex.

Sweep line construction:
 $O(n \log n)$

Graphsearch: $O(n \log n)$

Path: avoids obstacle
boundary but has high
curvature bends

Roadmap methods

Roadmaps

any roadmap RM must have three properties:

Connectivity:

path exists between any q'_{START} and q'_{GOAL} in RM

Accessibility:

exists a path from any $q_{START} \in Q_{free}$ to some $q'_{START} \in RM$

Departability:

exists a path from some $q'_{GOAL} \in RM$ to any $q_{GOAL} \in Q_{free}$

Staying away from Obstacles: Generalized Voronoi Graphs



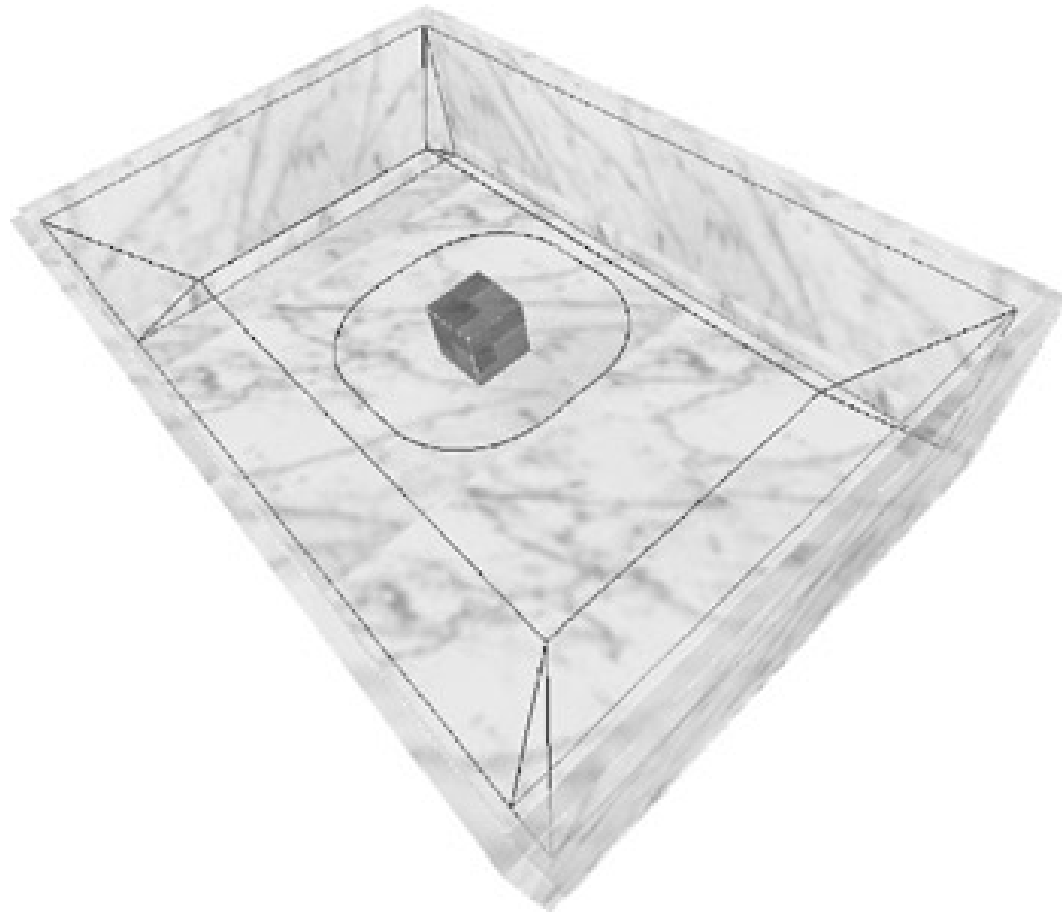
Voronoi Region of obstacle i :

$$\mathcal{F}_i = \{q \in \mathcal{Q}_{\text{free}} \mid d_i(q) \leq d_h(q) \quad \forall h \neq i\},$$

Voronoi diagram:

set of q equidistant from at least two obstacles

Generalized Voronoi Graphs



GVG Roadmaps

Accessibility / Deparability:

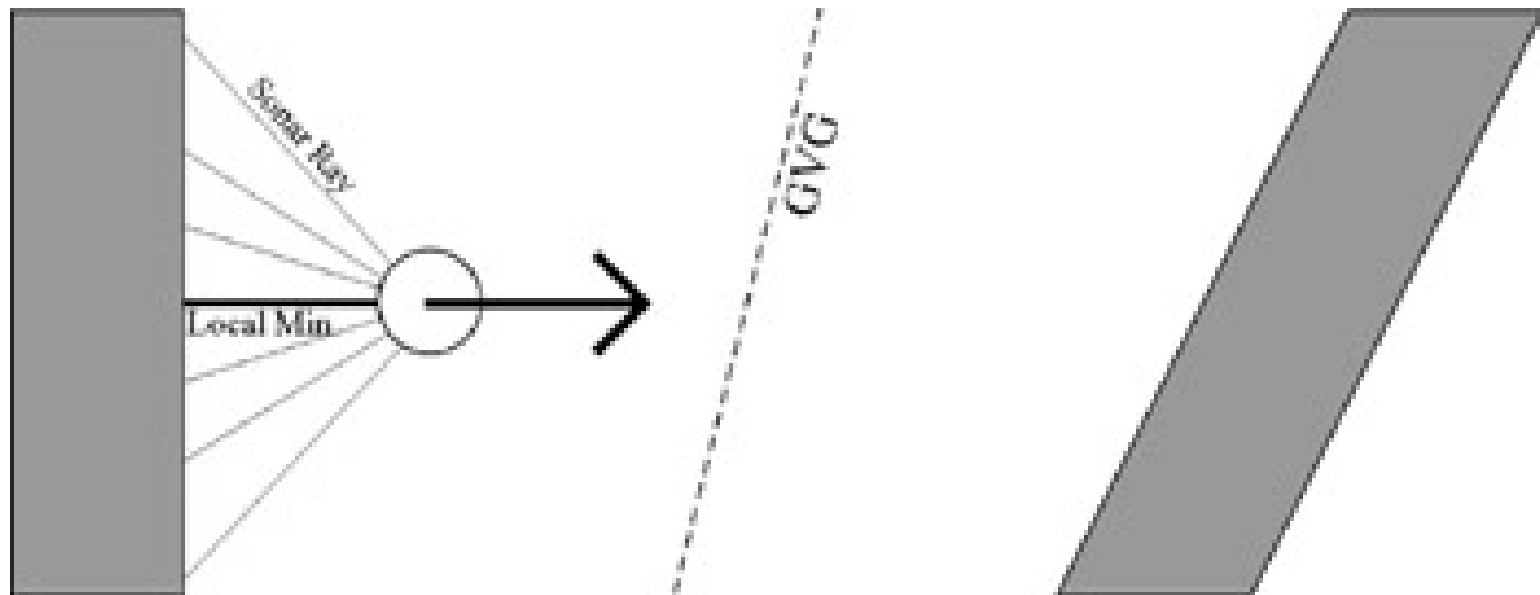
Gradient descent on distance from dominant obstacle :

- guaranteed to reach from any $q_{START} \in Q_{free}$ to some $q'_{START} \in RM$
- motion is along a “retract” or brushfire trajectory

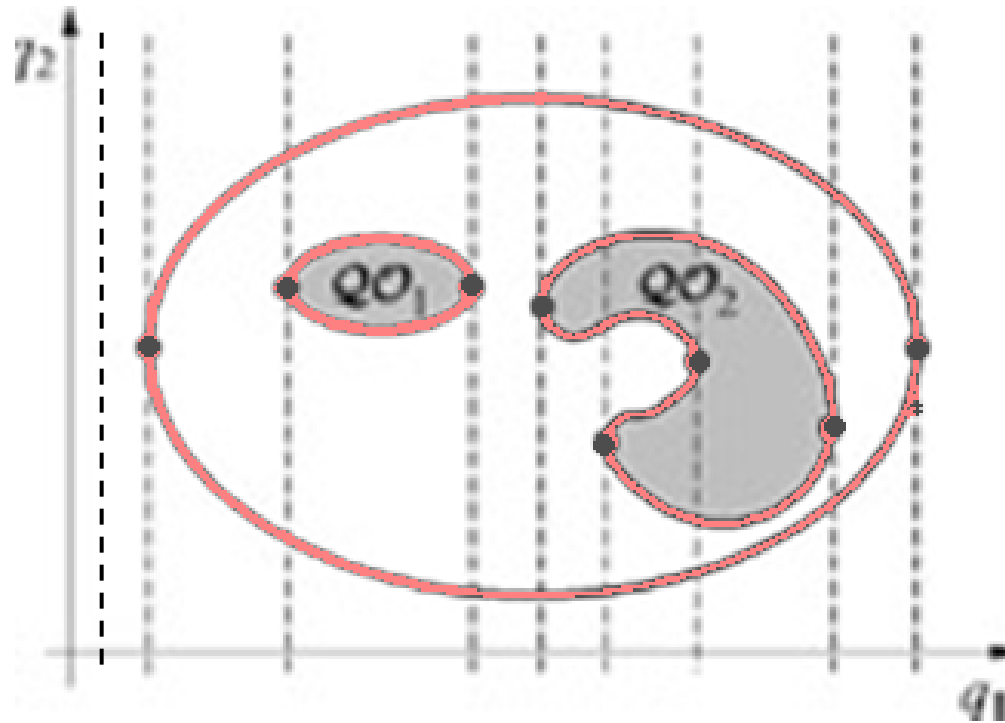
Connectivity:

GVG is Connected if path exists

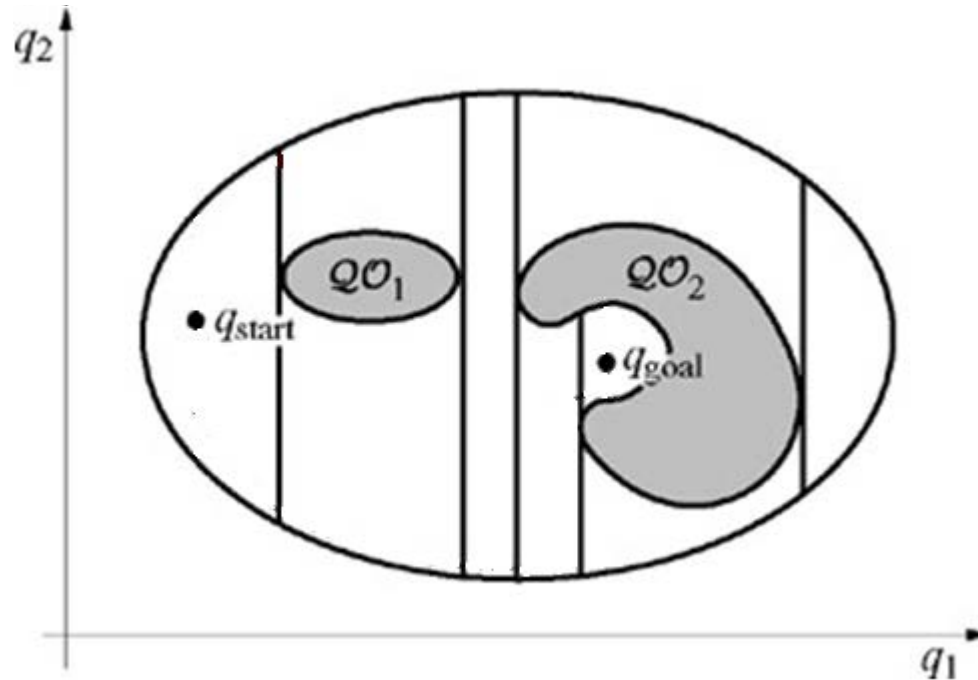
Sensor based Voronoi roadmap construction



Canny's Silhouette roadmap



Canny's Silhouette roadmap



Canny's Complexity Analysis

n : = degrees of freedom of robot (dim of C-space)

obstacles C-space boundaries represented as p polynomials of maximum degree w

Complexity:

any navigation path-planning problem can be solved in $p^n(\log p)w^{O(n^4)}$ time

Probabilistic Roadmap (PRM)

Probabilistic Roadmap

Sample n poses $q_1 \dots q_n$ in the WORKSPACE

Free space nodes: Reject q_i that intersect with an obstacle, remaining nodes q are in Q_{free}

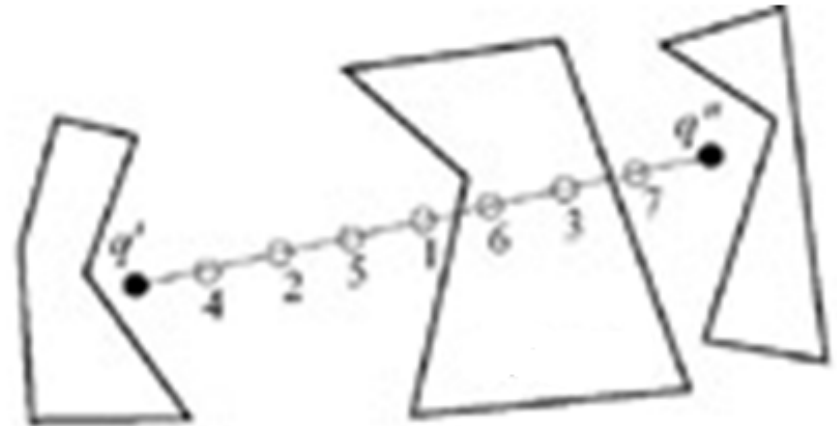
Local planning: in k -nearest neighbours, if path $\langle q_i, q_j \rangle$ collision-free, add edge to graph

Resulting graph = *Probabilistic Roadmap*

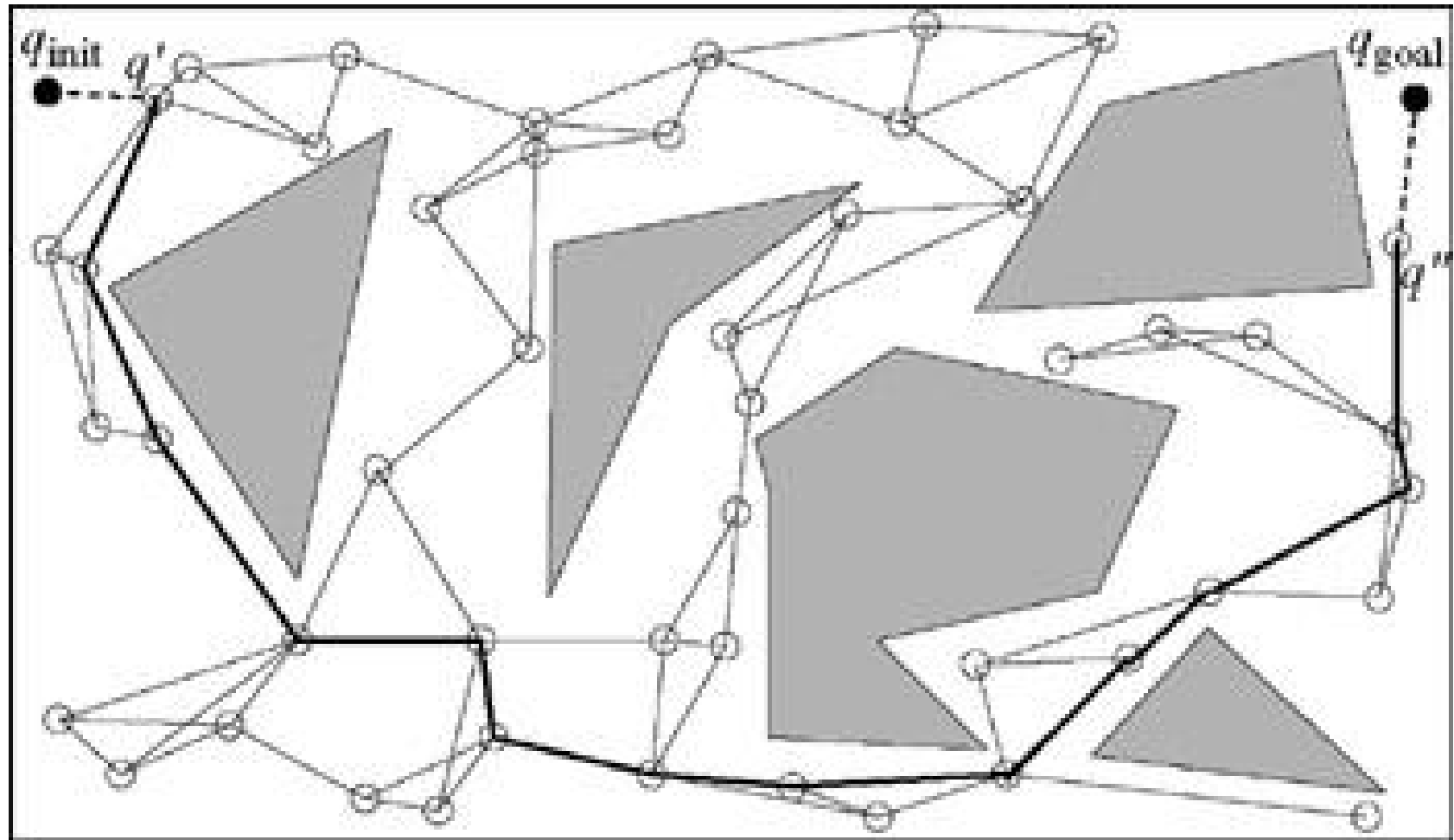
Local Planner

Objective: Test if path
 $\langle q_i, q_j \rangle$ is collision-free

Linear Subdivision
algorithm: start at
midpoint(q_i, q_j) ;
subdivide
recursively until
desired precision



Probabilistic Roadmaps (PRM)



Sampling-based motion planning

Sample n poses $q_1 \dots q_n$ in the workspace

Reject q_n that overlap with an obstacle,
remaining poses are in Q_{free}

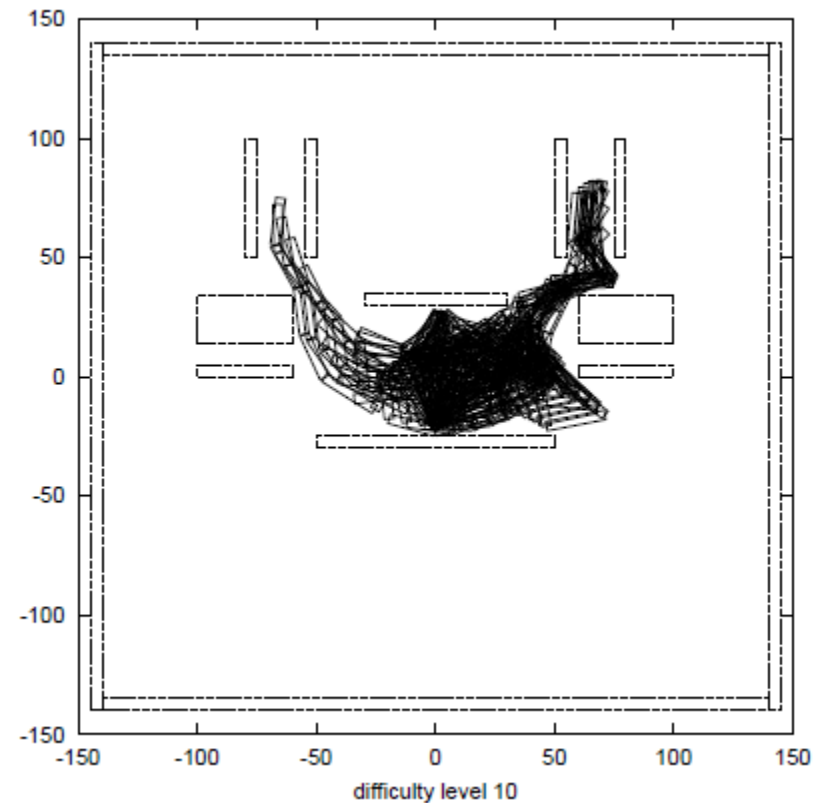
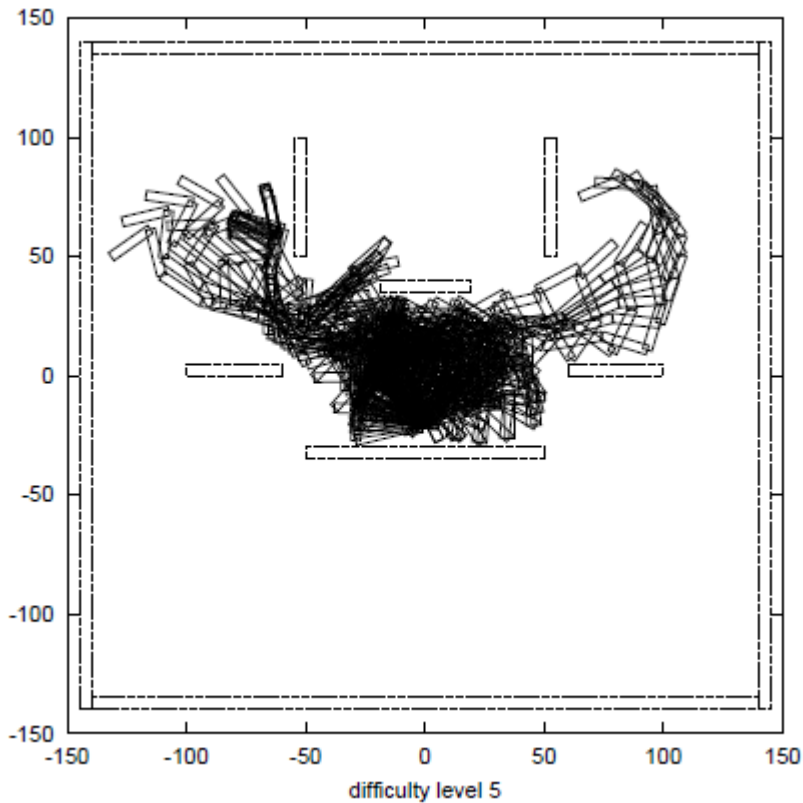
Use local planning to determine if a path
exists between neighbours q_i and q_j .

