Isothetic Covers for Digital Objects:

Algorithms and Applications

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RESEARCH PROMOTION WORKSHOP
INTRODUCTION TO GRAPH AND GEOMETRIC ALGORITHMS
NOVEMBER 1–3, 2011 (PDPM IIITDM JABALPUR)

Object and Isothetic Cover

image

Object and Isothetic Cover

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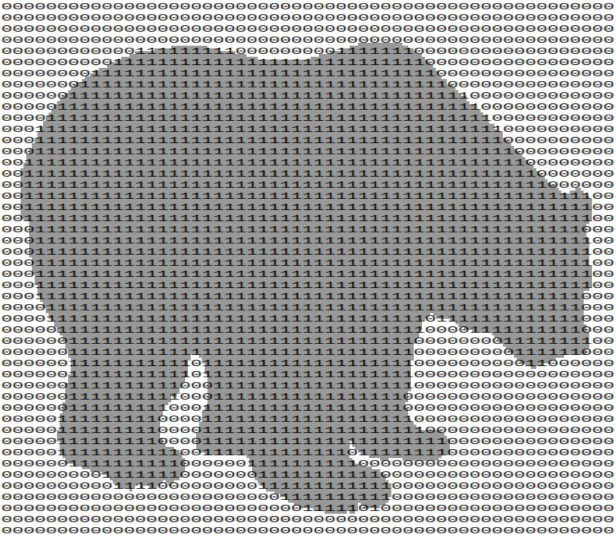
object = set of 1s

Object and Isothetic Cover



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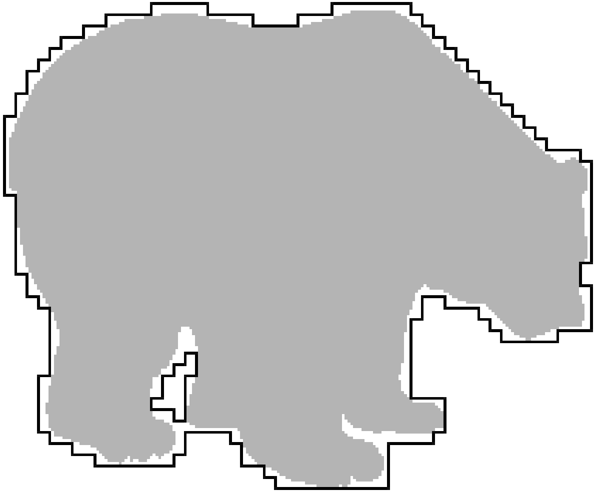
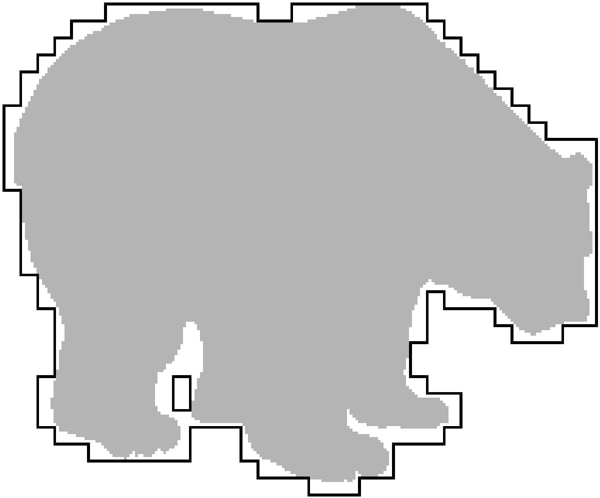


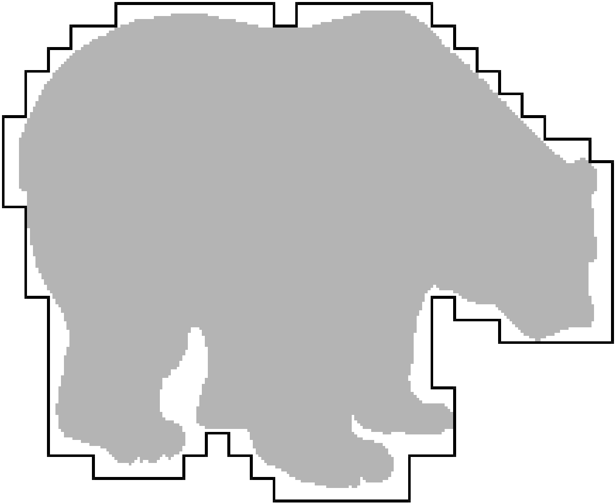
$g = 4$: Isothetic Cover

Object and Isothetic Cover



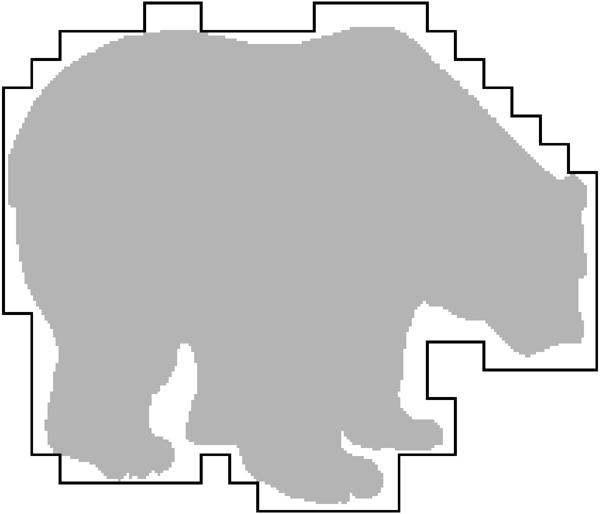
$g = 6$: Isothetic Cover

Object and Isothetic Cover



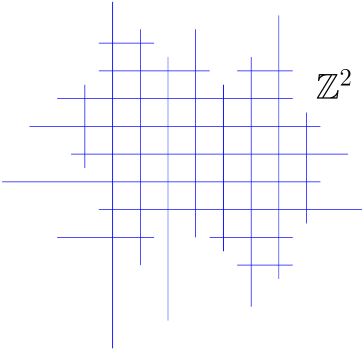
$g = 8$: Isothetic Cover

Object and Isothetic Cover



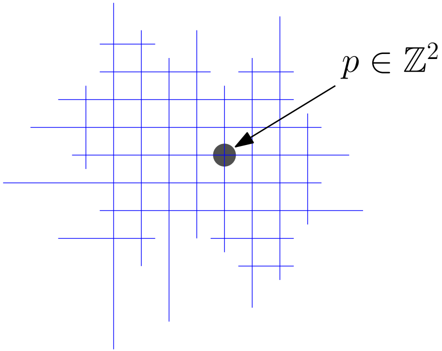
$g = 10$: Isothetic Cover

Definitions



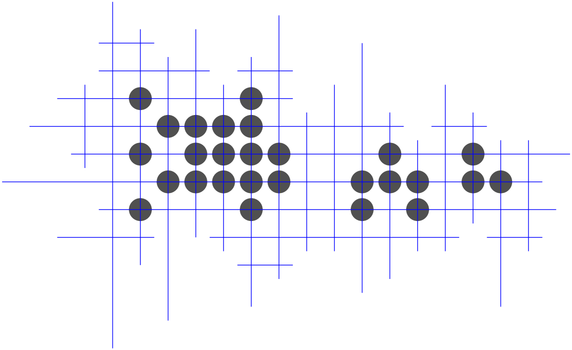
Digital plane, $\mathbb{Z}^2$ = set of all points having integer coordinates.

Definitions



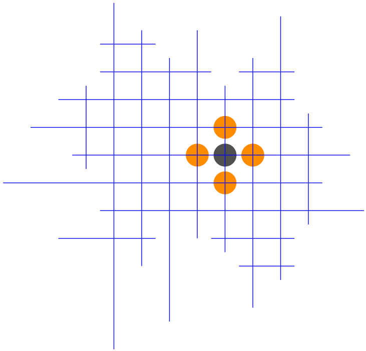
Digital point (pixel) = a point in $\mathbb{Z}^2$.

Definitions



Digital object = a set S of digital points.