Resulting graph = *Probabilistic Roadmap*

Probabilistically complete:

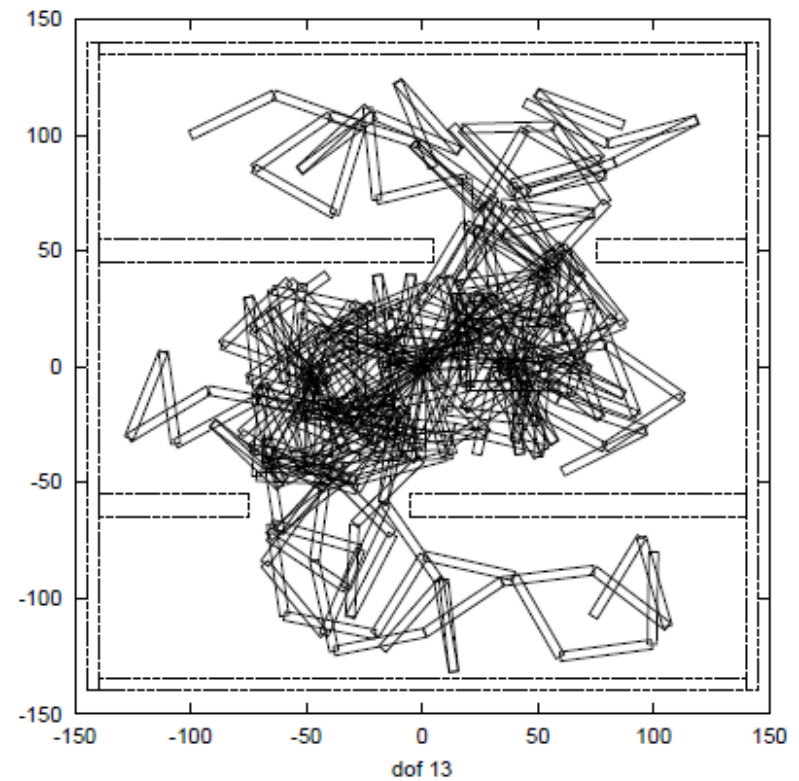
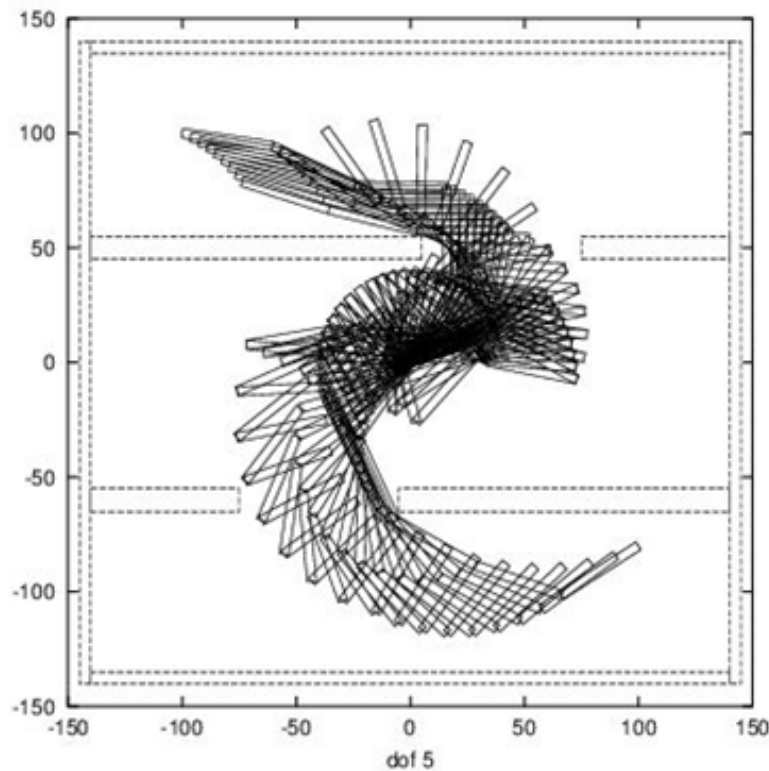
As #samples $n \rightarrow \infty$, Prob (success) $\rightarrow 1$

Hyper-redundant robot motion planning using PRM



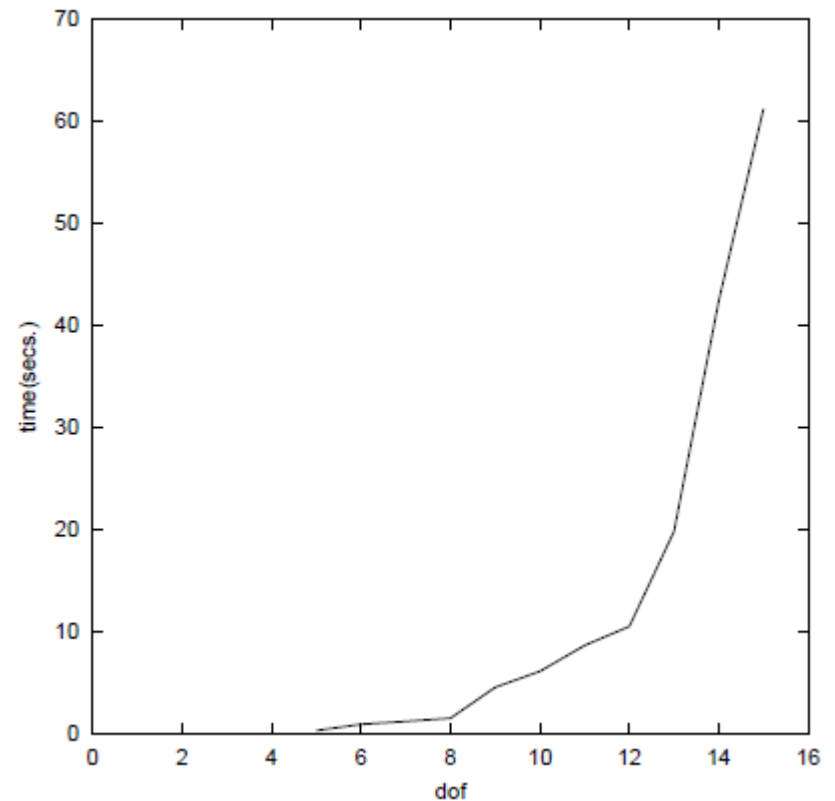
[sinha mukerjee dasgupta 02]

Hyper-redundant robot motion planning using PRM



[sinha mukerjee dasgupta 02]

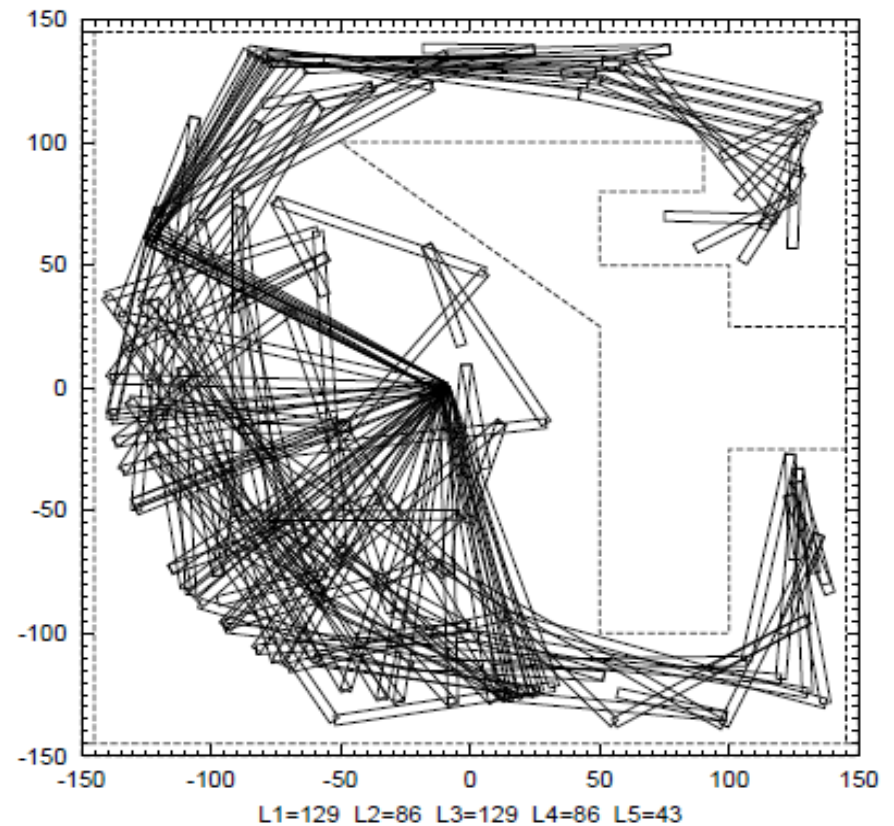
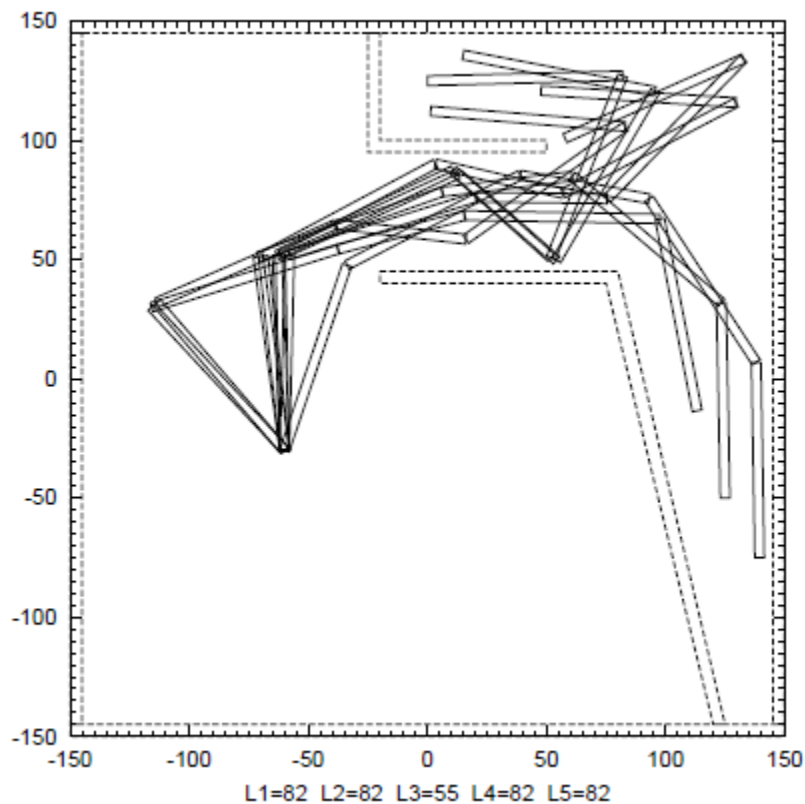
Hyper-redundant motion planning



Time:
Exponential in DOFs

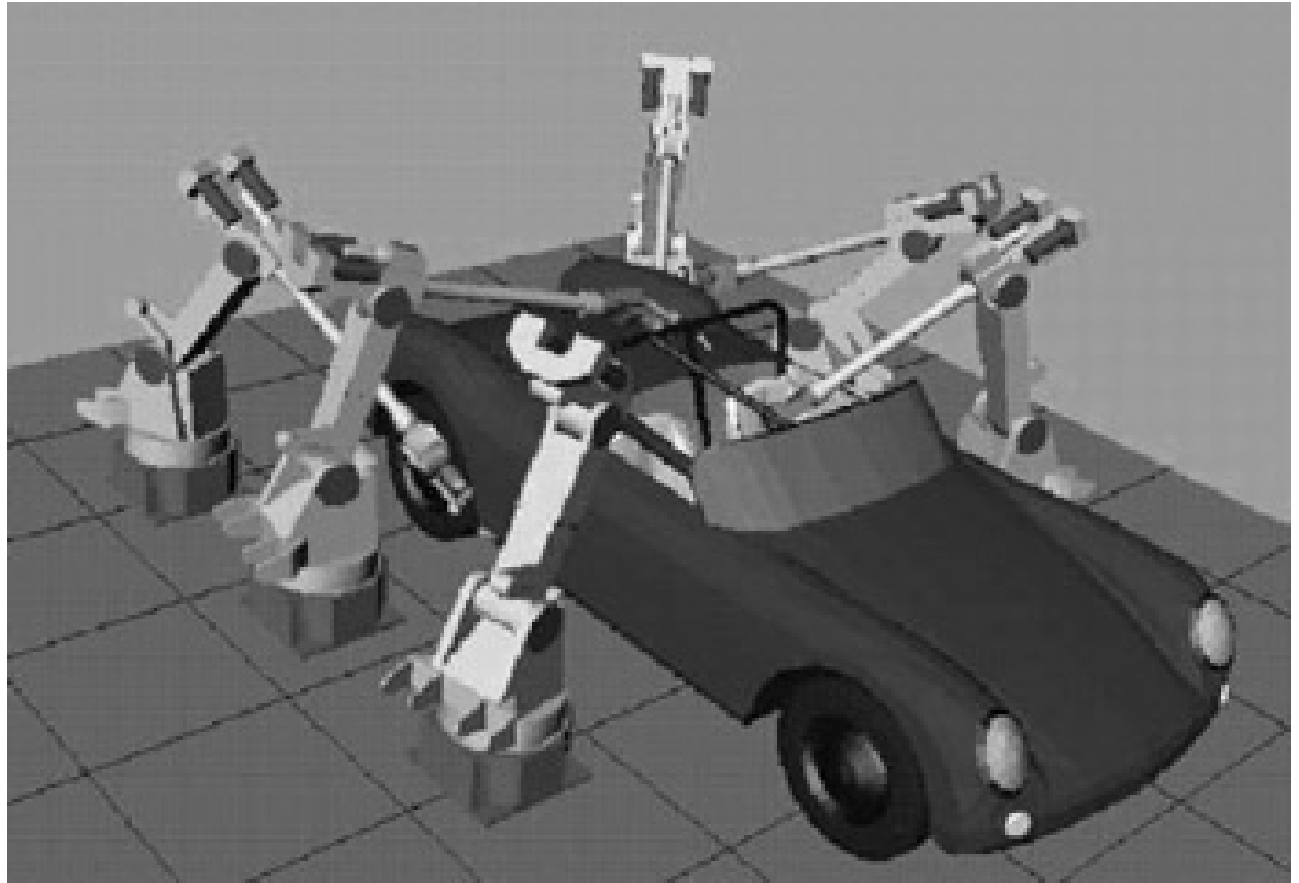
[sinha mukerjee dasgupta 02]

Design for manipulability



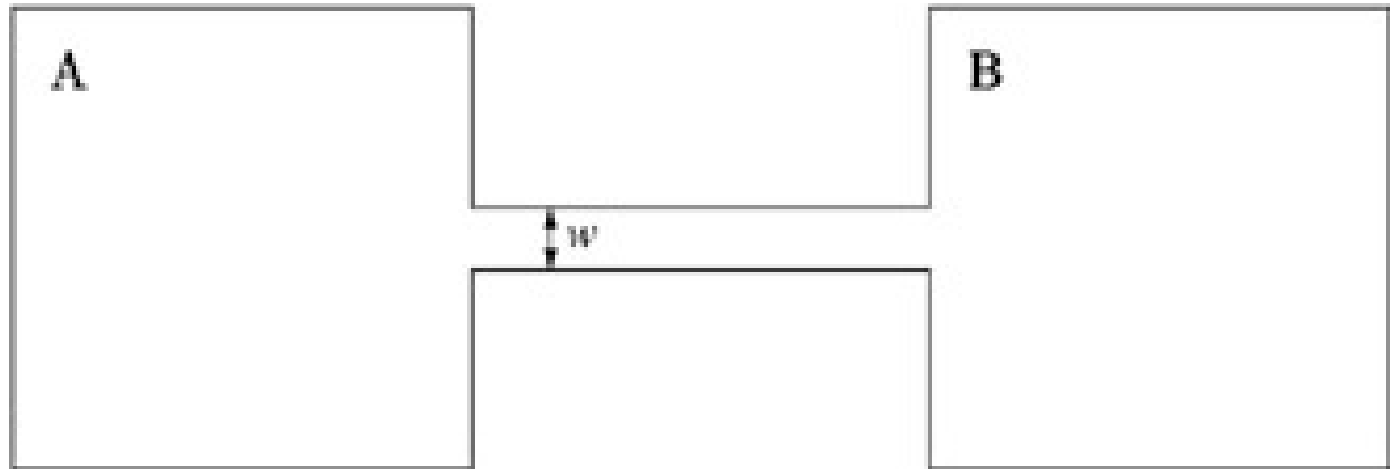
[sinha mukerjee dasgupta 02]

PRM applications



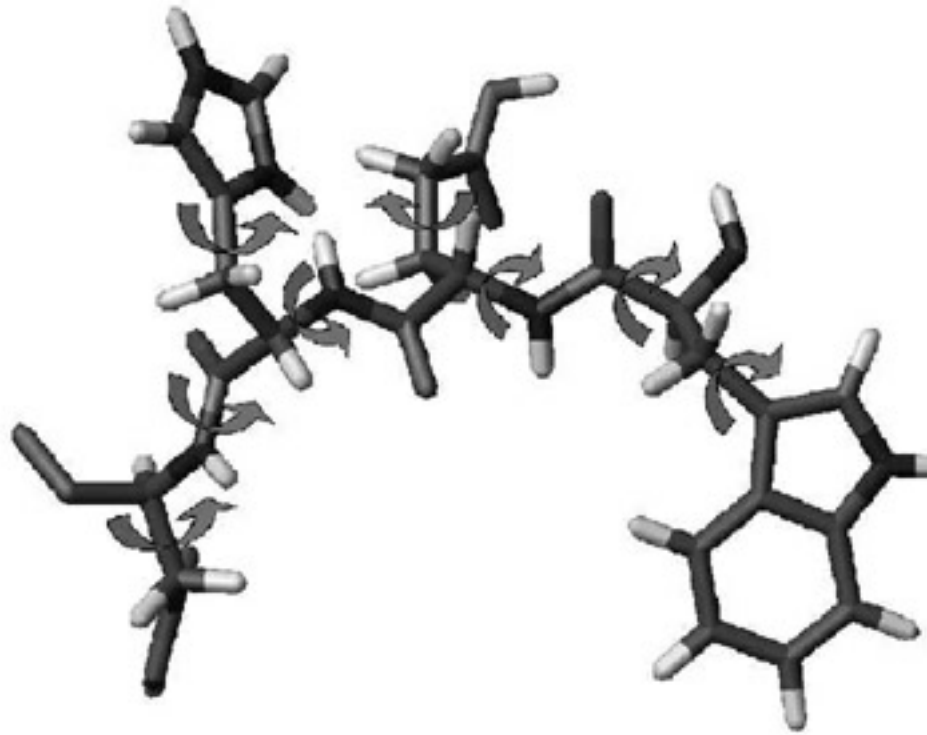
42 DOFs: [Sánchez and J. C. Latombe 02]

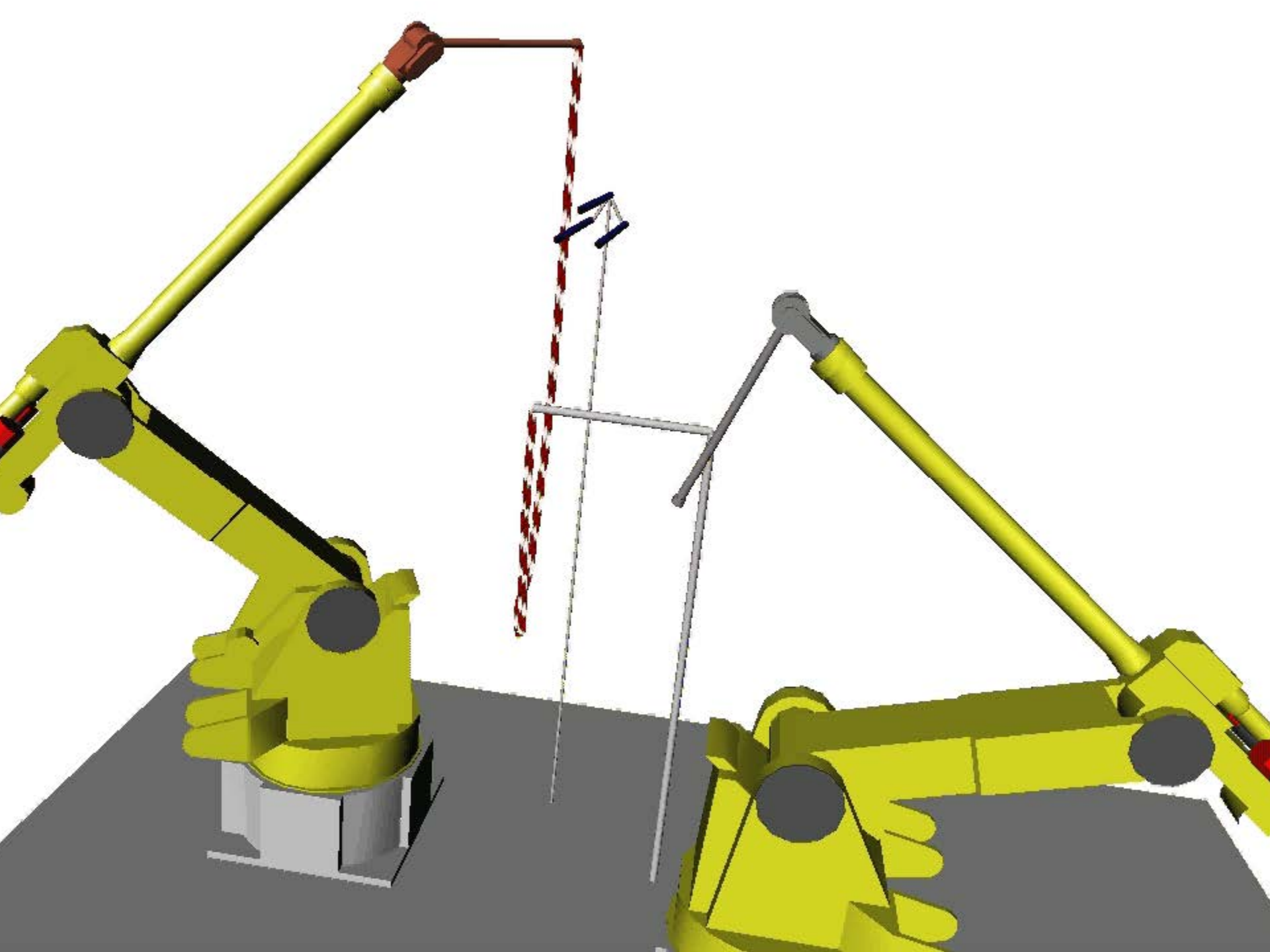
Narrow corridor problem



- Solution: generate more samples near boundary
- bias the sample towards boundary region
 - if midpoint between two obstacle nodes is free, add

PRM applications : Protein folding





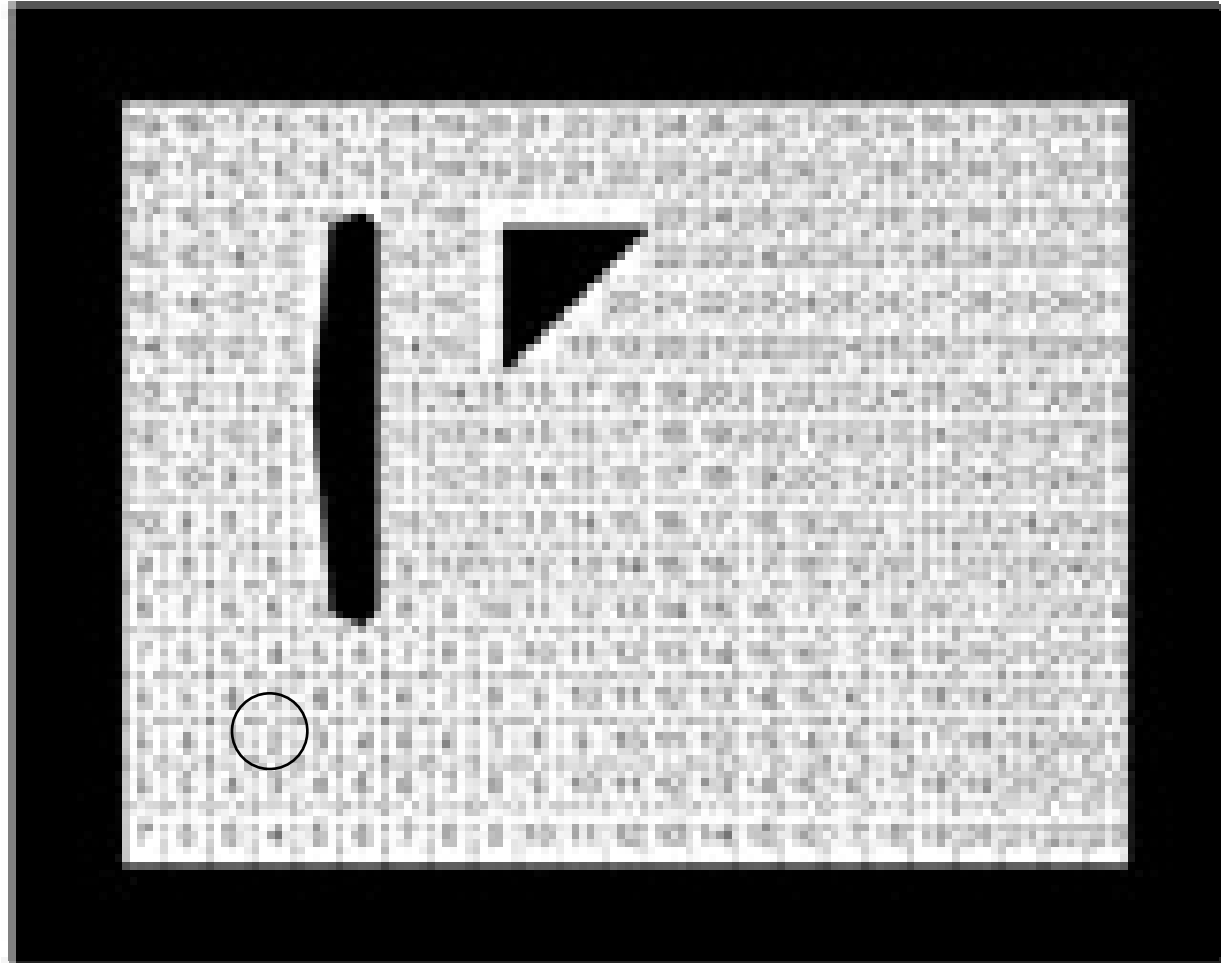
Continuum methods: Overcoming Local minima

Grid-based: Wave-front

- Grid-based model
- given a start grid cell q_s assign it the value “2”
 - Every neighbour gridcell gets +1
 - Until grid is filled
- Given a goal cell q_G use greedy search to find path back to goal

Grid-based: Wave-front

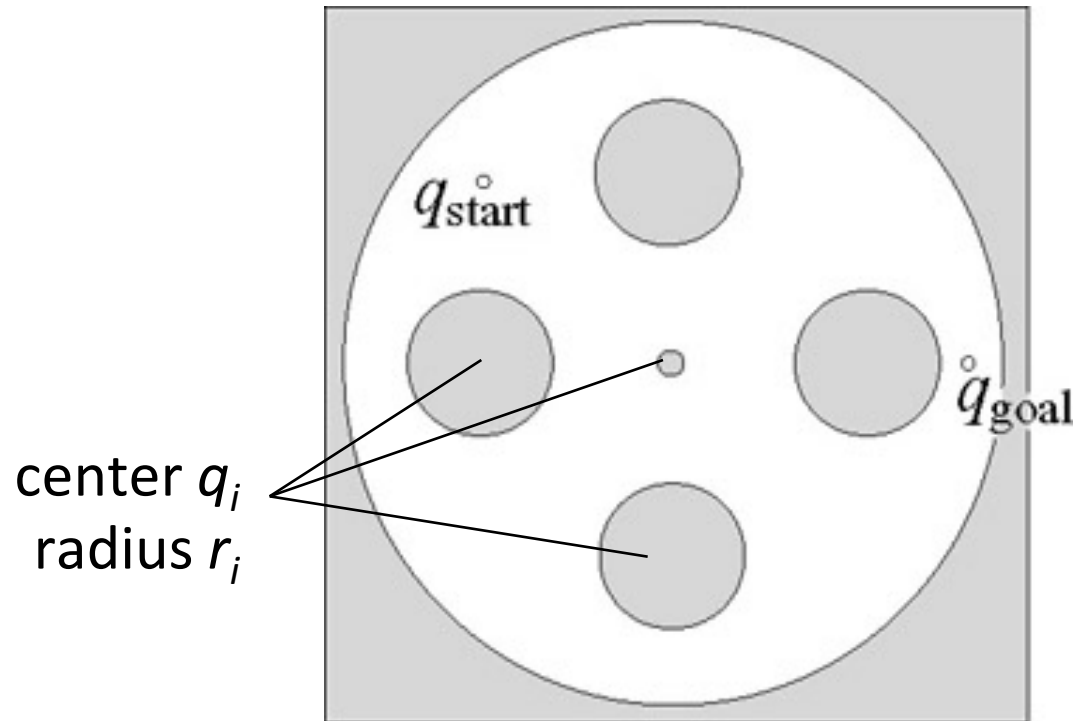
$O(k^d)$ space /
time



Navigation Function : Sphere space

- Spherical wall (r_0), with spherical obstacles inside
- Obstacle distance
 $\mathcal{QO}_i = \{q \mid \beta_i(q) \leq 0\}$
 $\beta_0(q) = -d^2(q, q_0) + r_0^2, \quad \text{— wall}$
 $\beta_i(q) = d^2(q, q_i) - r_i^2, \quad \text{— obstacles}$

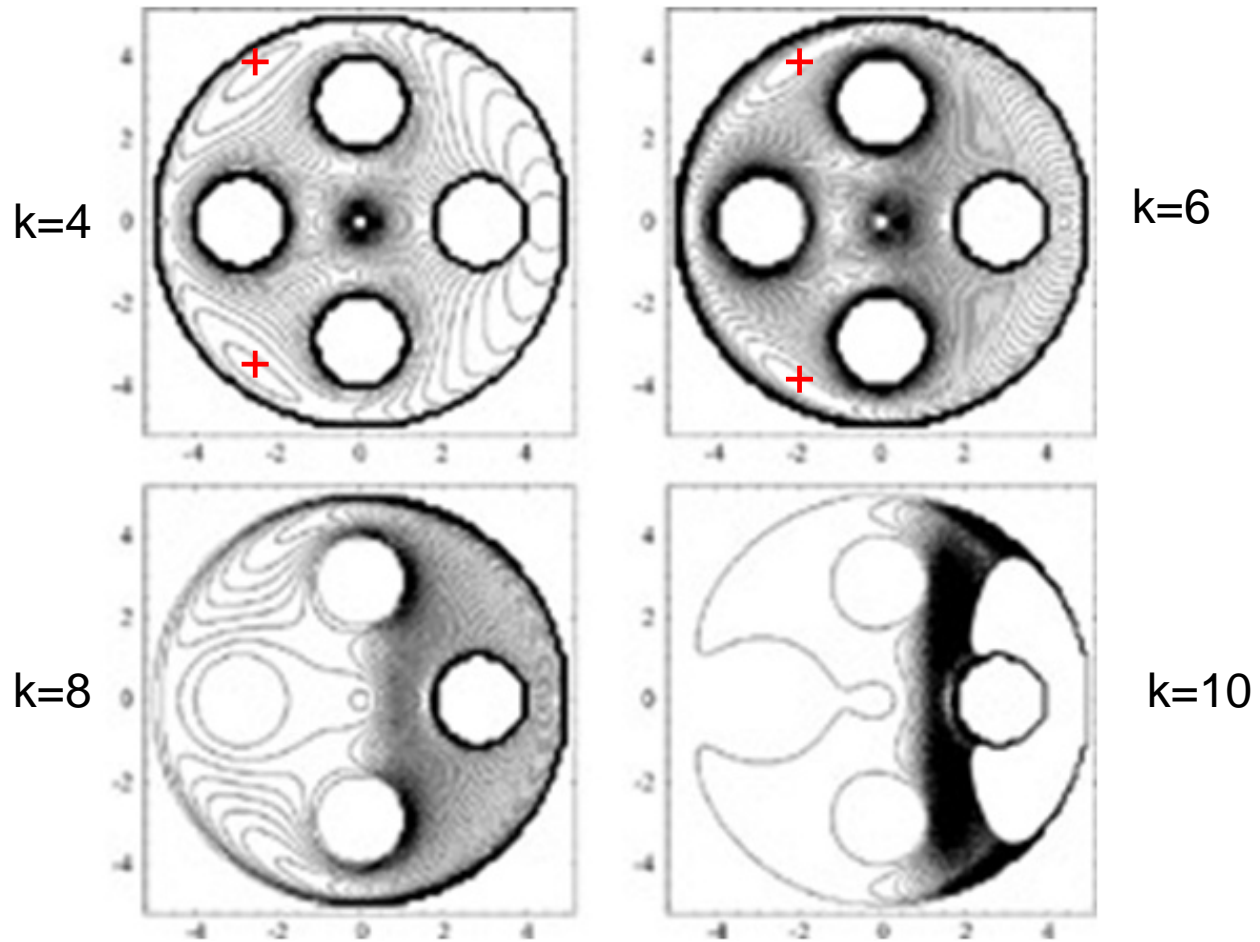
Sphere space



Navigation Function : Sphere space

- Spherical wall (r_0), with spherical obstacles inside
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 $\mathcal{QO}_i = \{q \mid \beta_i(q) \leq 0\}$
 $\beta_0(q) = -d^2(q, q_0) + r_0^2, \text{ — wall}$
 $\beta_i(q) = d^2(q, q_i) - r_i^2, \text{ — obstacles}$
- Goal potential with high exponent $\gamma_\kappa(q) = (d(q, q_{\text{goal}}))^{2\kappa}$
- Instead of sum, use product to combine obstacle potentials
$$\beta(q) = \prod_{i=0}^n \beta_i(q).$$
- For high k , $\frac{\gamma_\kappa}{\beta}(q)$ has unique minima at goal

Navigation Function

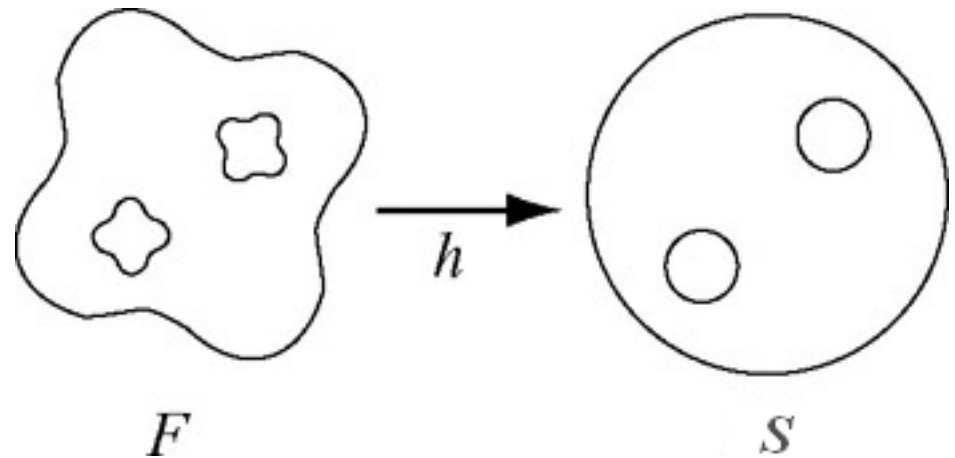


Navigation Function

$\varphi : S \rightarrow [0, 1]$:
navigation function on
sphere space S .