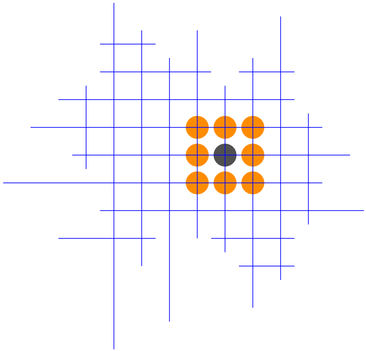
Definitions



4-neighborhood of p:

$$N_4(p) = \{(x', y') : (x', y') \in \mathbb{Z}^2 \wedge |x - x'| + |y - y'| = 1\}$$

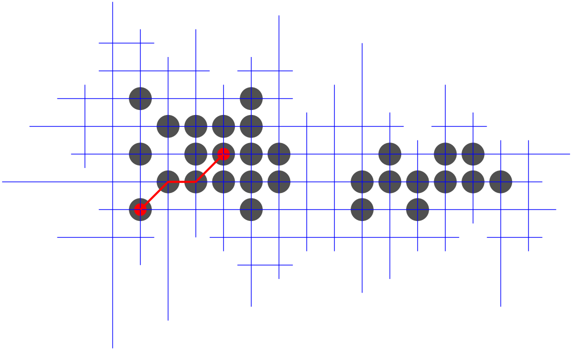
Definitions



8-neighborhood of p:

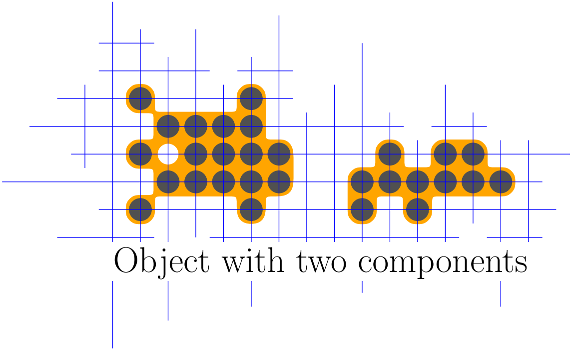
$N_8(p) = \{(x', y') : (x', y') \in \mathbb{Z}^2 \wedge \max(|x - x'|, |y - y'|) = 1\}$

Definitions



Two points p and q are k-connected in S if there exists a sequence $\langle p := p_0, p_1, \ldots, p_n := q\rangle \subseteq S$ such that $p_i \in N_k(p_{i-1})$ for $1 \leqslant i \leqslant n$.

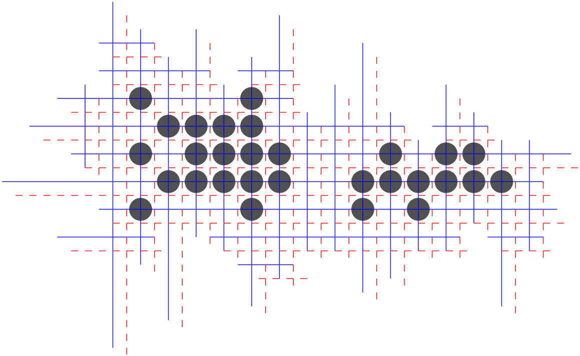
Definitions



Object with two components

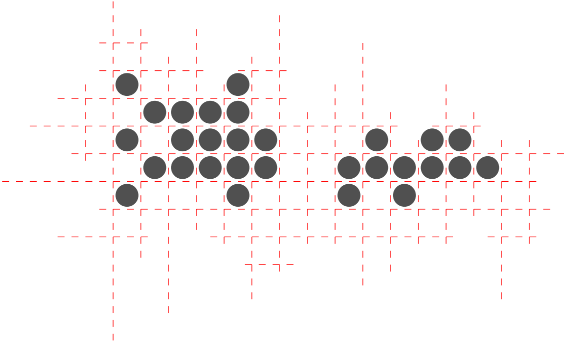
For any point $p \in S$, the maximum-cardinality set of points that are k-connected to p forms a k-connected component of S.

Definitions



Grid $\mathbb{G}$ with grid size $g = 1$ (red dashed lines)

Definitions



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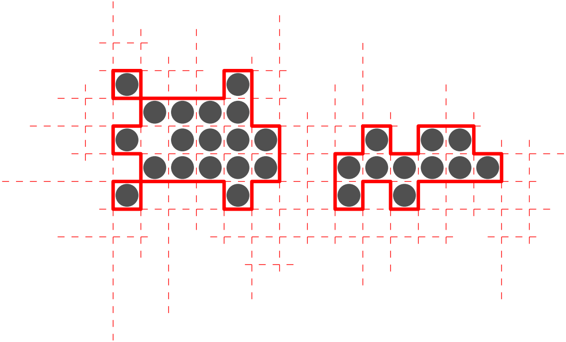
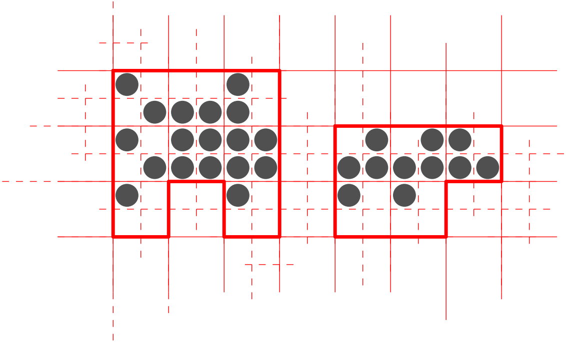