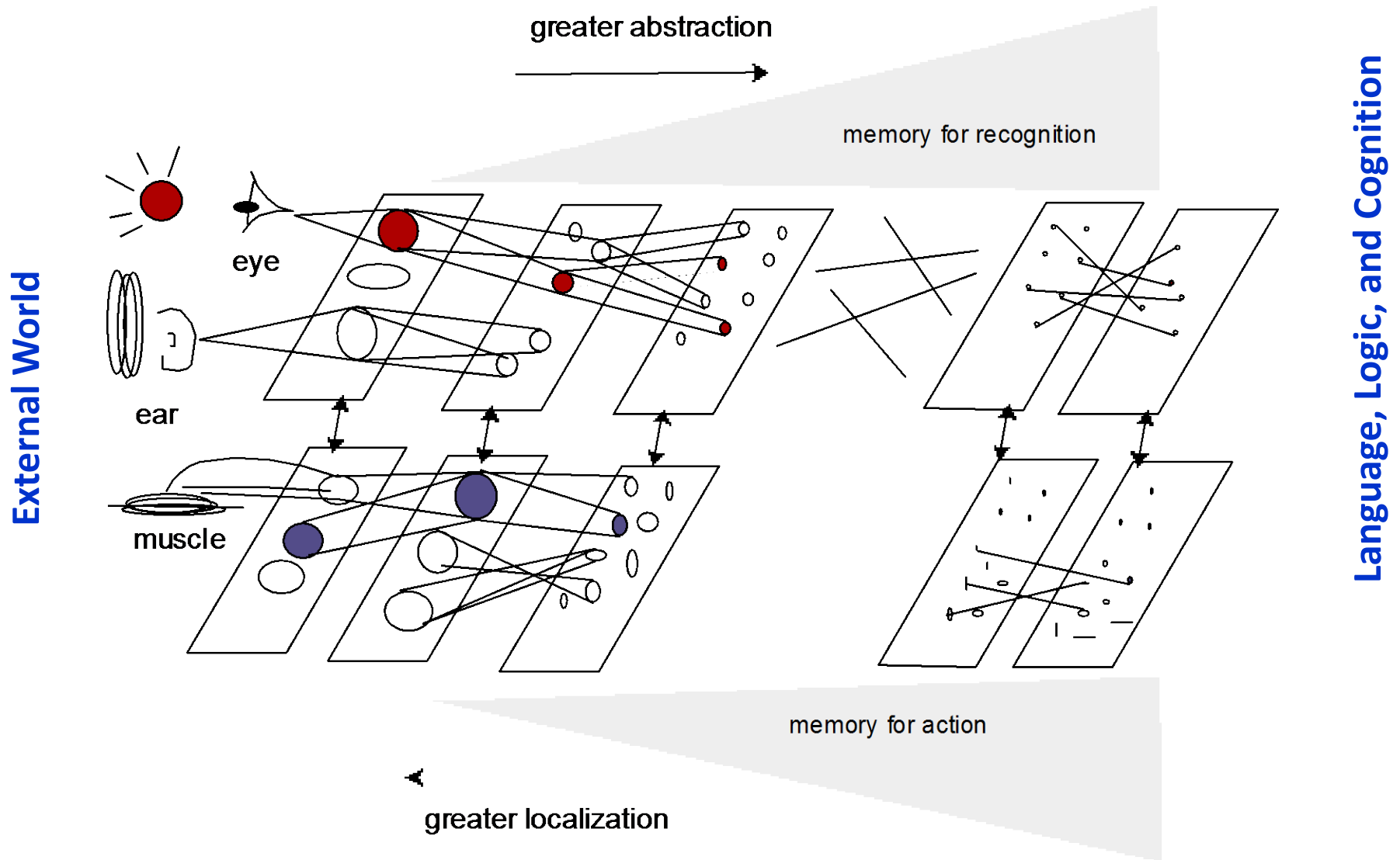
For any space F if exists
diffeomorphic
mapping $h : F \rightarrow S$
(i.e. h is smooth, bijective, and
has a smooth inverse),

then $\varphi = \varphi \circ h$ is a
navigation function on F



Sensori-motor map learning

Cognitive Architecture: Levels of Abstractions



Visuo-Motor expertise

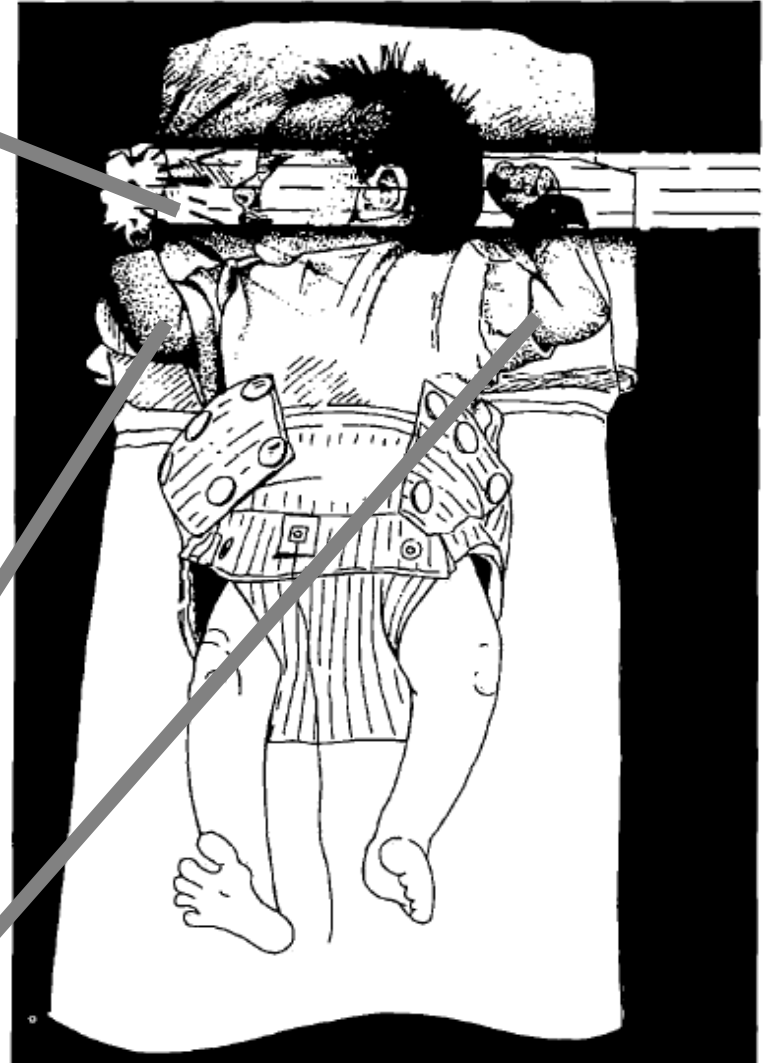
in darkened room,
works hard to position arm
in a narrow beam of light

Newborns
(10-24 days)

Small weights
tied to wrists

Will resist weights to move
the arm they can see

Will let it droop if
they can't see it

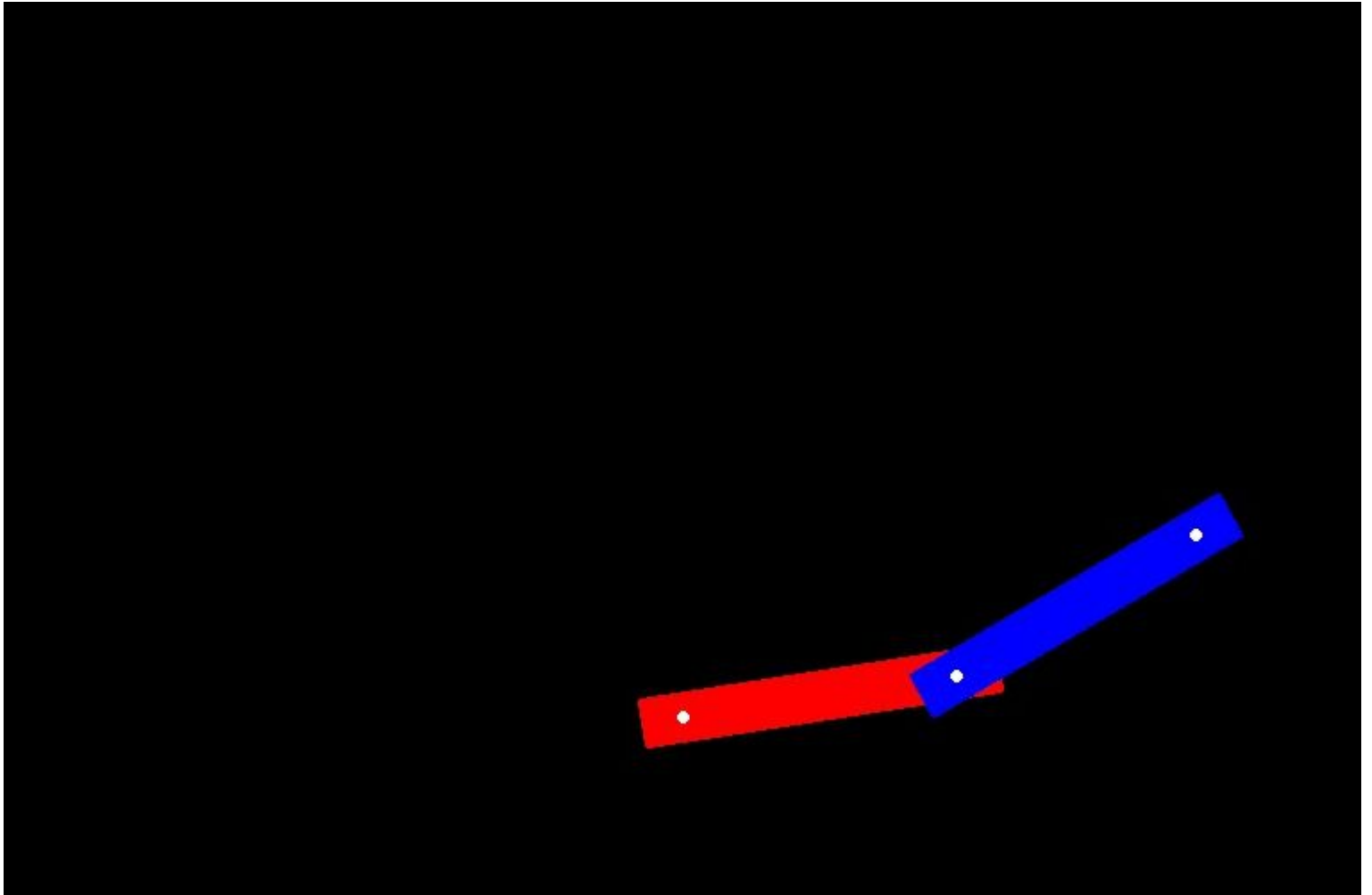


[A. van der Meer, 1997: Keeping the arm in the limelight]

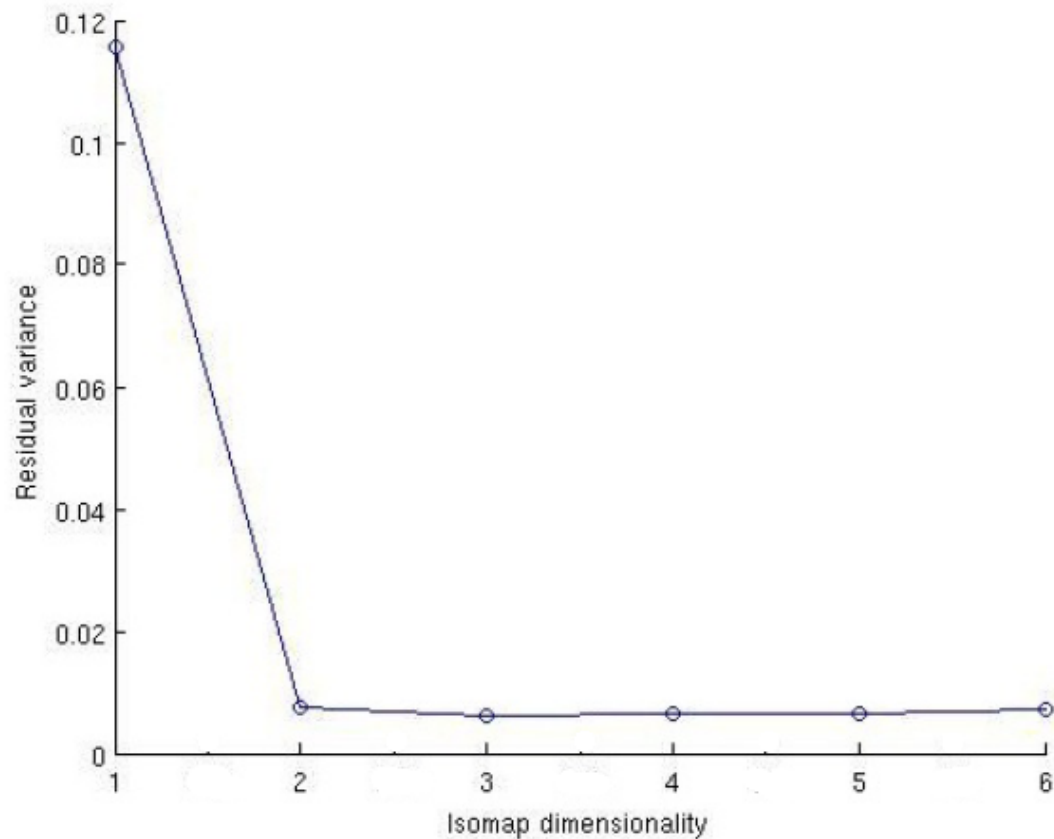
Observing self motions



Simulation



Manifold dimension



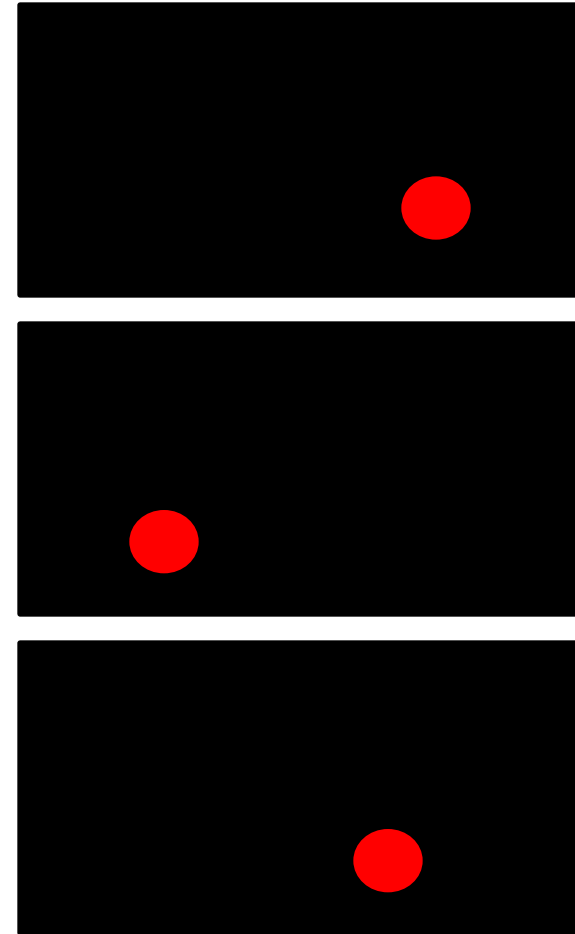
residual variance = $1 - r^2(D_g, D_y)$; r = linear correlation coefficient
 D_g = geodesic distance matrix; D_y = manifold distance

Discovering Configuration Space

Collect a set of images of the robot
at random configurations

Reduce the image set to a low
dimensional representation

Latent variables: (y_1, y_2, \dots, y_d)
for $d=2$, each image represented as a
pair of values



Smoothly Deformable objects

Object S = set of connected points S in $G \subset \mathbb{R}^d$.

Deformation function $h : G \rightarrow G$ is a function of a parameter vector $q = \{q_1 \dots q_k\}$.

Smooth Deformation : $S \rightarrow hS(\mathbf{q})$, is a diffeomorphism from G to G

Smoothly Deformable objects

Ex.1

Articulated chain with N rigid links, and k joints, each with 1 DOF each.

hS determined by $\{q_1 \dots q_k\}$, where each q_i is the parameter associated with each joint.

Deformable objects

LEMMA:

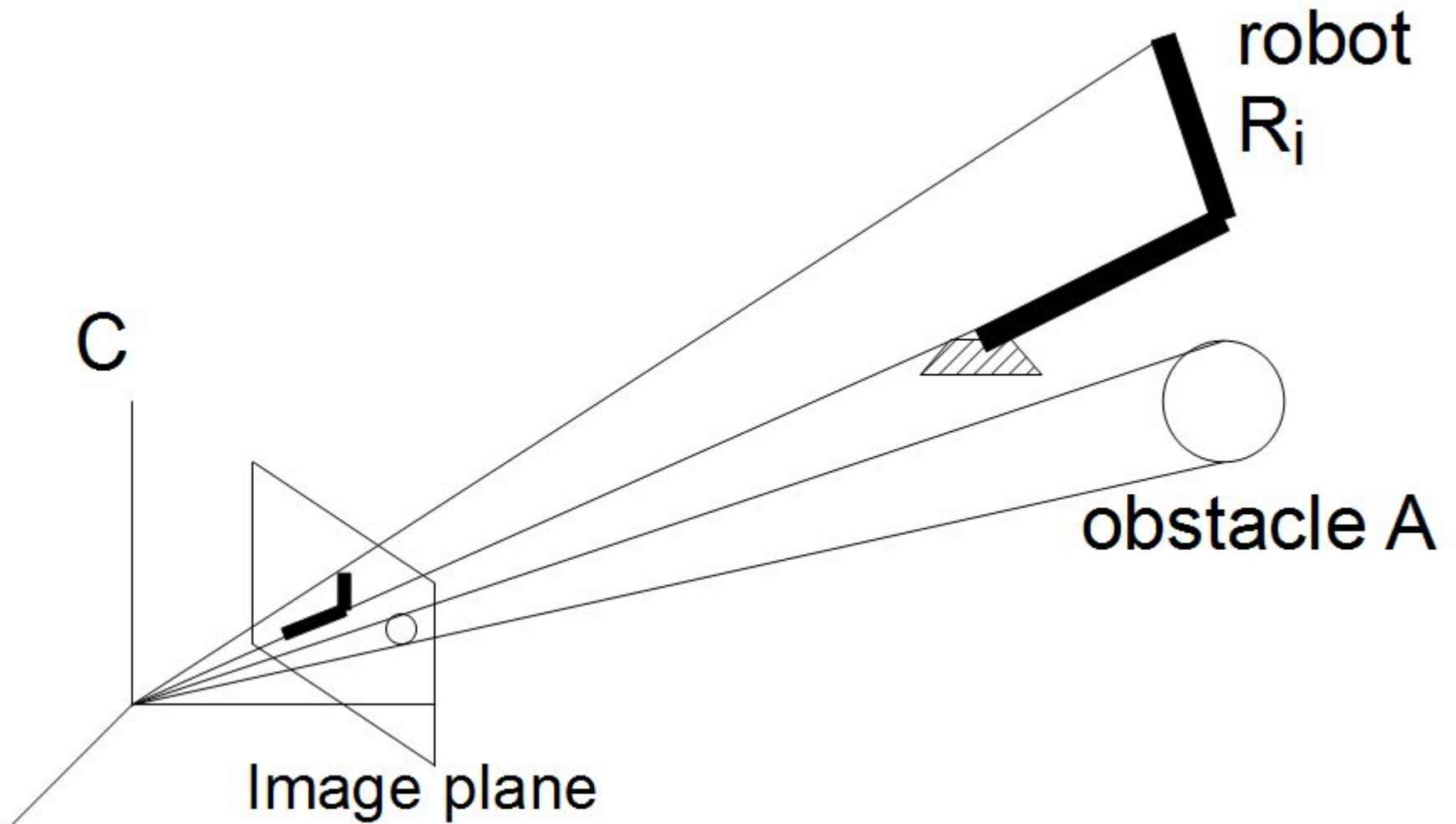
The space of shapes and poses of S is a manifold of at most k dimensions.
(k = dimensionality of \mathbf{q}).

Any $x \in S(\mathbf{q})$ has a neighbourhood N , s.t. there is a homeomorphism ϕ from N onto the space Q of $\{q_1 \dots q_N\}$. In \mathbb{R}^N

the map ϕ can be composed of the motion transformation $T(q_1 \dots q_m)$ [a special euclidean group] and the smooth deformation shape function $h_S(q_1 \dots q_d)$

Both of these transformations being diffeomorphic.

Imaging transformation



Visual Manifold theorem

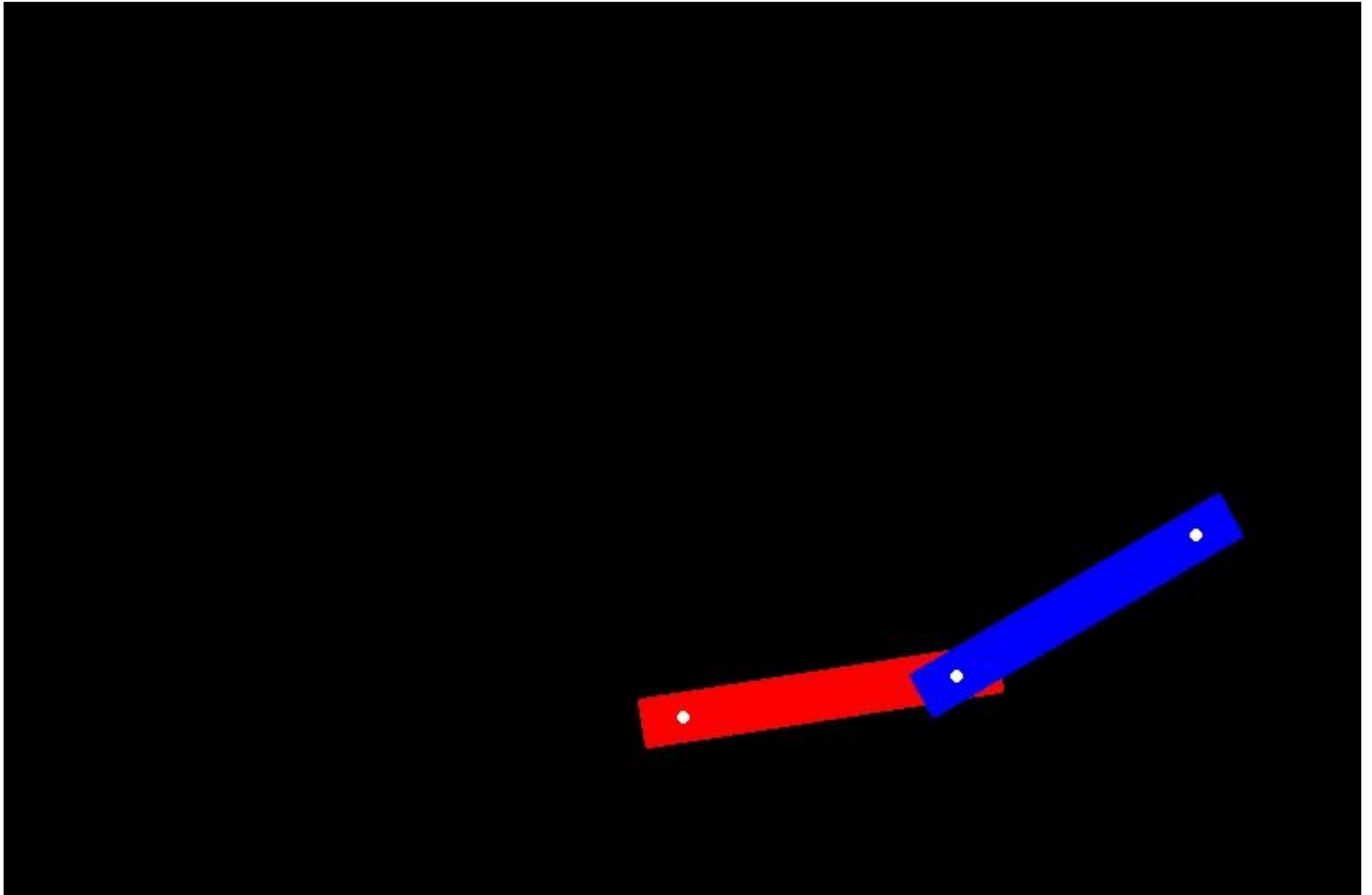
If S is a smoothly moving, visually distinguishable, deformable object, then any image of S , taken from a fixed camera, would lie on a manifold of at most N dimensions in the image space.

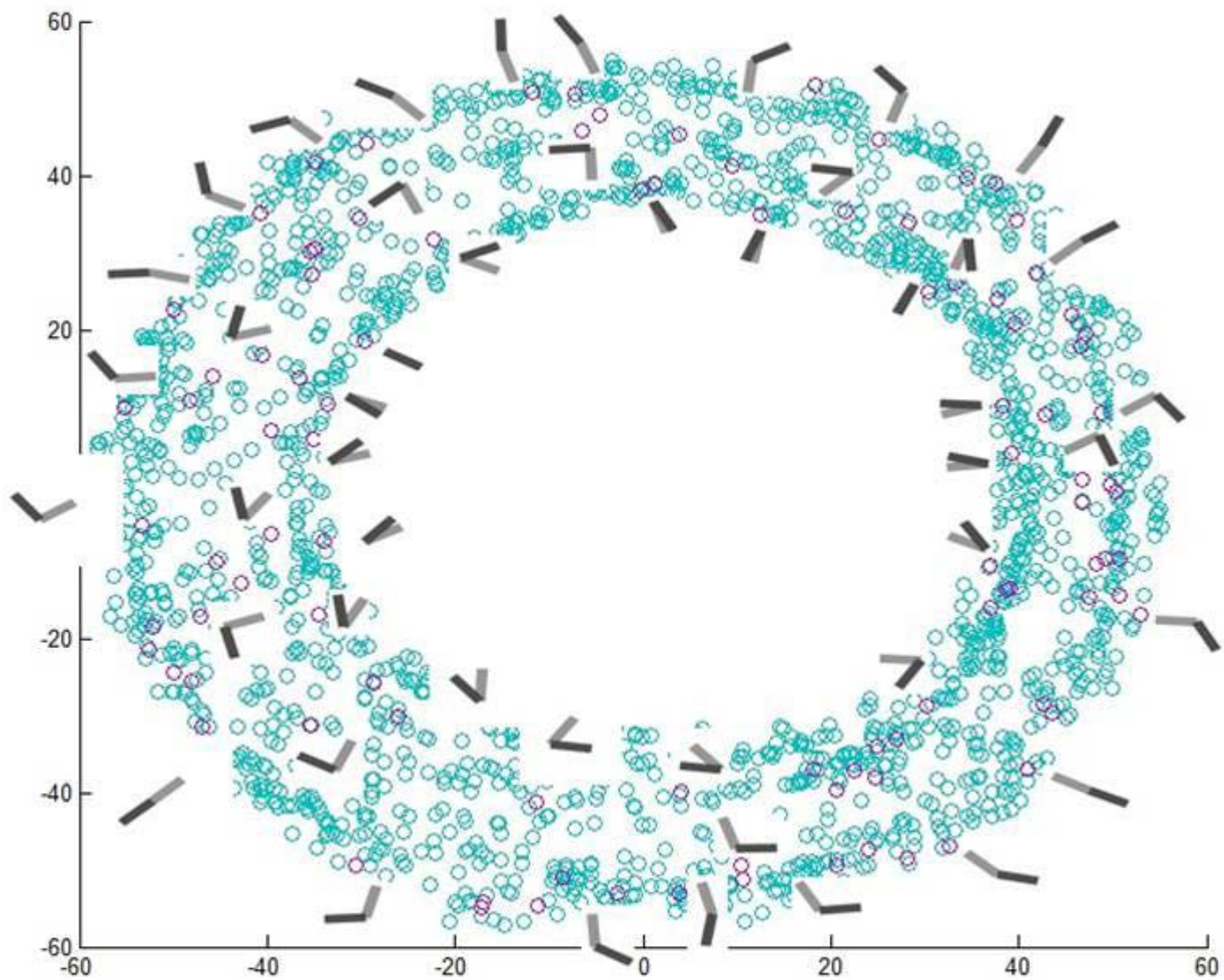
Visual Distinguishability



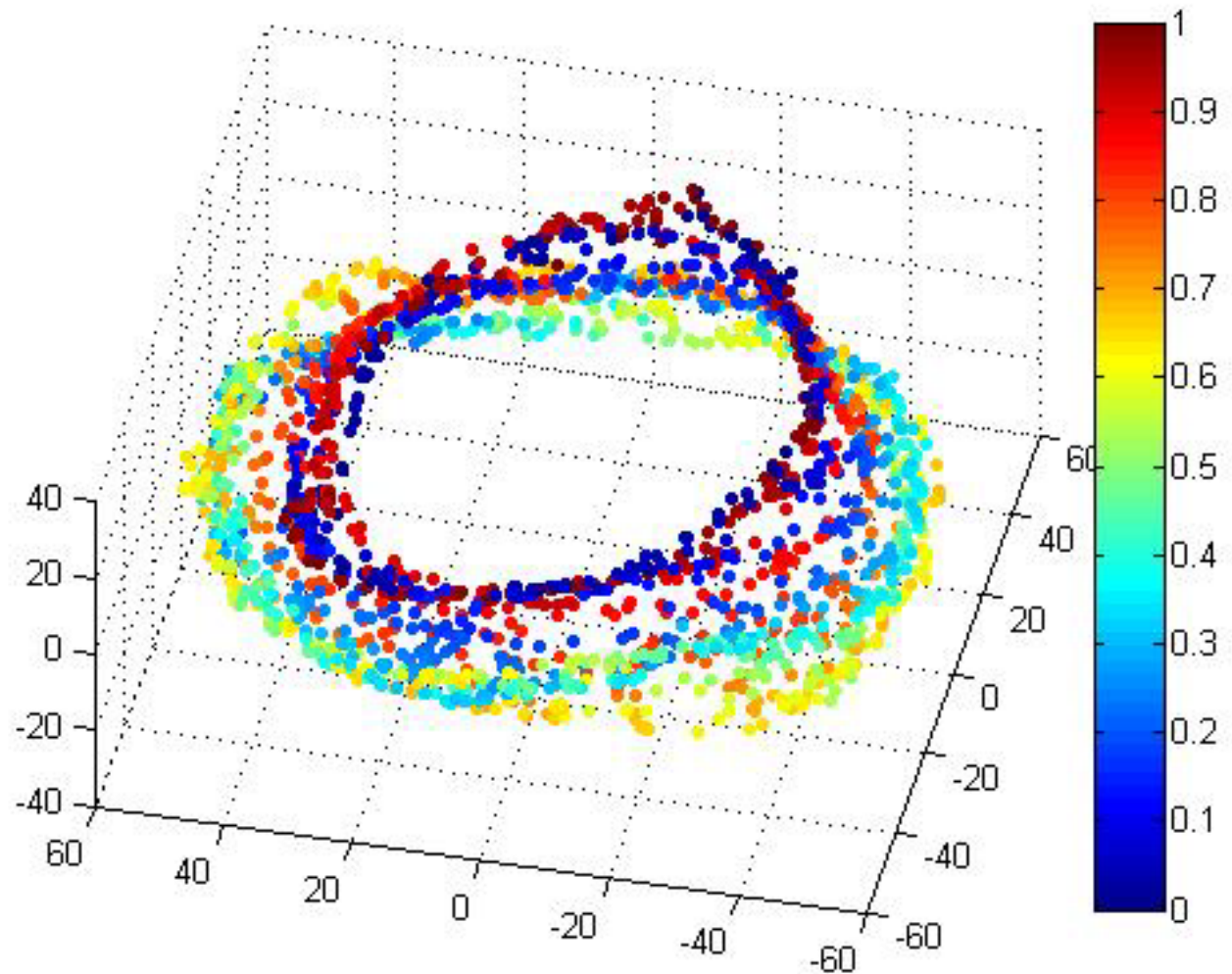
Robot Structure Discovery

Simulation

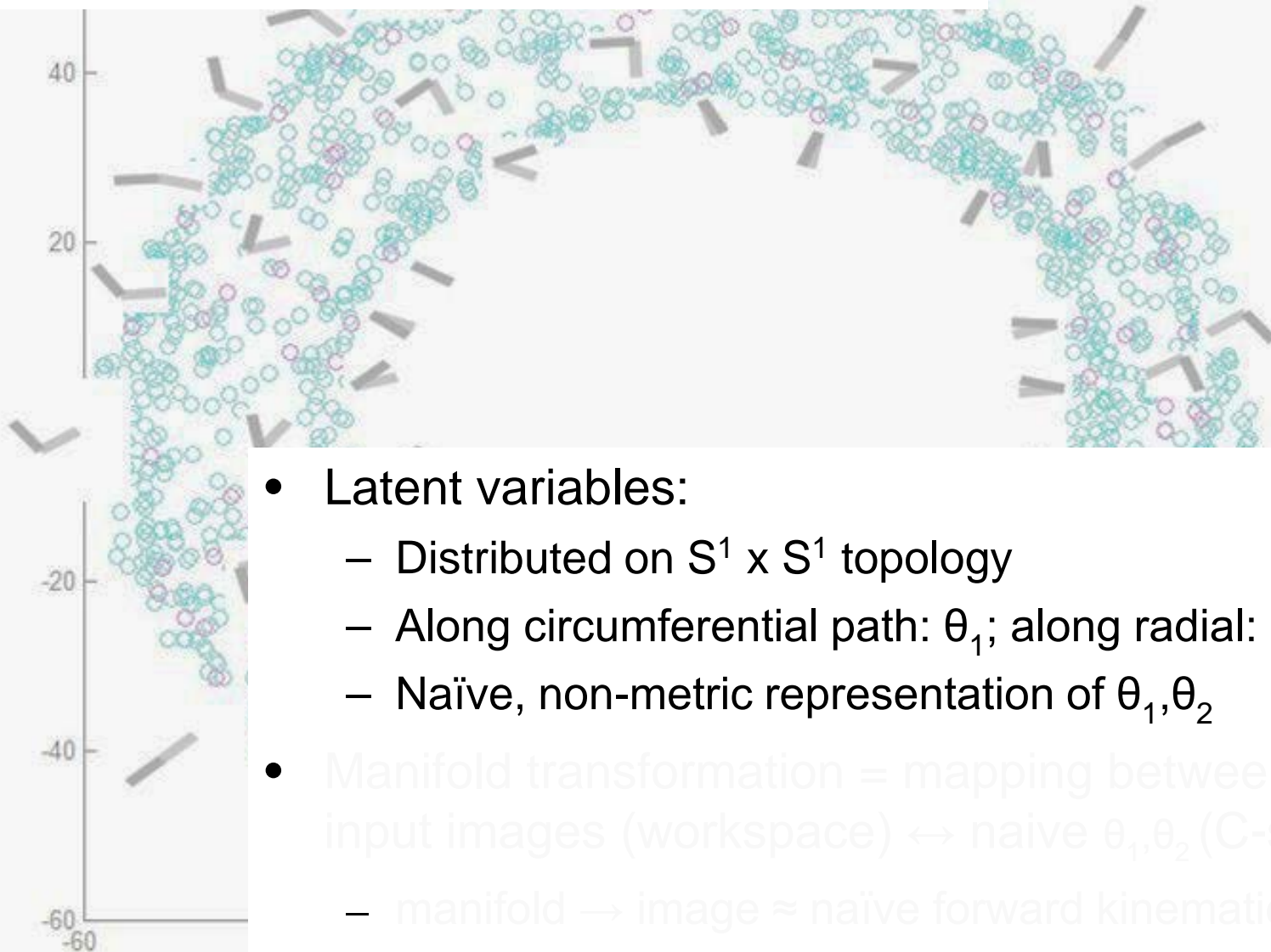




Robot Structure Learning



Robot Structure Learning



- Latent variables:
 - Distributed on $S^1 \times S^1$ topology
 - Along circumferential path: θ_1 ; along radial: θ_2
 - Naïve, non-metric representation of θ_1, θ_2
- Manifold transformation = mapping between input images (workspace) \leftrightarrow naive θ_1, θ_2 (C-space)
 - manifold \rightarrow image \approx naïve forward kinematics
 - image \rightarrow manifold \approx naïve inverse kinematics