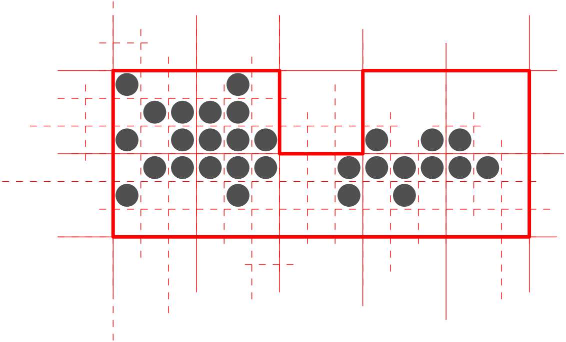
Isothetic cover for $g = 1$

Definitions



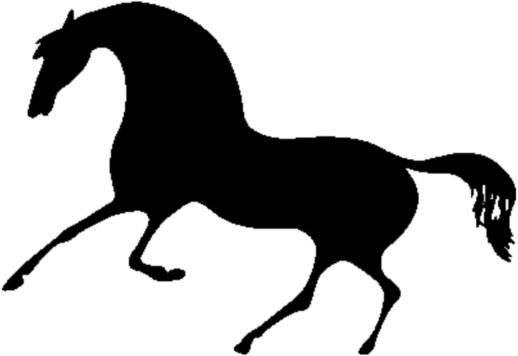
Isothetic cover for $g = 2$

Definitions

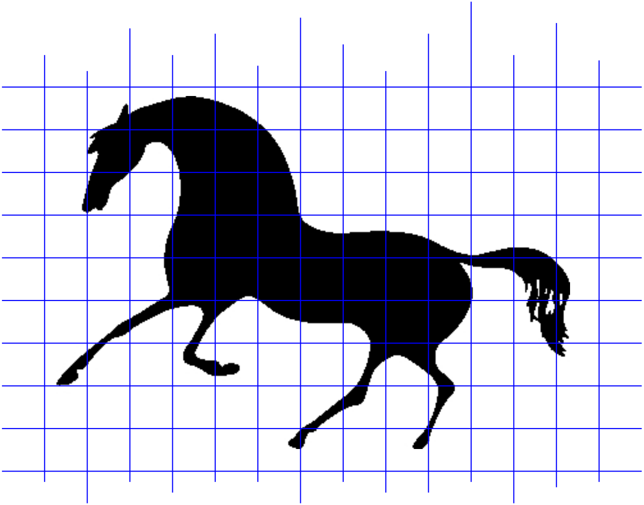


Isothetic cover for $g = 3$

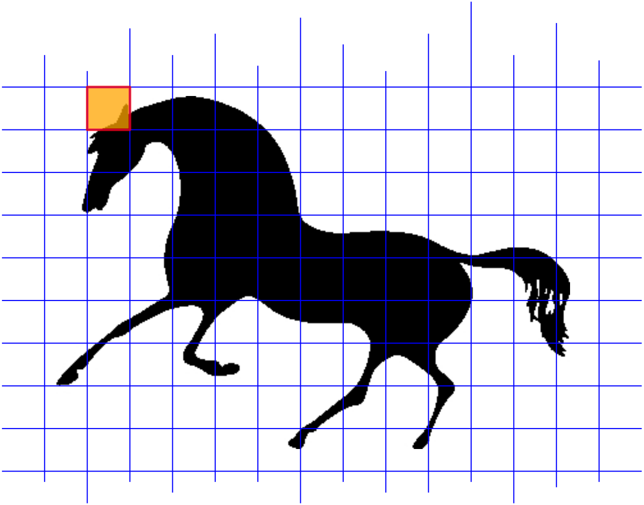
Naive algorithm



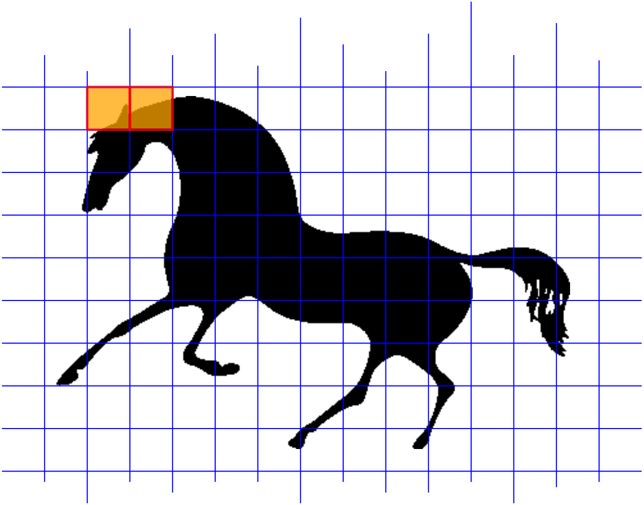
Naive algorithm



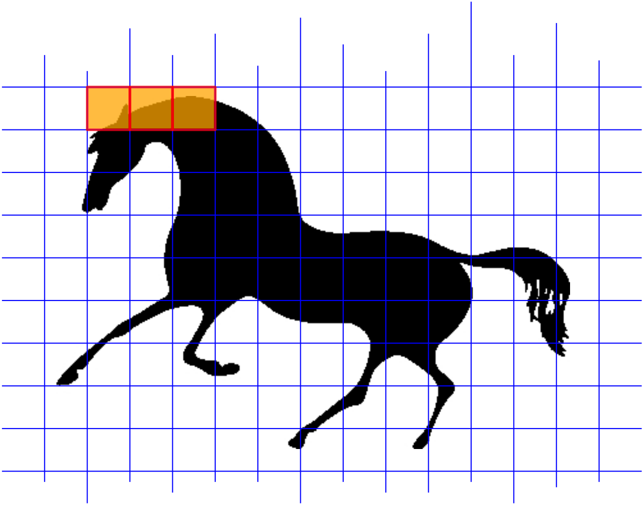
Naive algorithm



Naive algorithm



Naive algorithm



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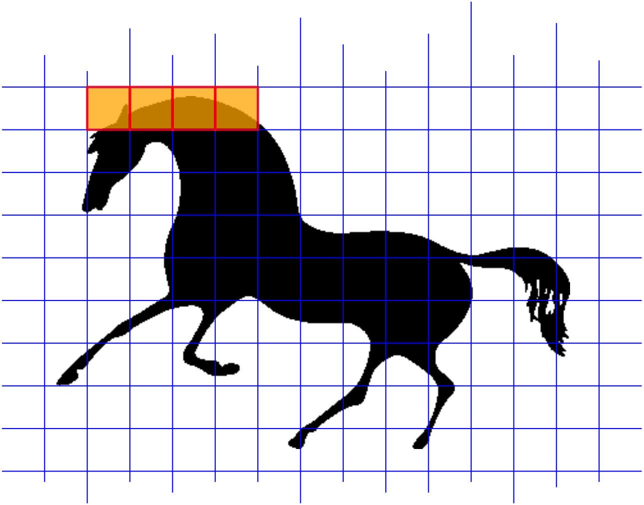

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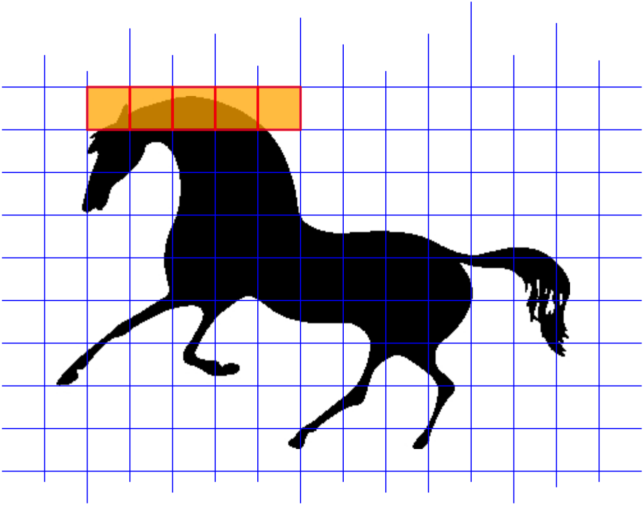

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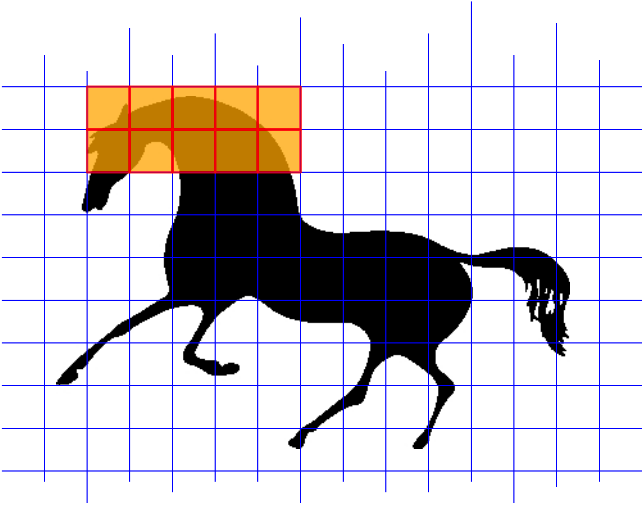

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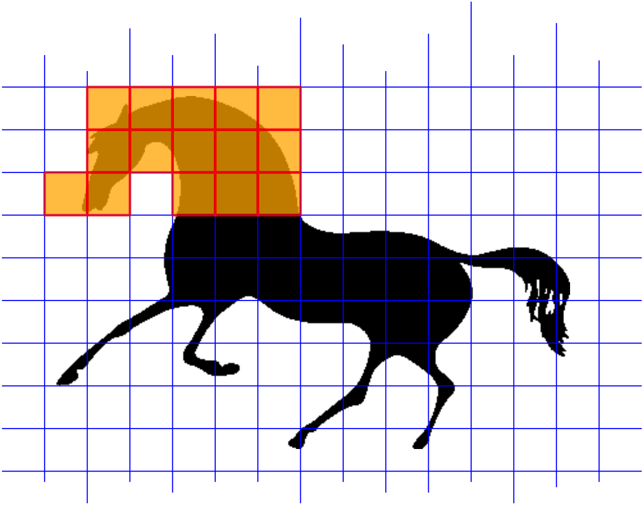

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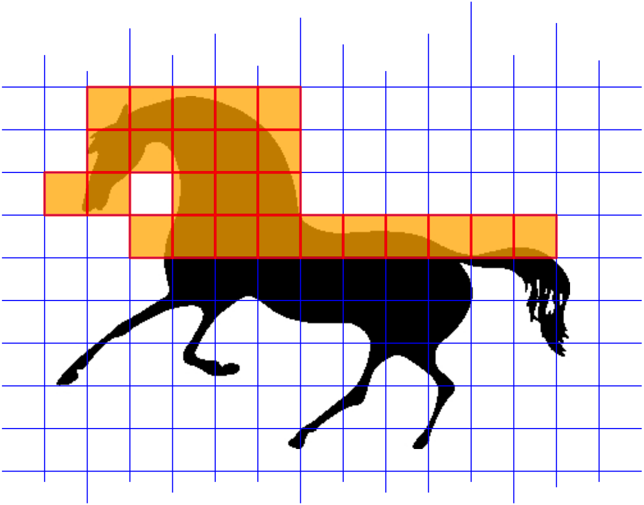

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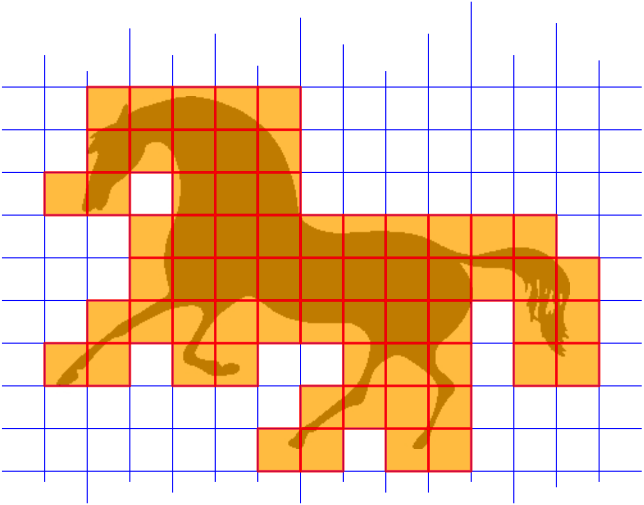

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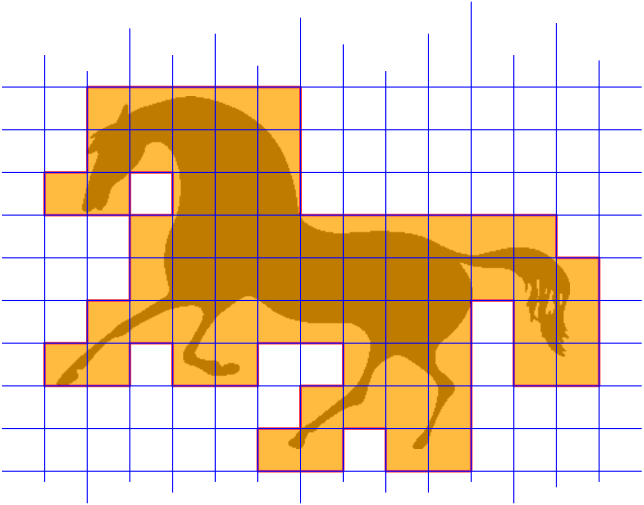

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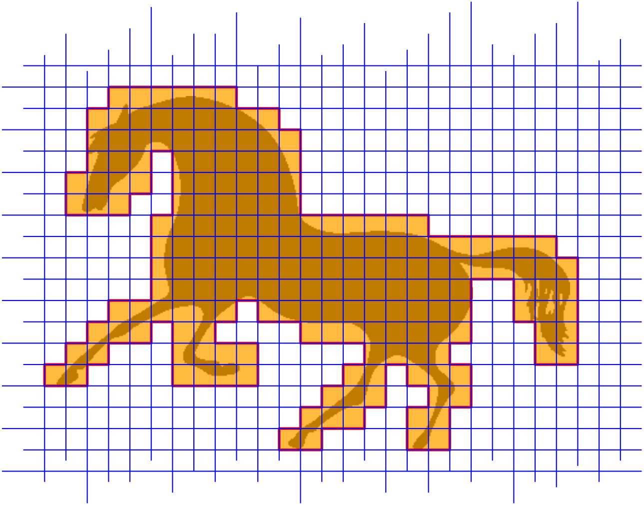

Naive algorithm

Disadvantages

- Scans the entire image
- Cell joining required to output the vertex sequence

Alternative solution: Combinatorial algorithm.