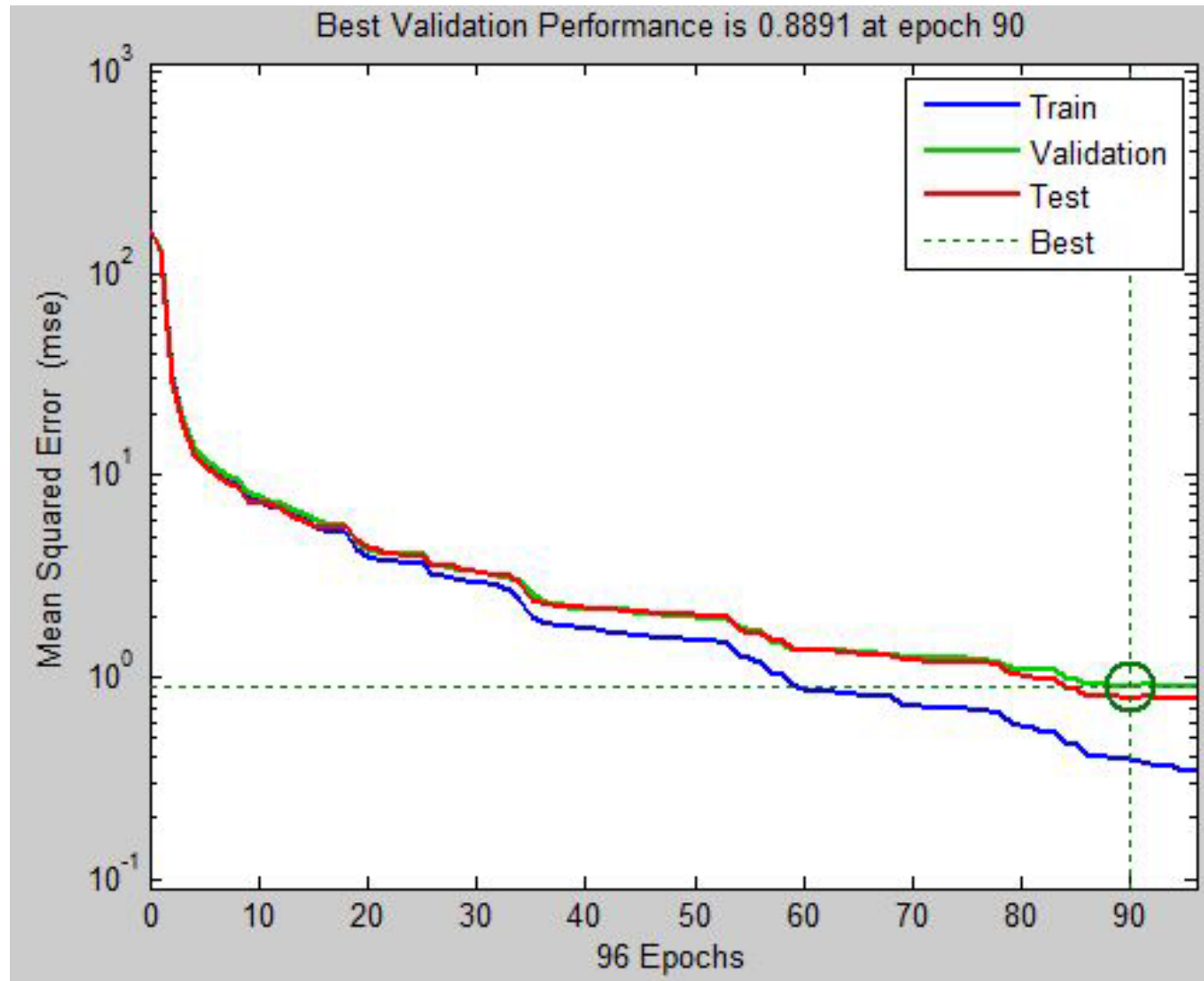
Robot Structure Learning



- Latent variables:
 - Distributed on $S^1 \times S^1$ topology
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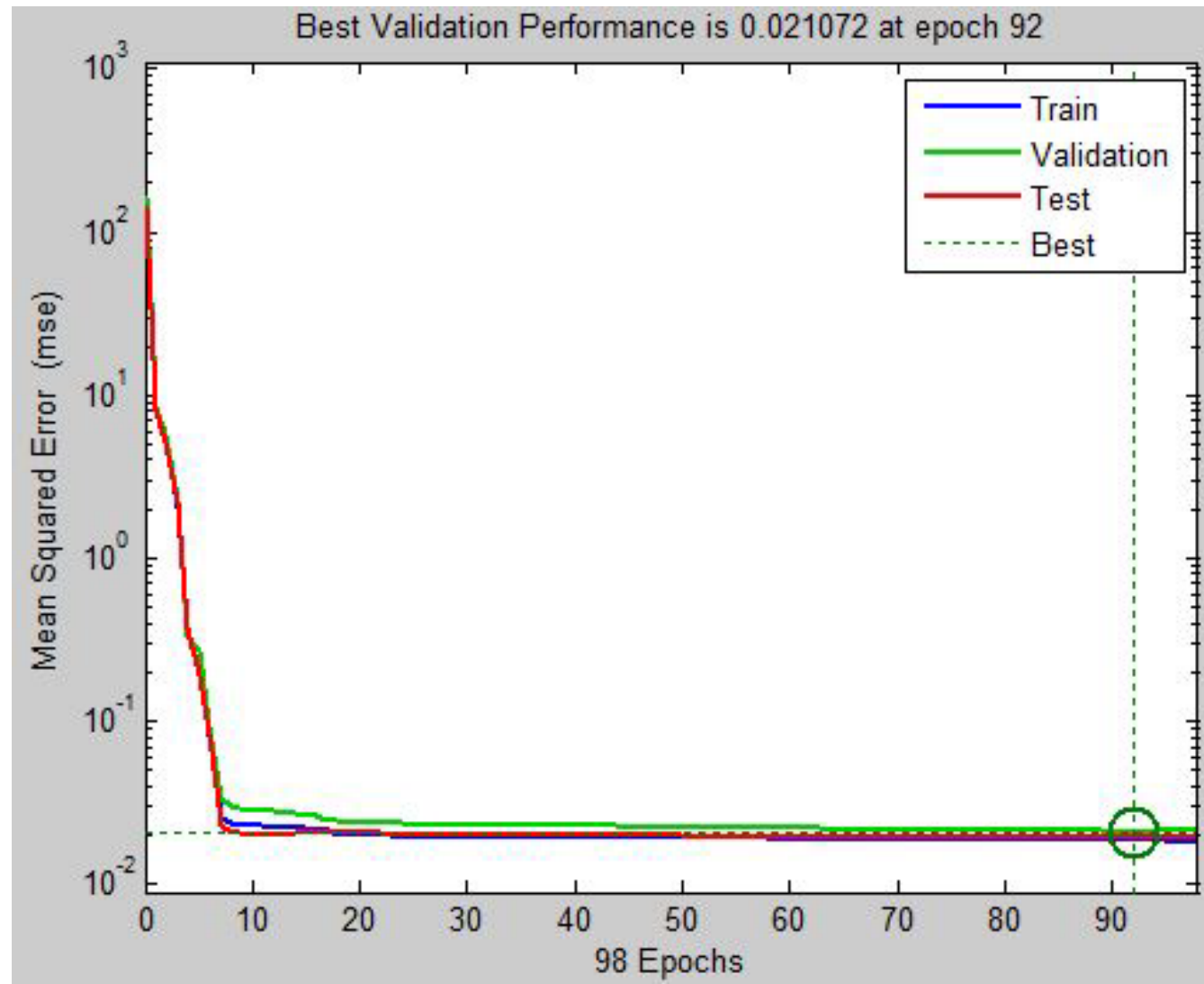
Mapping to control parameters

Supervised:
Image to (θ_1, θ_2)



Mapping to control parameters

Supervised:
Manifold (y_1, y_2)
to (θ_1, θ_2)



Formal and Naïve Representations

Formal Representation

{

dofs - 2

parameters - Θ : $\Theta_1, \Theta_2 \Theta \in Q \subseteq S^1 \times S^1$

forward mapping - $\text{workspace}(\Theta_1, \Theta_2): Q \rightarrow I_m \subset R^D$,

I_m : image space

inverse mapping - $\text{Cspace}(I): I_m \rightarrow Q$

motion plan - $\text{wspace}(I_s, I_g) \rightarrow I_s, I_1, I_2, \dots, I_n, I_g$

motion plan - $\text{Cspace}(q_s, q_g) \rightarrow q_s, q_1, q_2, \dots, q_n, q_g$

}

Discovered (Naïve) Representation

Naive Representation

{

dimensionality : 2

discovered parameters: $\Phi: \phi_1, \phi_2 \in Y \subseteq S^1 \times S^1$

forward mapping - workspace(ϕ_1, ϕ_2): $Y \rightarrow I_m \subset R^D$,

I_m : image space

motion plan - wspace(I_s, I_g) $\rightarrow I_s, I_1, I_2, \dots, I_n, I_g$

motion plan - Cspace(q_s, q_g) $\rightarrow q_s, q_1, q_2, \dots, q_n, q_g$

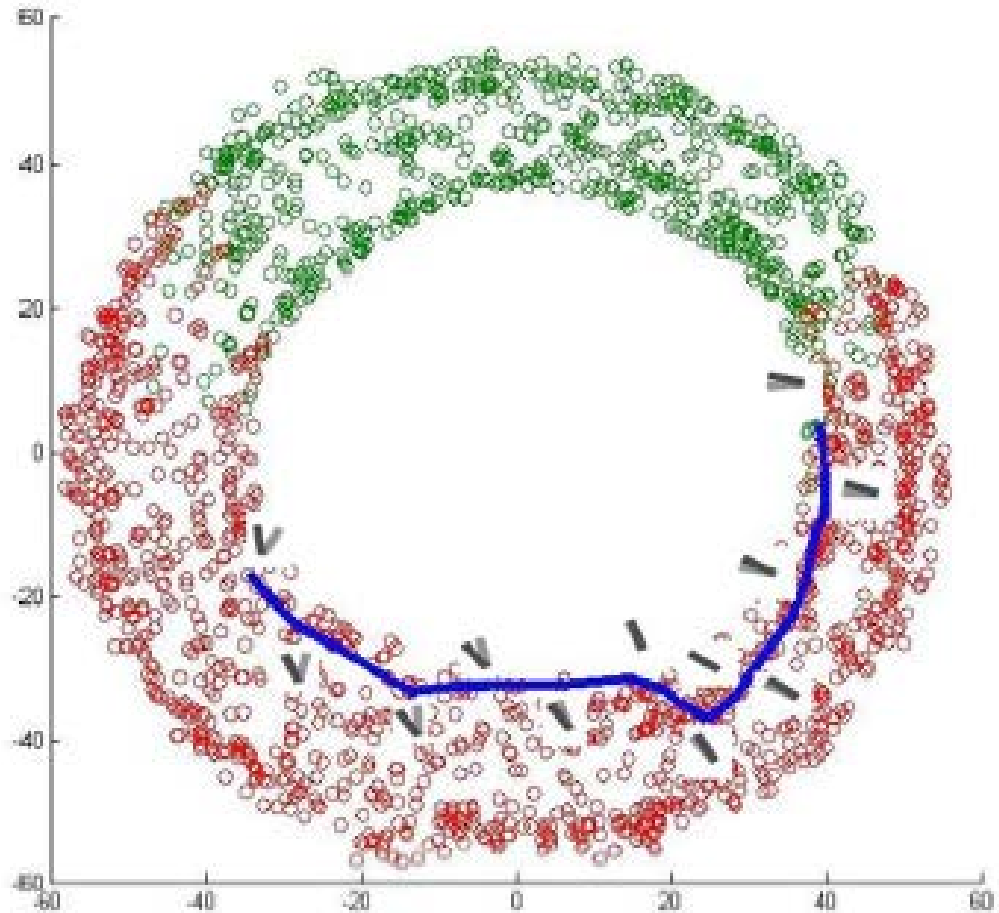
}

Motion Planning

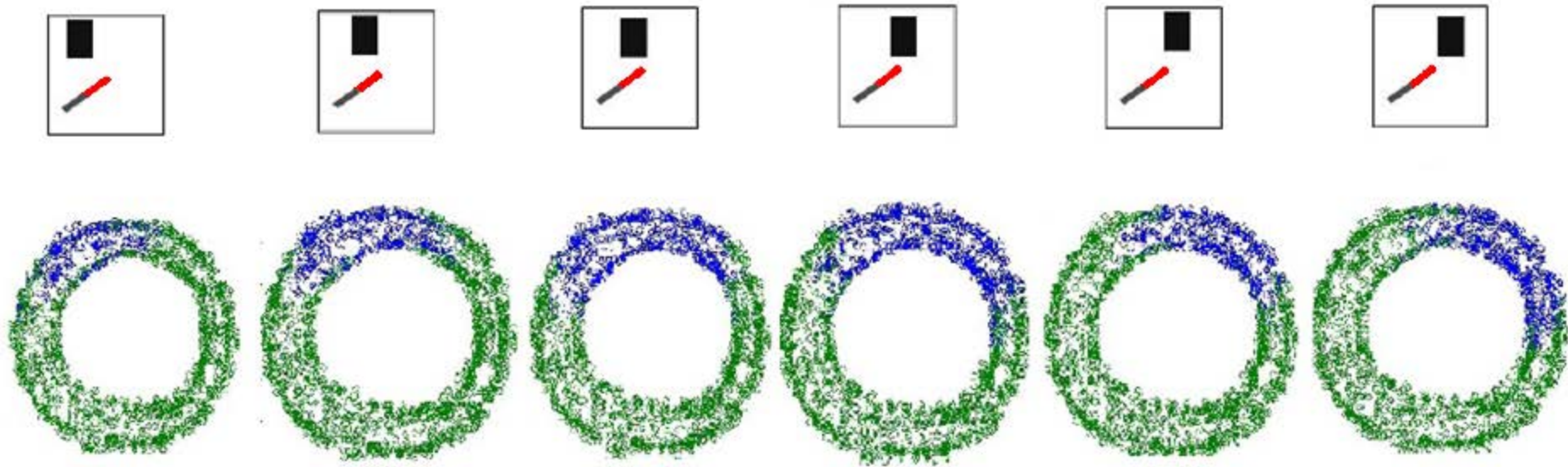
Motion planning

Given start / goal image,
map to manifold using
local interpolation

Use k-nn connectivity in
manifold as “roadmap”
for motion planning

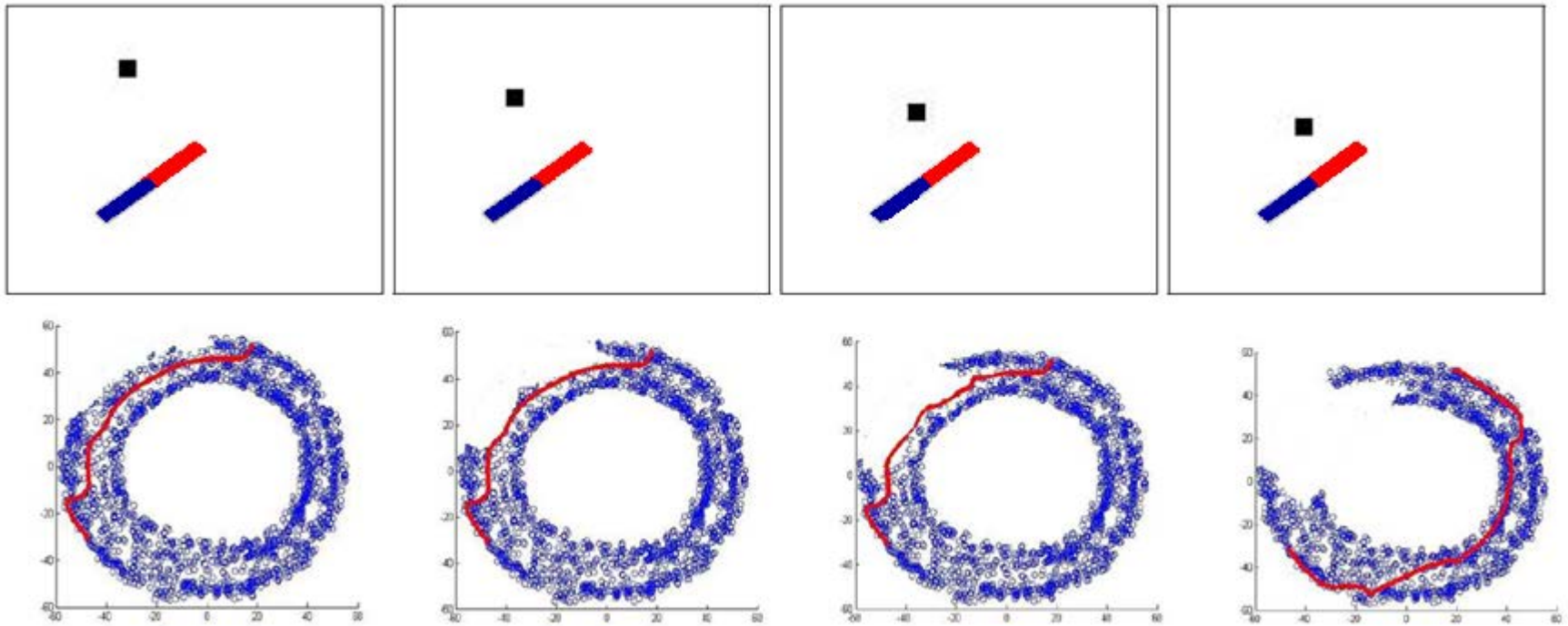


Obstacle modeling by node deletion



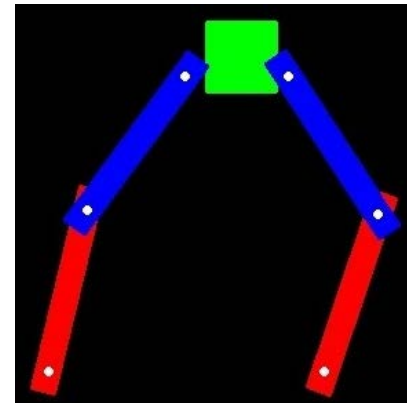
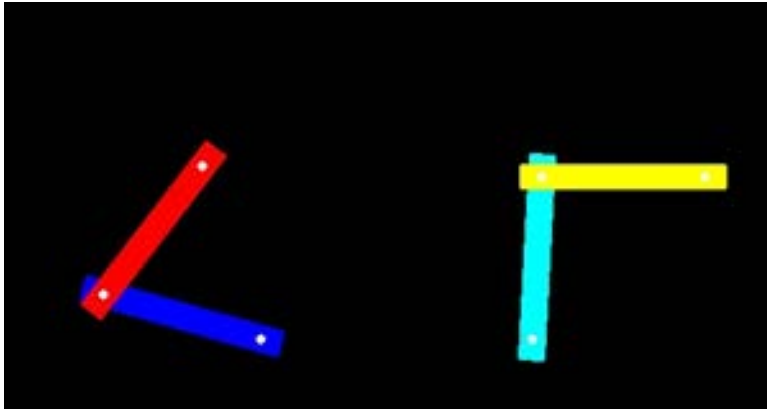
If obstacle intersects robot in image space →
delete corresponding nodes from “visual roadmap”