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Isothetic Cover

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Introduction

Naive

Combinatorial

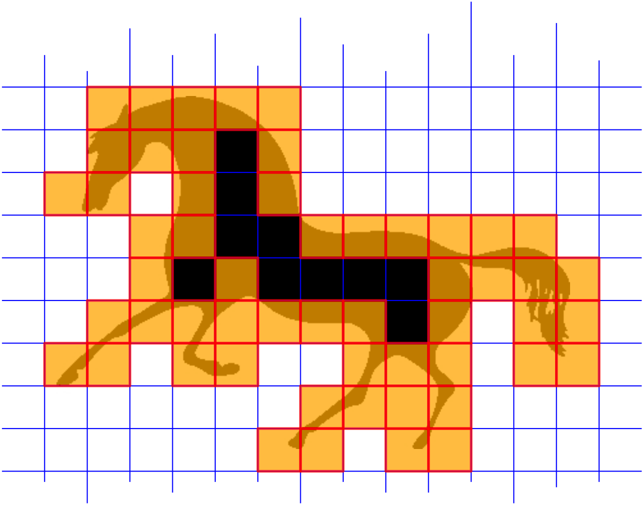
Applications

Hull

Shape

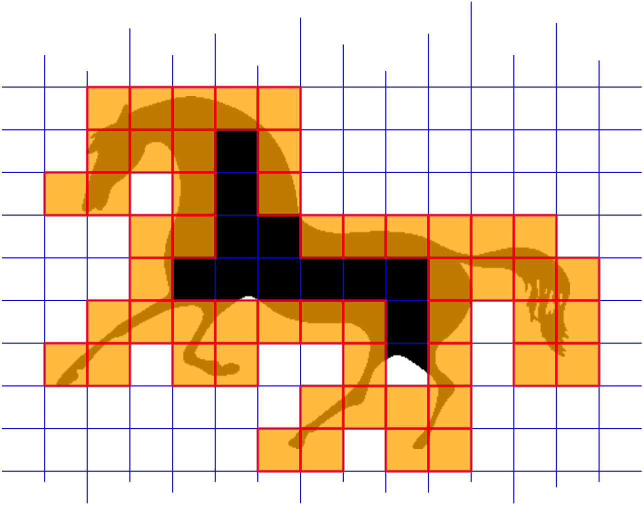
3D

Vertex types



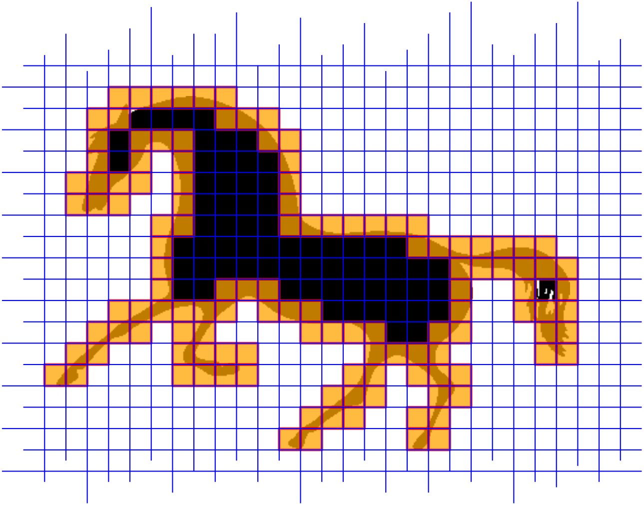
Fully black cells can be disregarded

Vertex types



Avoid also some partly black cells. Just consider the border cells.

Vertex types



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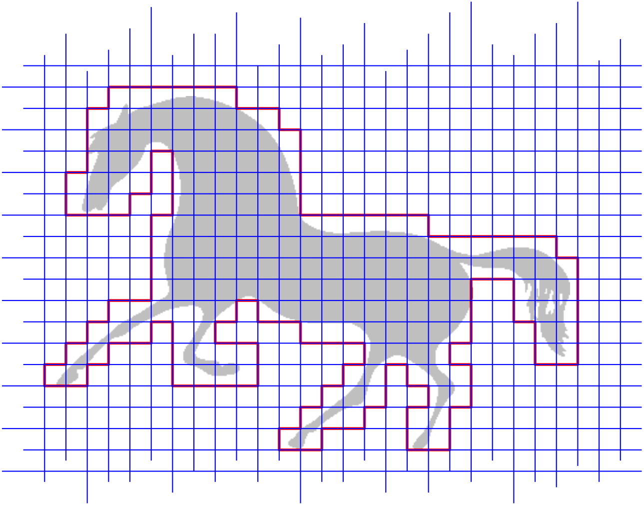
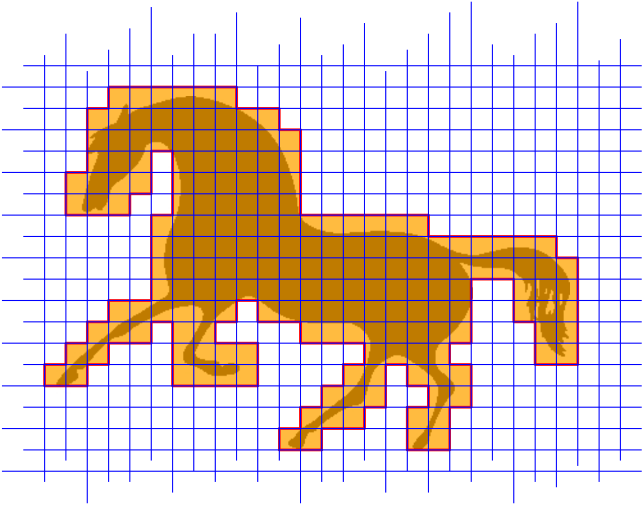


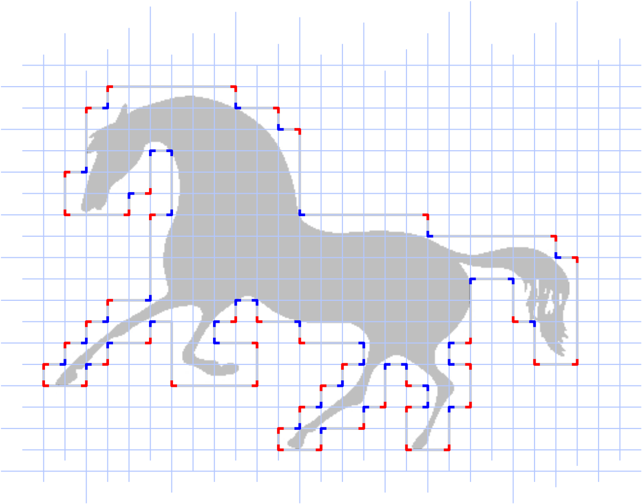
Avoid the concept of cell joining

Vertex types



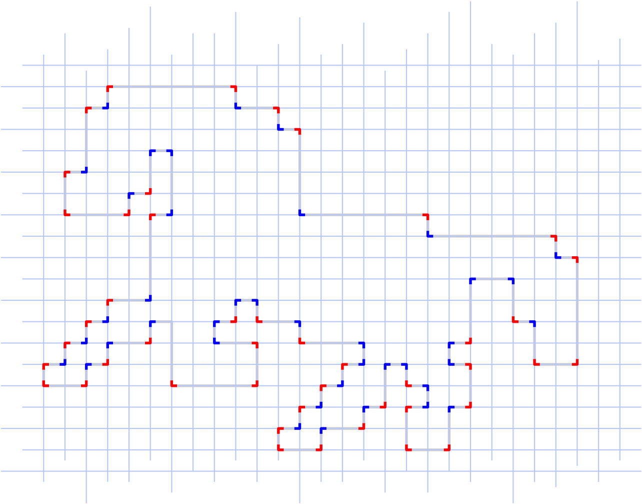
The isothetic polygon contains the object

Vertex types



Vertex angles are 90^0 and 270^0

Vertex types



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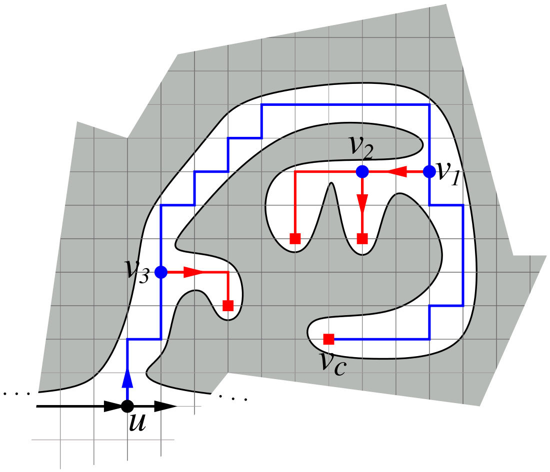
Backtracking—A serious issue



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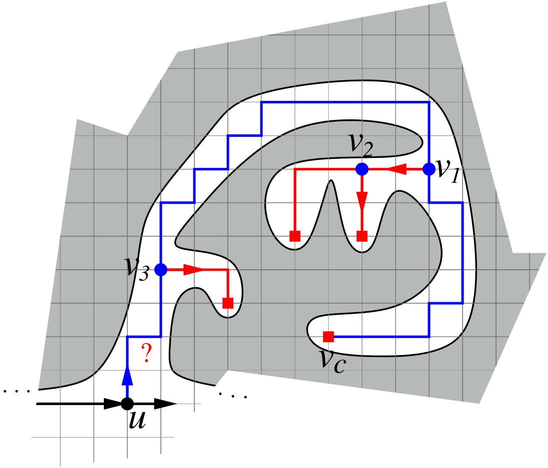
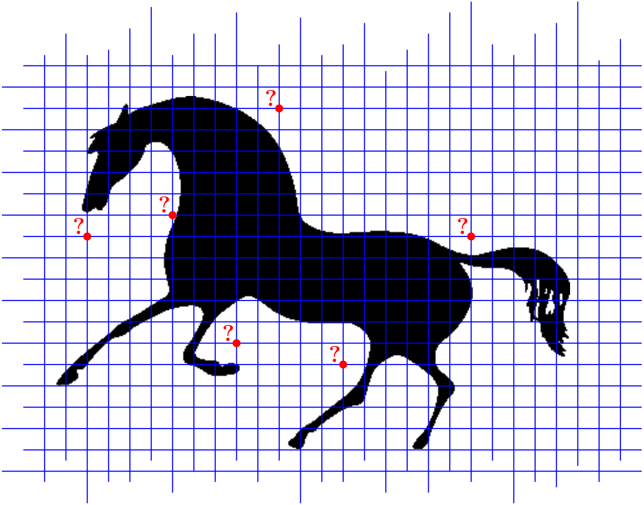


Grid point classification



Grid point classification

Isothetic Cover

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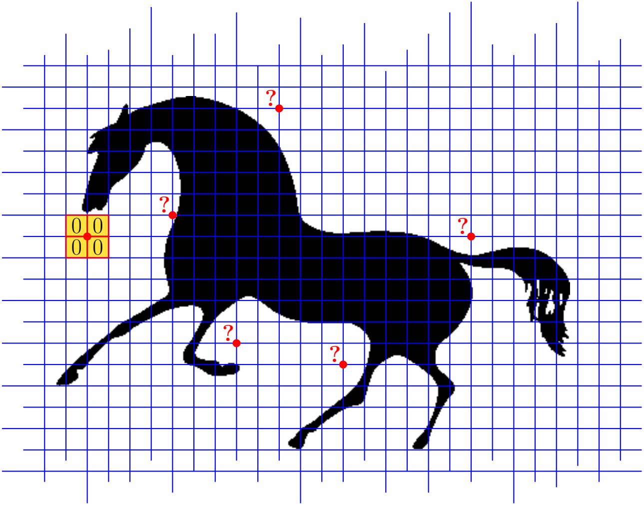

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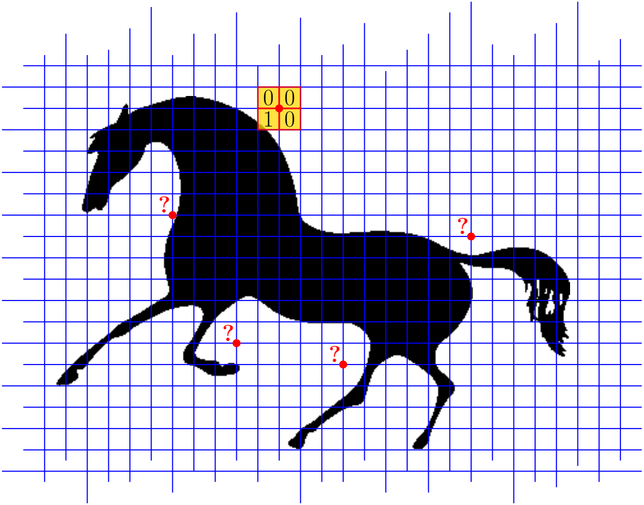

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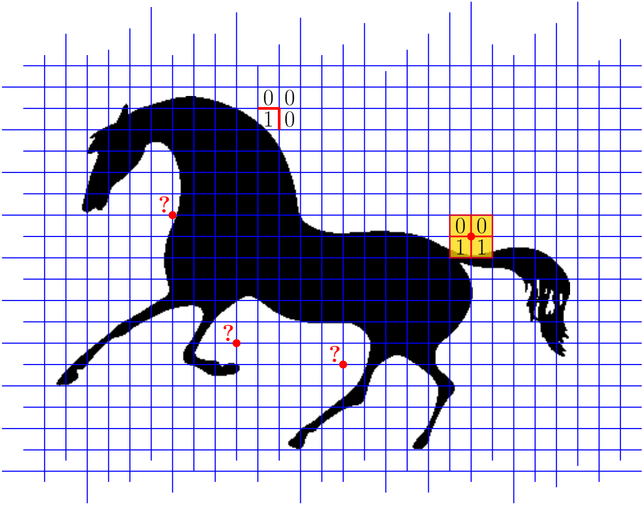

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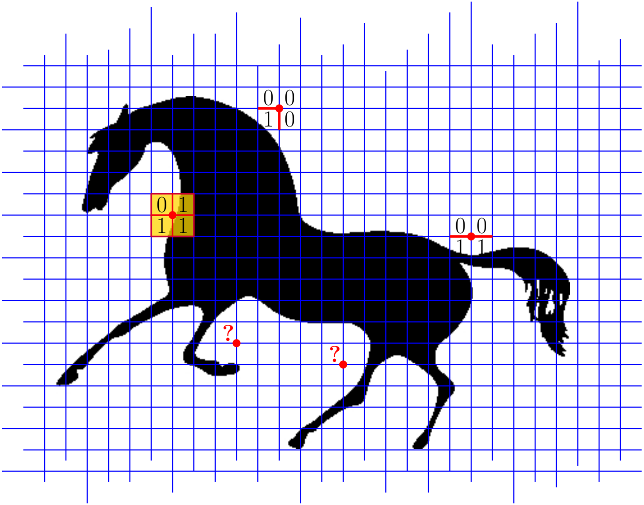

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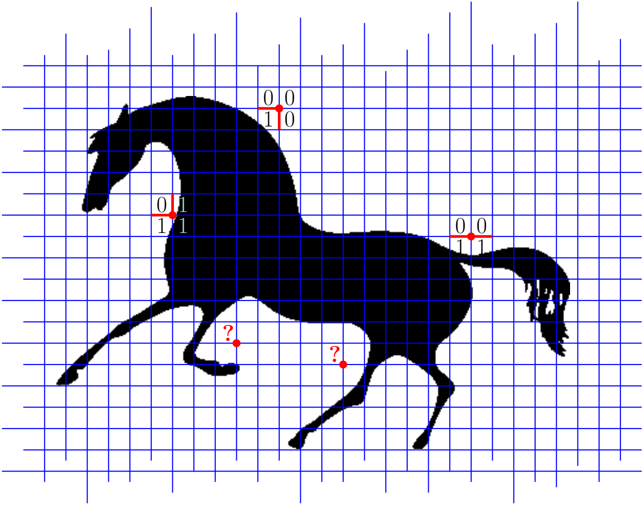

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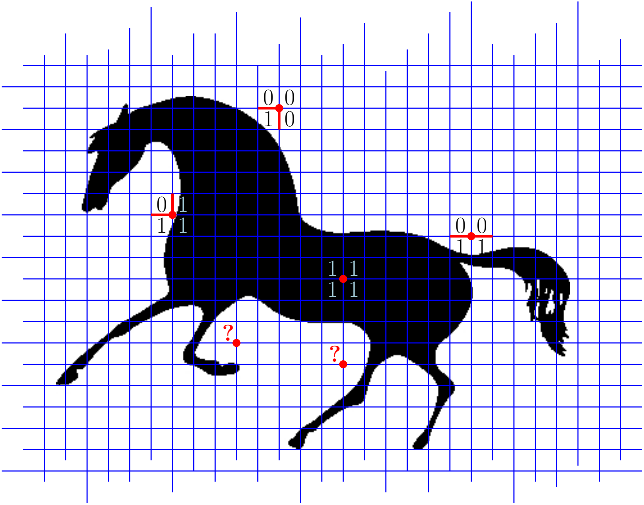