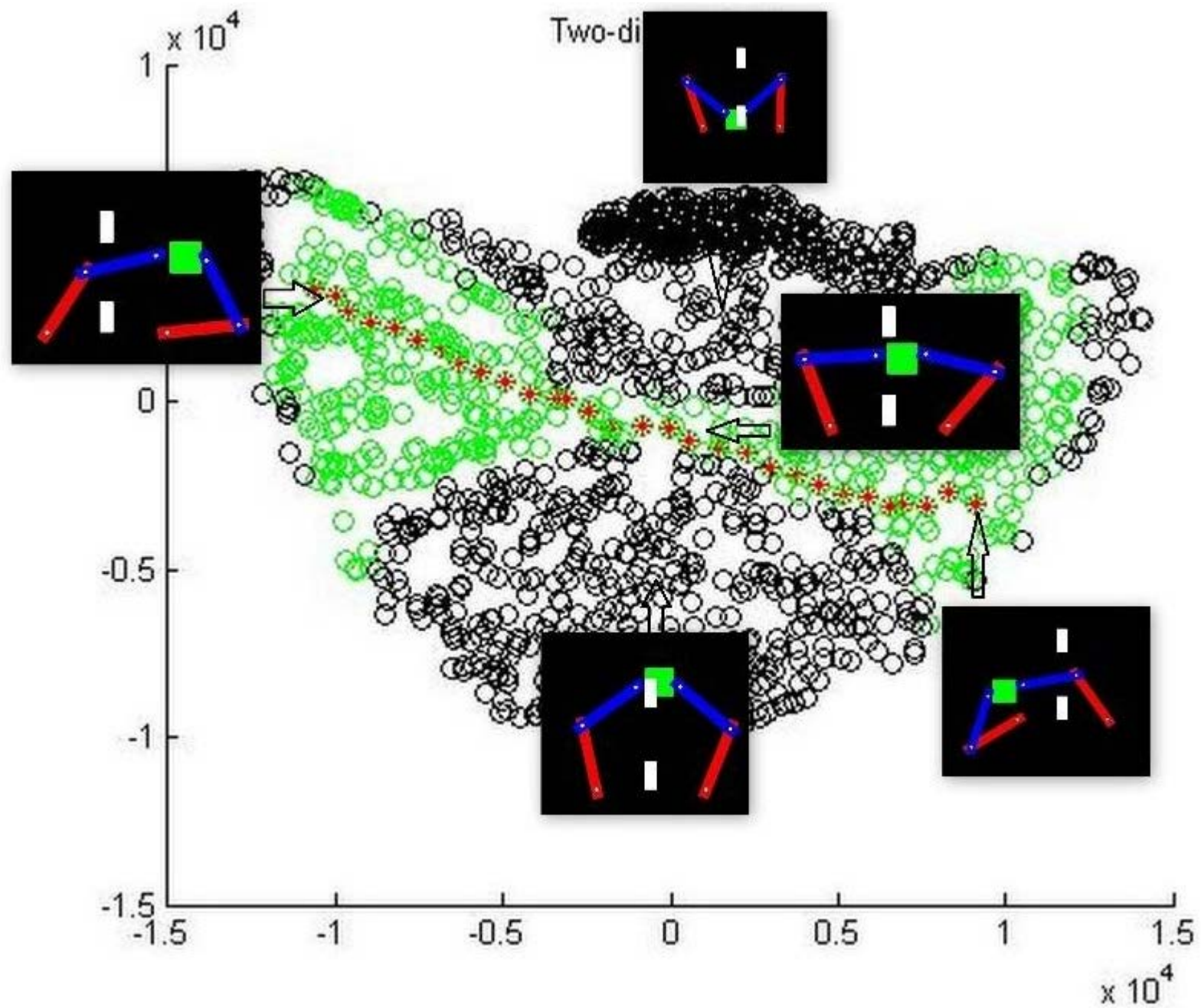
Path planning as obstacle moves



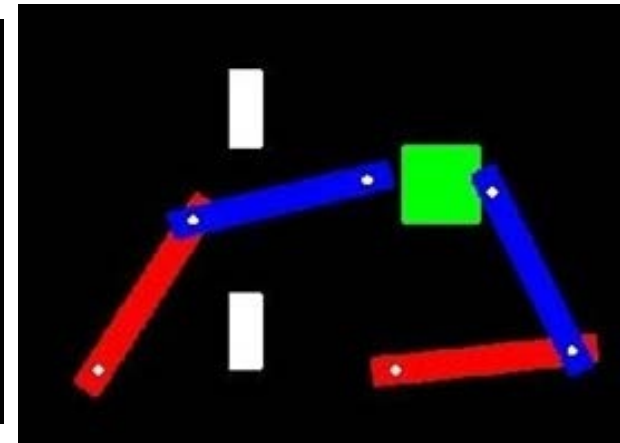
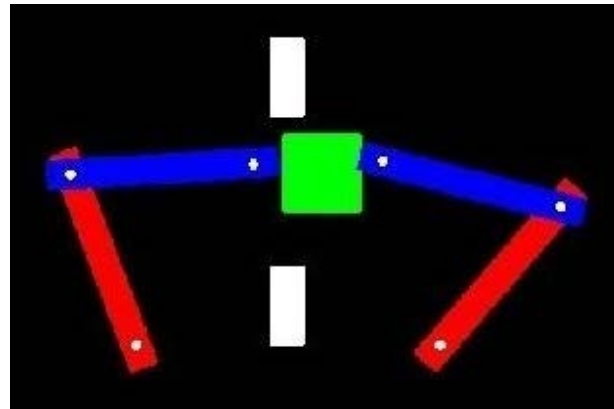
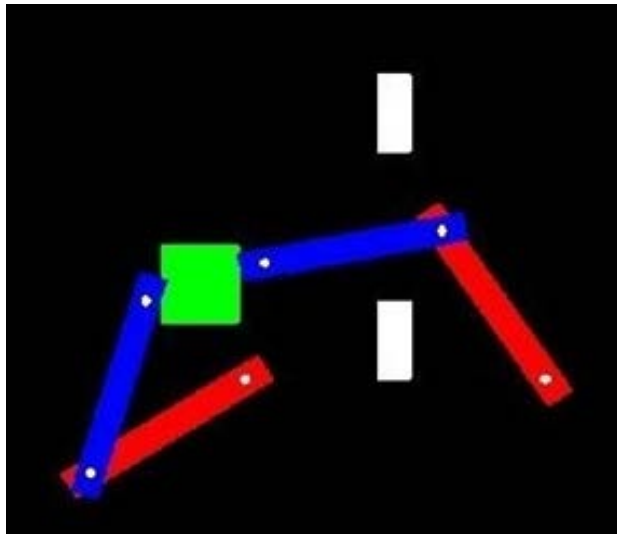
Constrained Motion

Obstacle modeling by node deletion

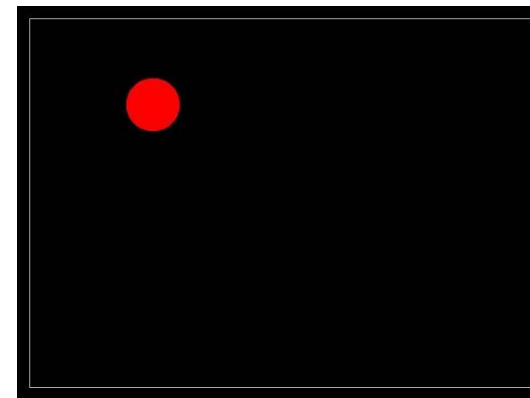
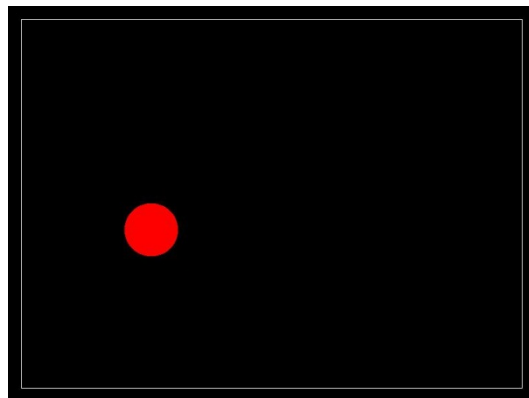
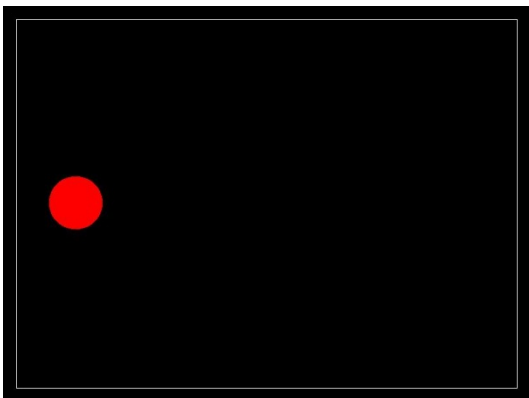
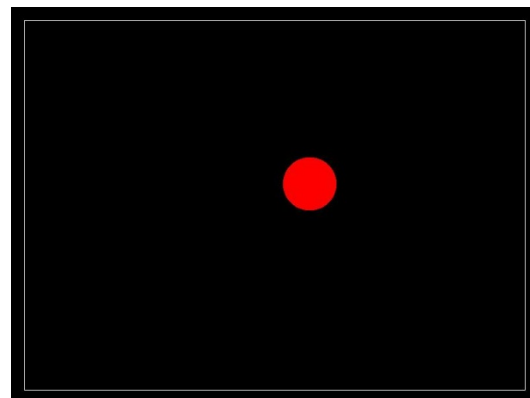
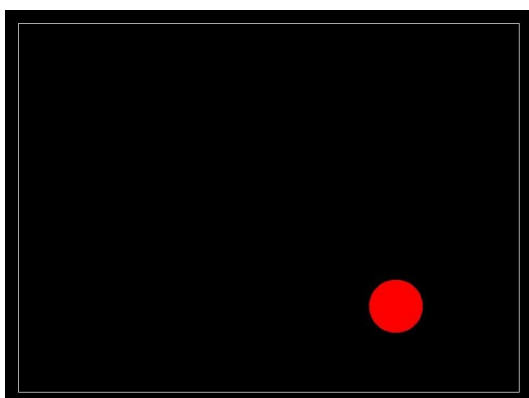
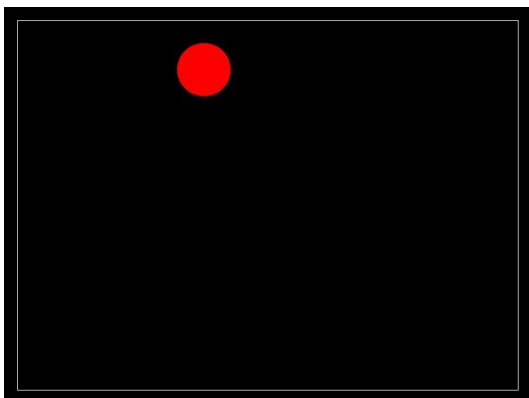
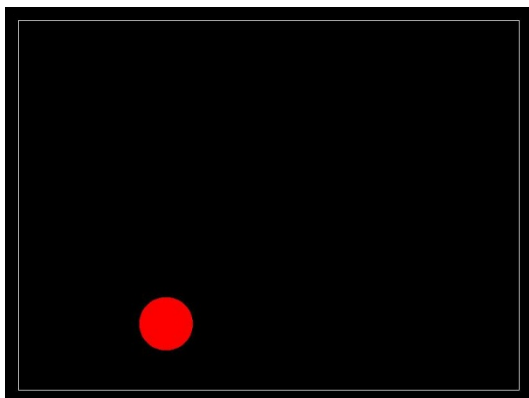
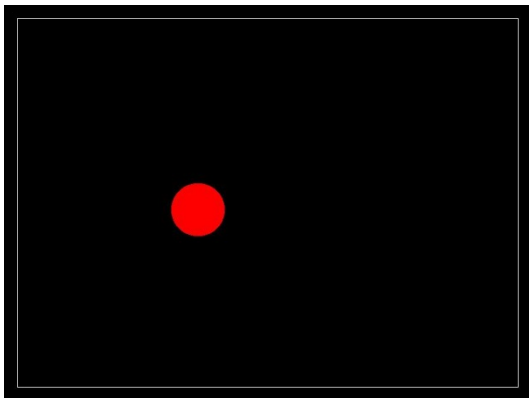