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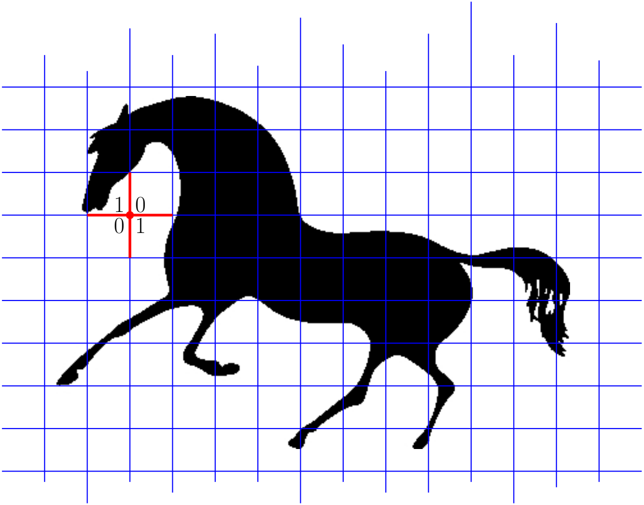

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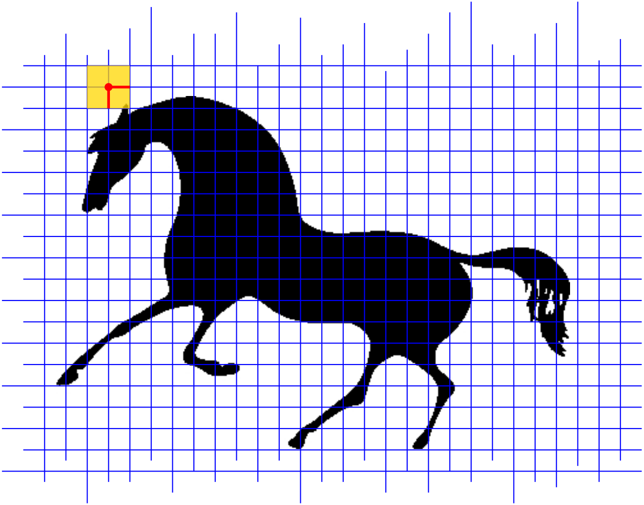

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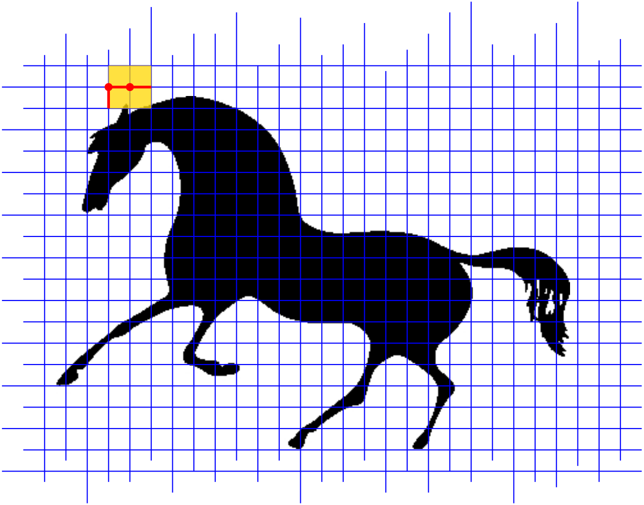

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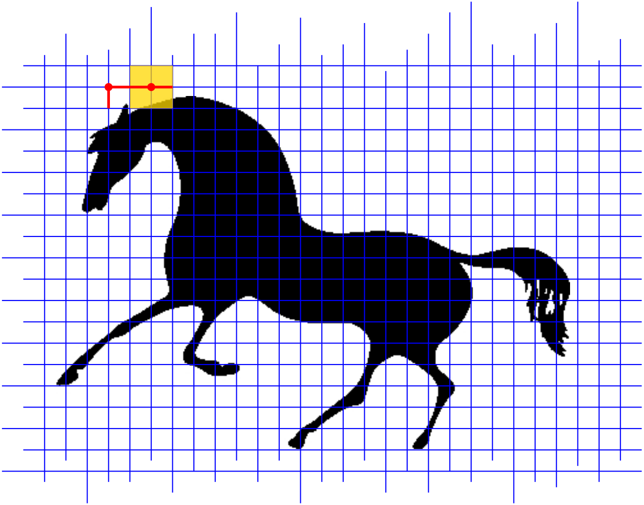

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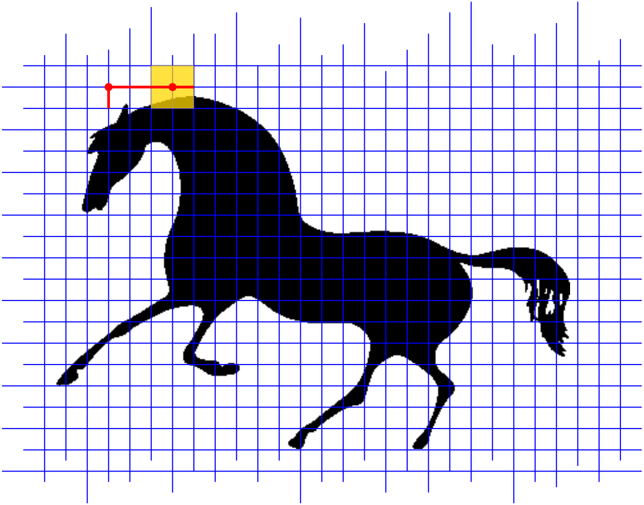

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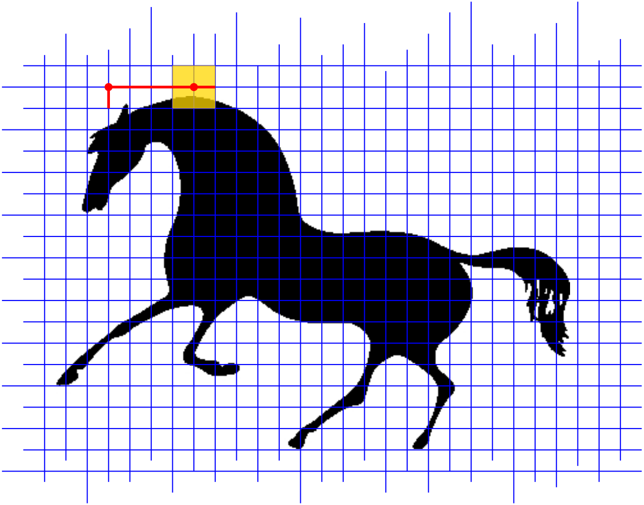

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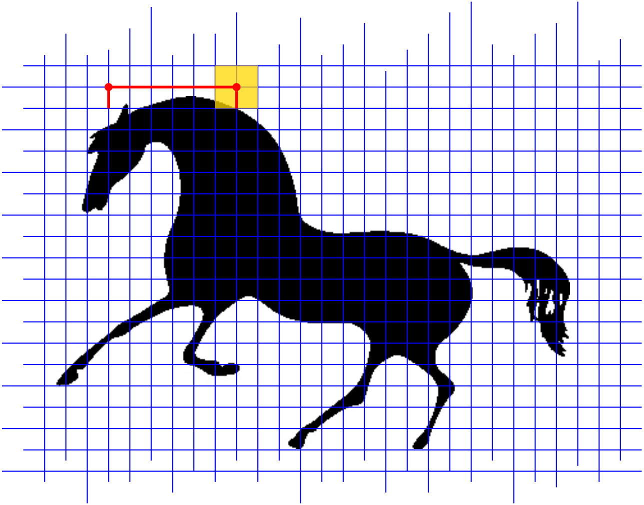

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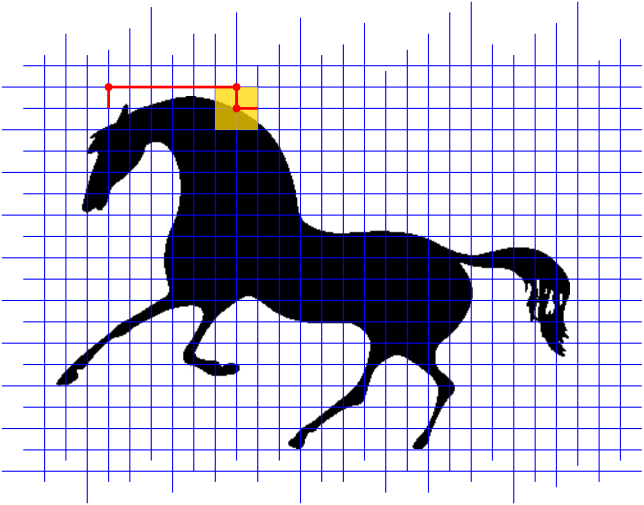

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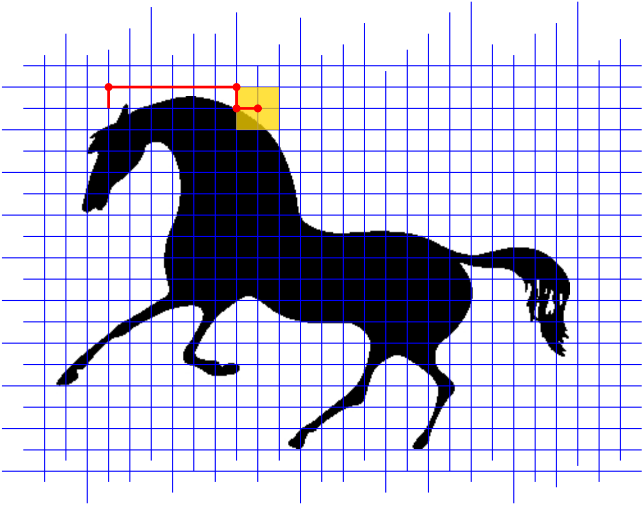

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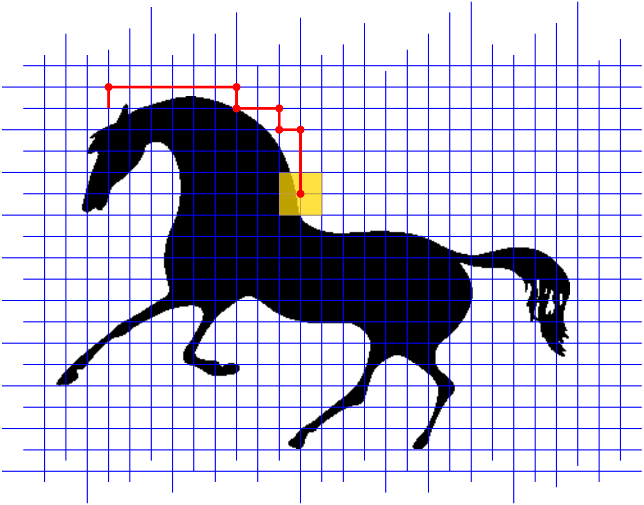

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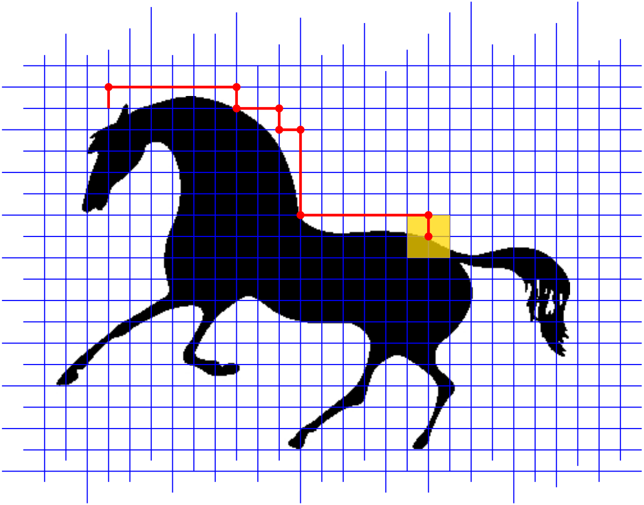

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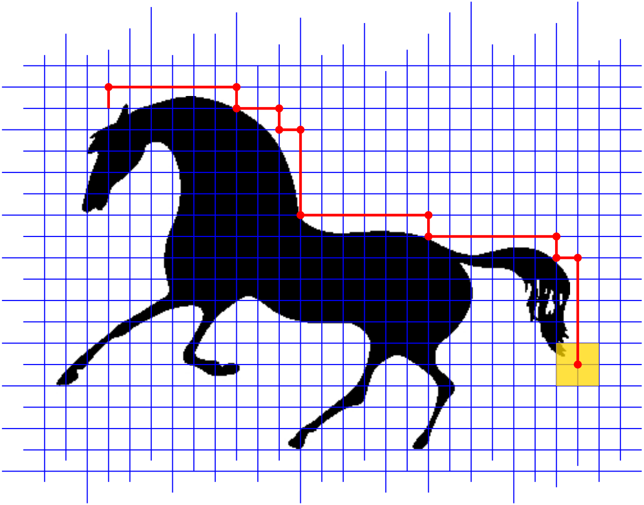

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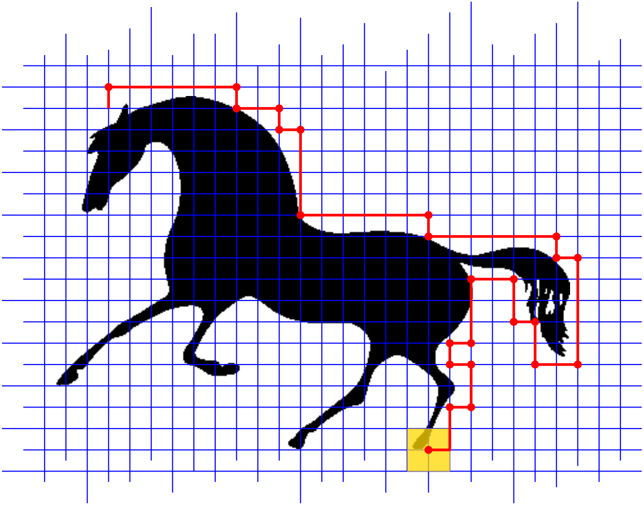

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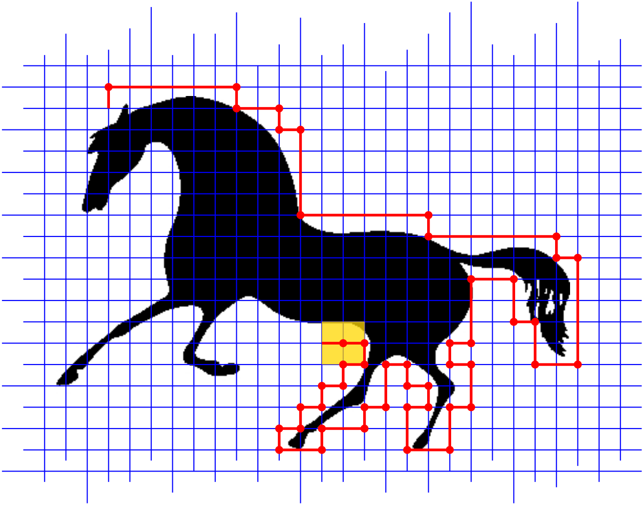

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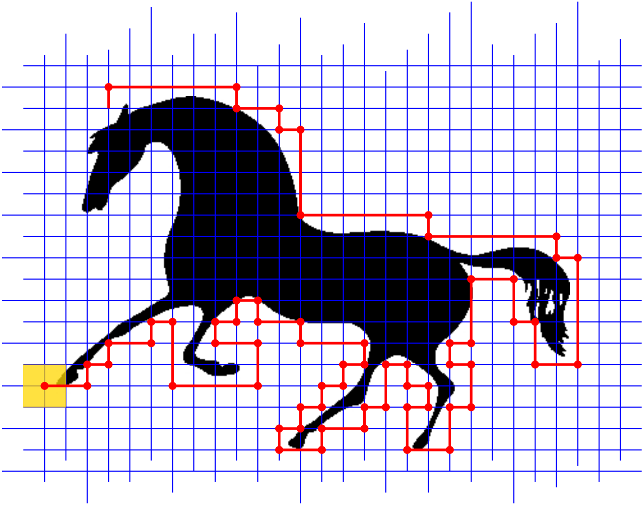

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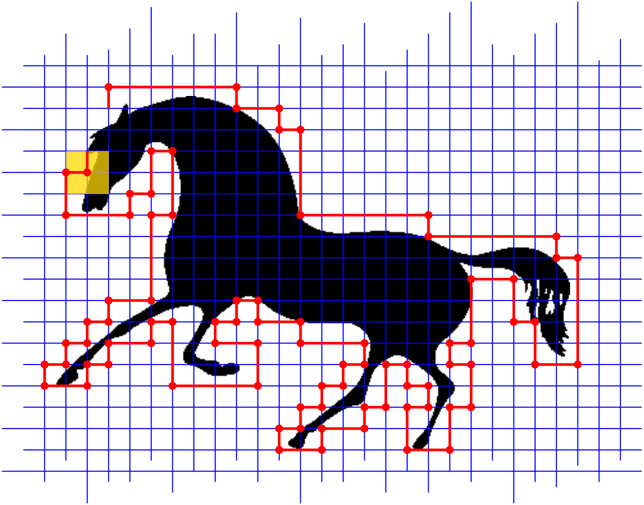