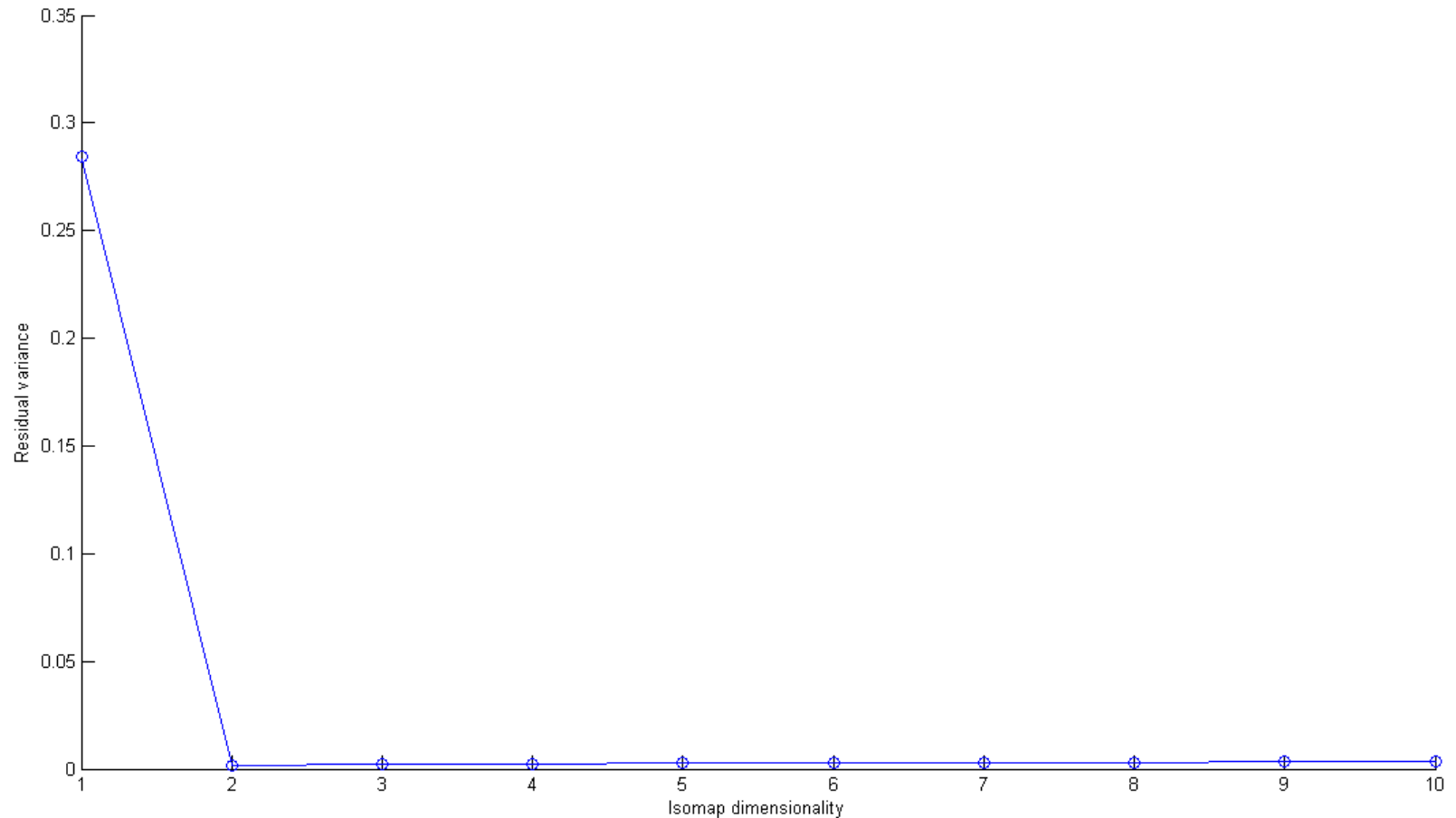
Obstacle modeling by node deletion



Mobile robots

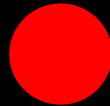


Residual error : disk robot

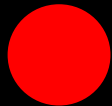


Robot Motion Planning

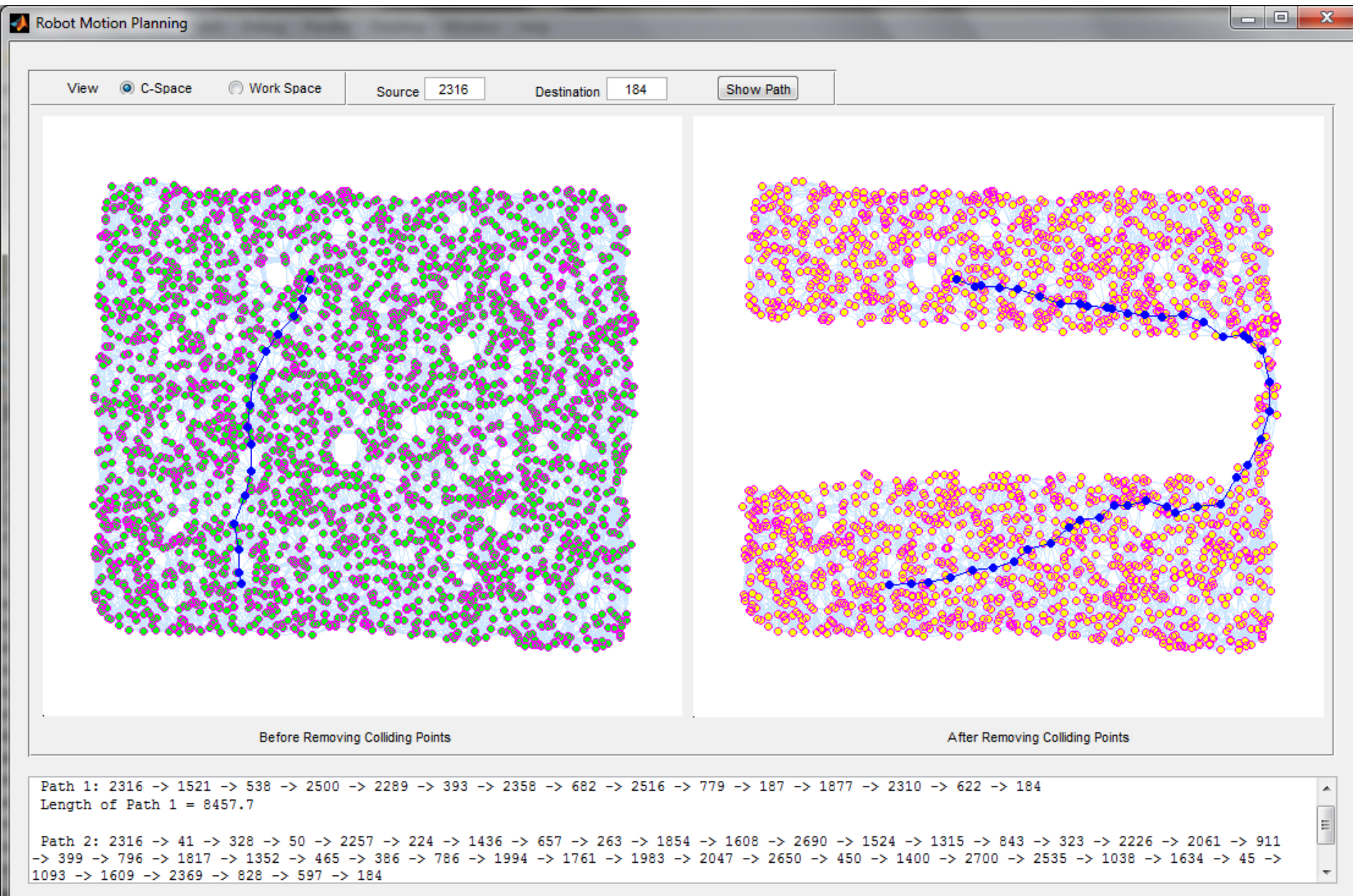
Destination



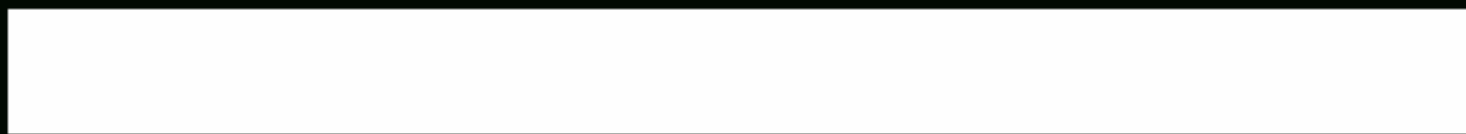
Source



Path Planning Interface

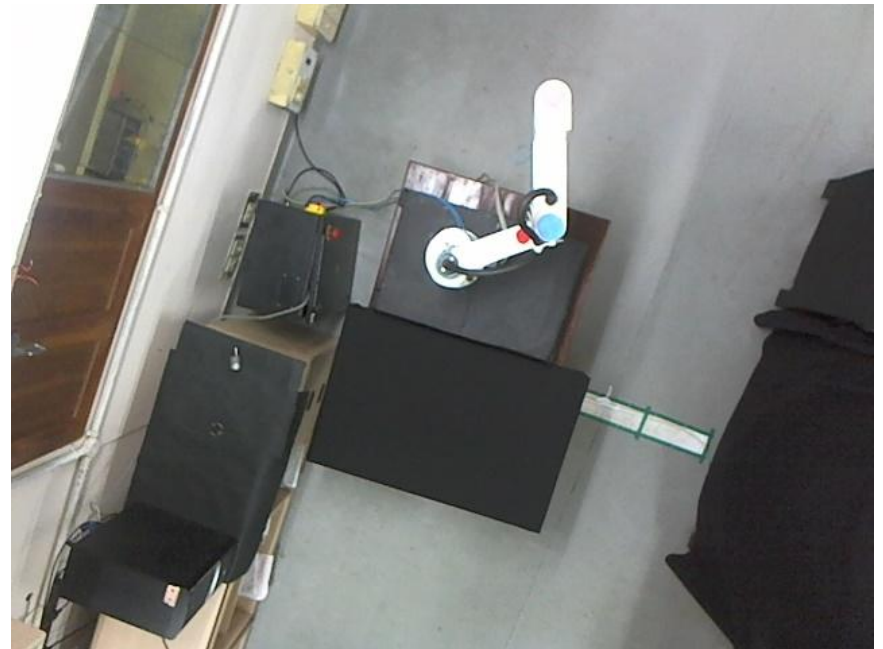




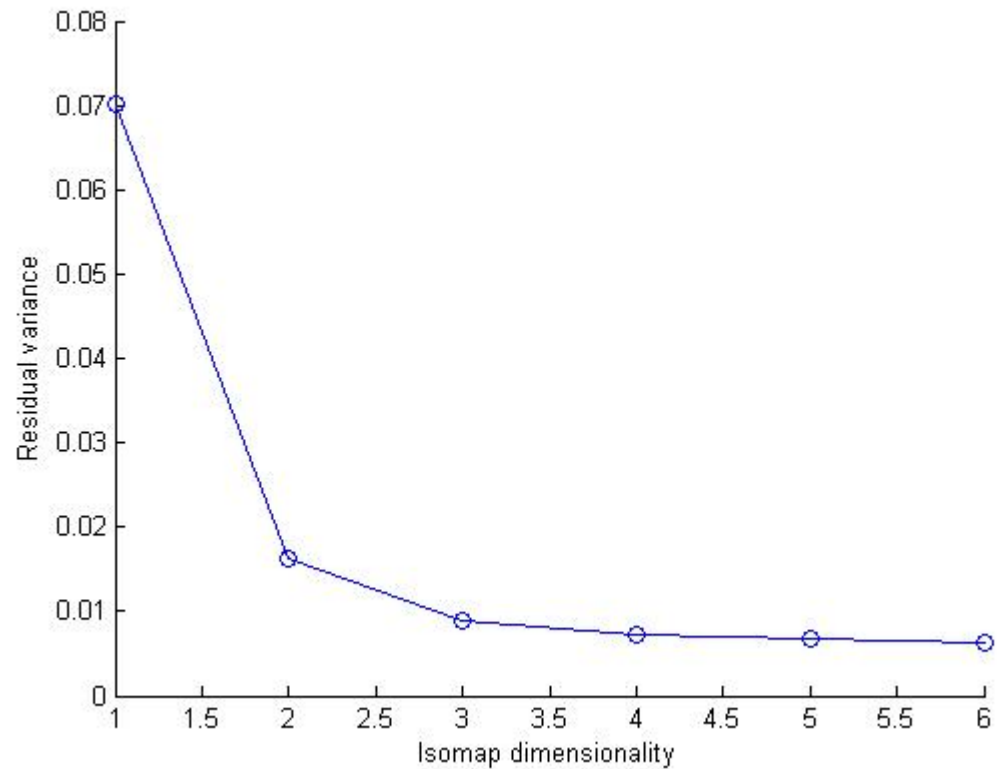


Real robots

SCARA arm



SCARA arm : degrees of freedom

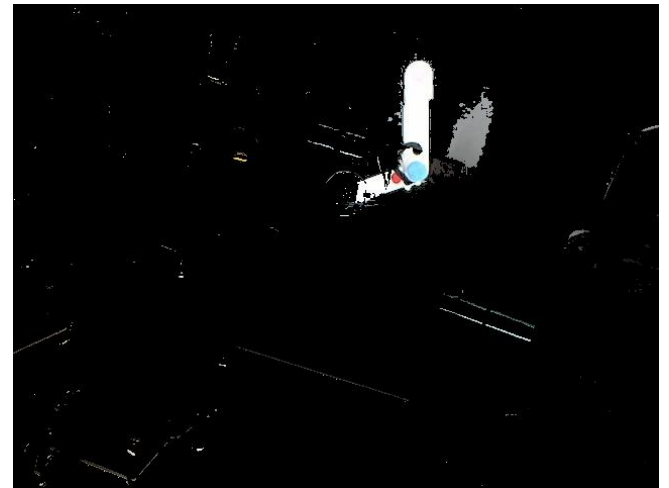


Background Subtraction : Robot



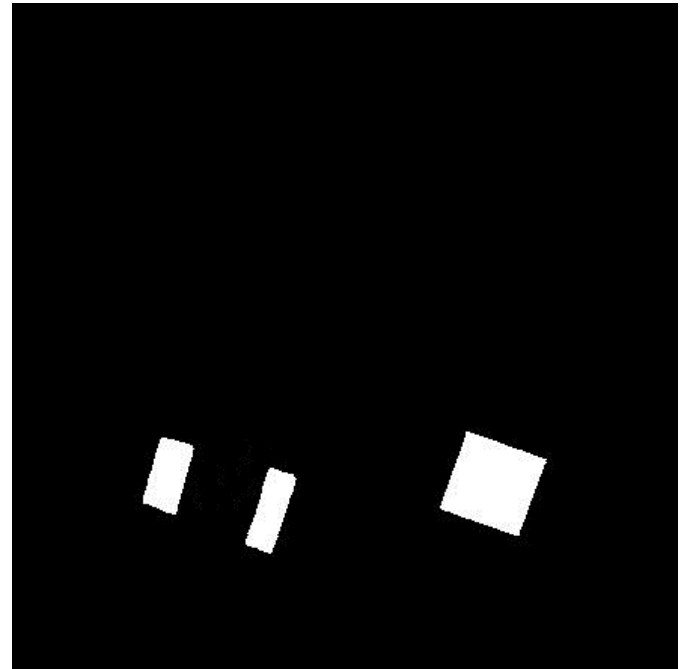
image

foreground (moving part)

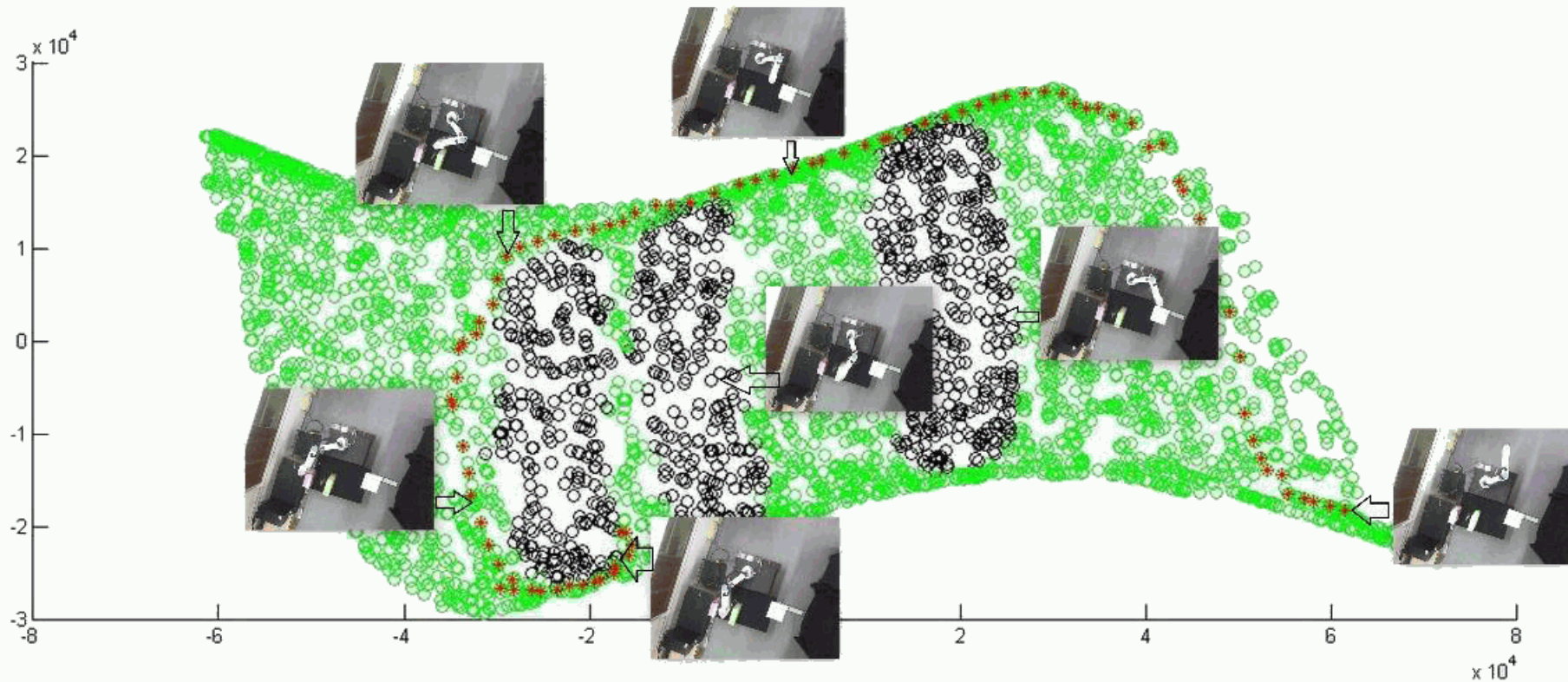


learned background

Background Subtraction : Obstacle

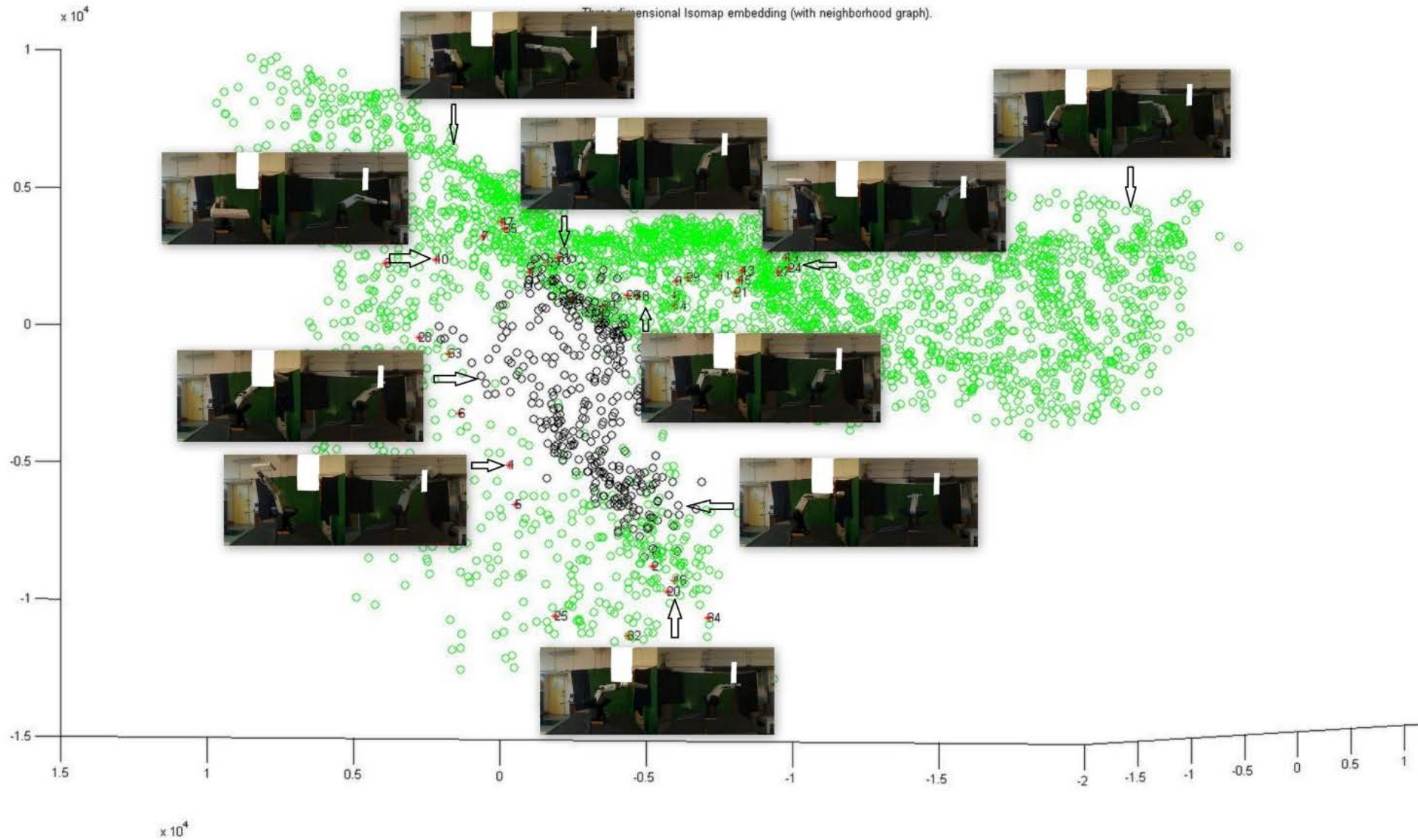


Visual Configuration Space



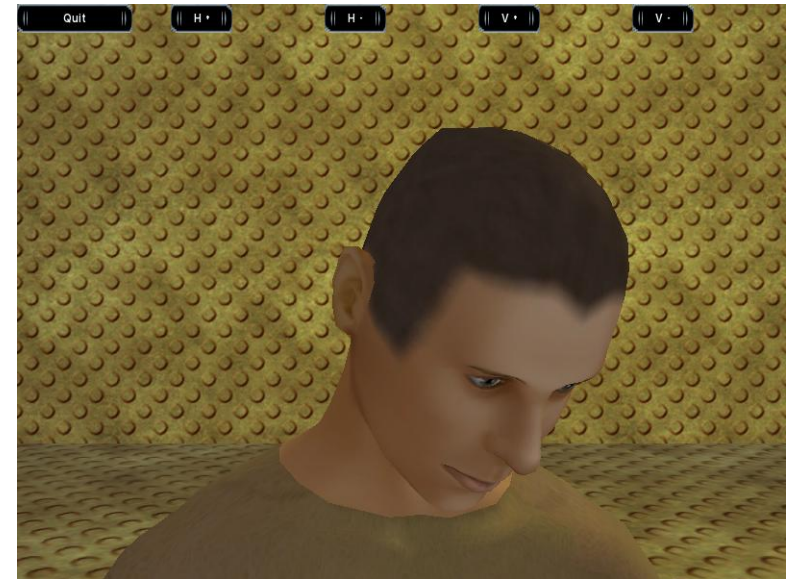


5-DOF Adept CRS manipulator



Application to Graphics

Head Motion

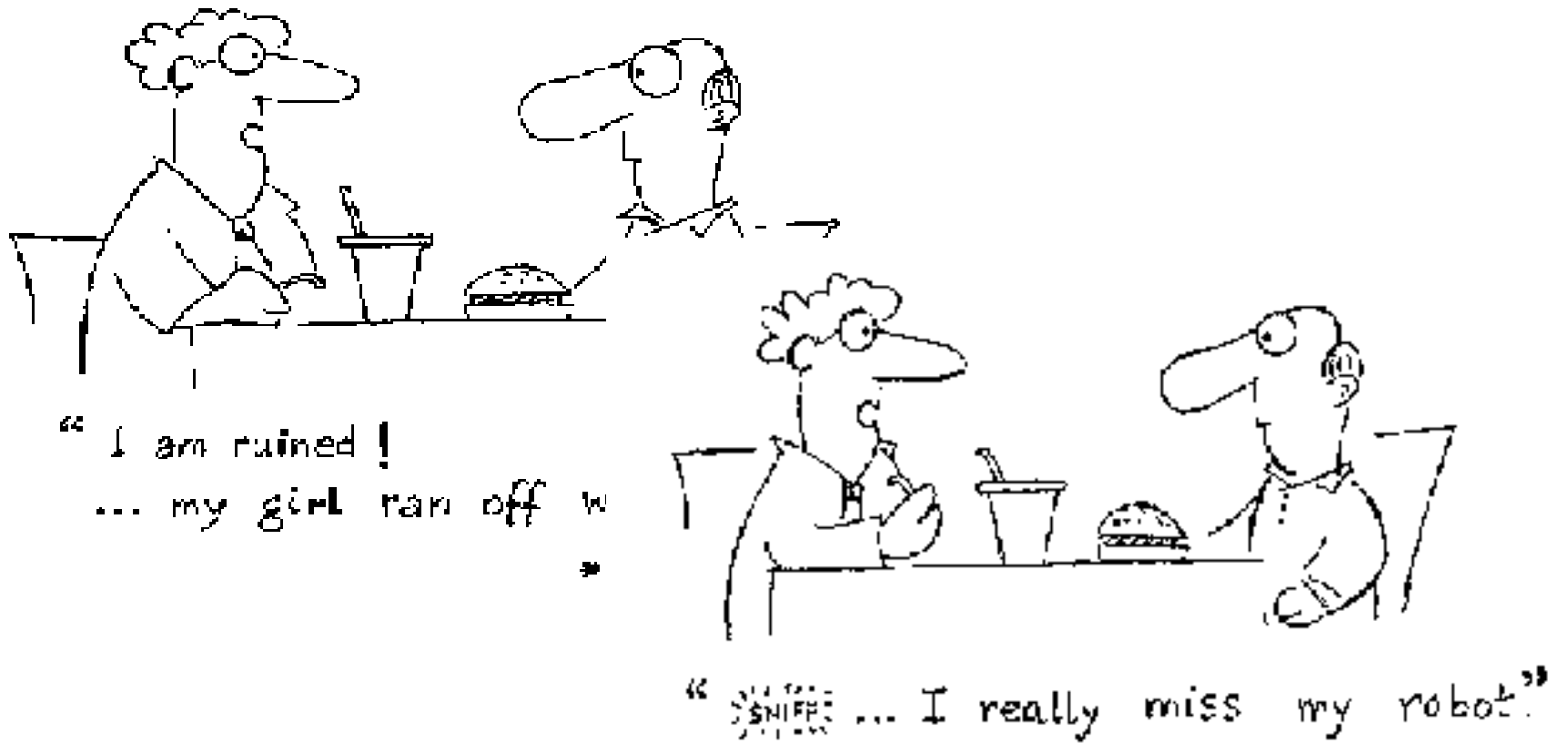


Conclusion

Beyond Geometry

- Geometry is not everything!!
- Real robots have limitations on acceleration owing to torque / inertia → **Dynamics**
- **Learning** to plan motions?
 - Babies learn to move arms
 - Learn low-dimensional representations of motion
- **Grasping** / Assembly : Motions along obstacle boundary

Humans and Robots



Ambastha
24 Jan 02