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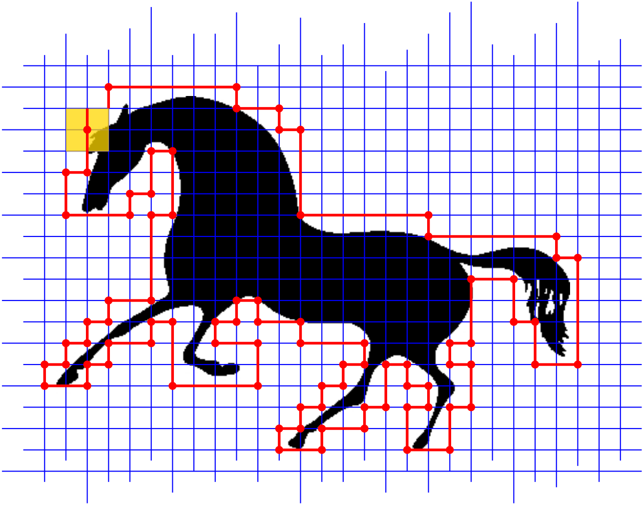
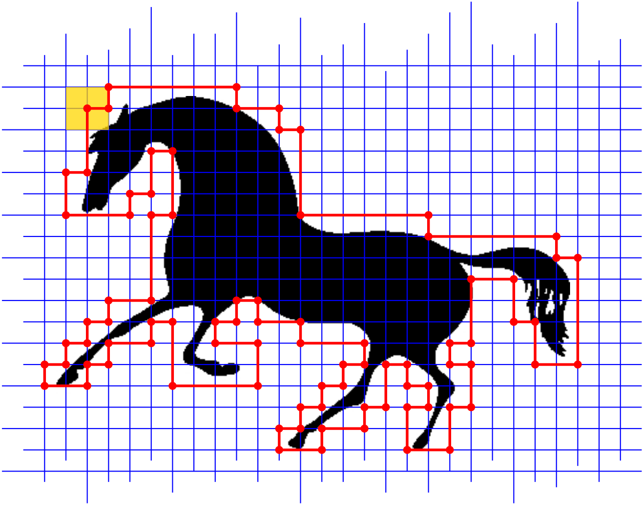


Grid point classification



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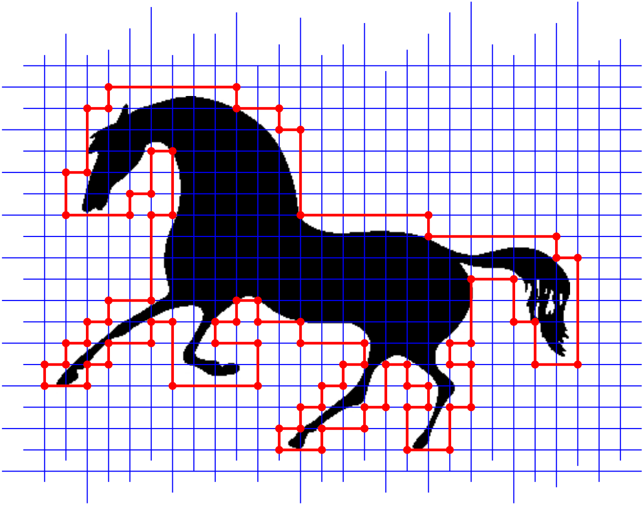
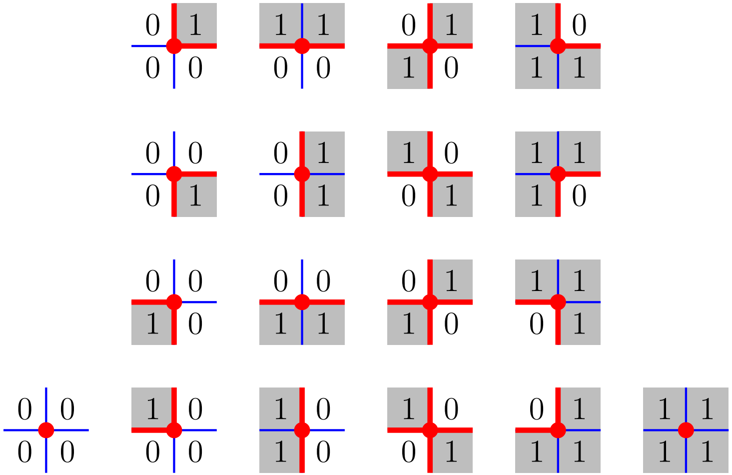


Grid point classification



Class 0 Class 1 Class 2A Class 2B Class 3 Class 4

Correctness & Runtime

The line of proof:

- The interior of a cell lies outside $P_{\mathbb{G}}(S)$ if and only if the cell has no object occupancy.
- All vertices are detected and correctly classified.
- If p is a point lying on $P_{\mathbb{G}}(S)$, then $0 < d_{\top}(p, S) \leqslant g$.
- The construction of $P_{\mathbb{G}}(S)$ always concludes at the start vertex.

Runtime:[1]

- Best case: $O(|P|/g)$ $\leftarrow$ found in practice
- Worst case: $O(|P|)$

[1] $|P|$ = perimeter of $P_{\mathbb{G}}(S)$

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Orthogonal convex hull

$H_{\mathbb{G}}(S)$ = smallest-area orthogonal polygon such that

- S lies inside $H_{\mathbb{G}}(S)$
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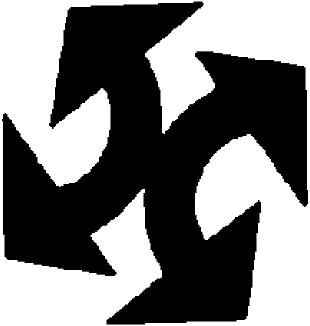
- intersection of $H_{\mathbb{G}}(S)$ with any horizontal or vertical line is either empty or exactly one line segment.

Algorithm—Uses combinatorial rules over vertex subsequences.

Runtime—Linear on perimeter of $P_{\mathbb{G}}(S)$.



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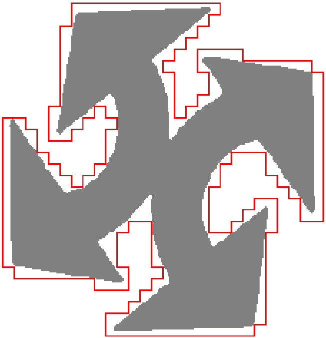

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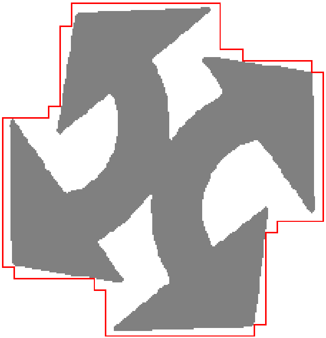

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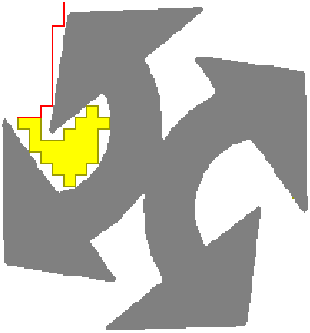

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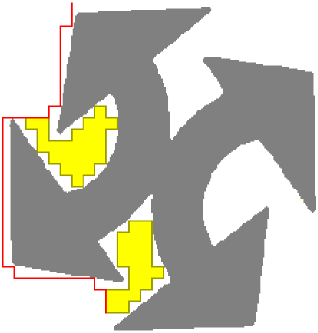

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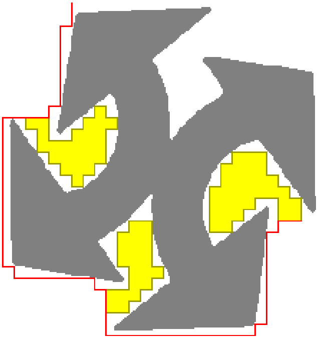

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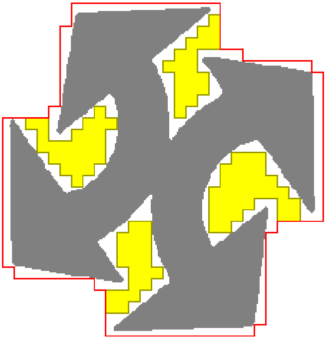

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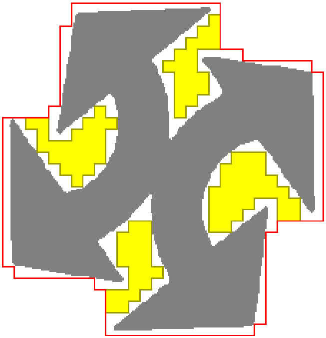

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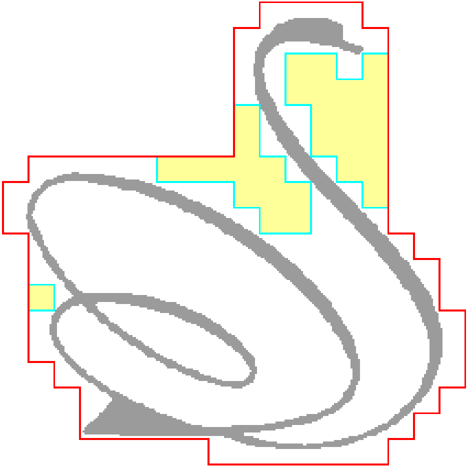
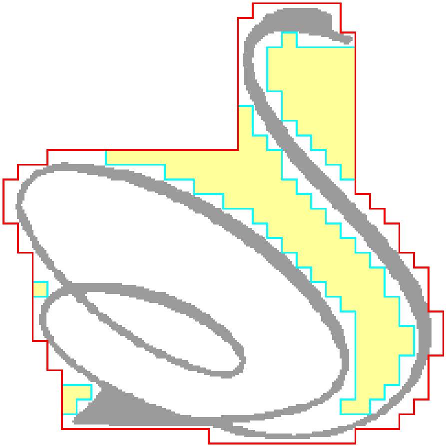


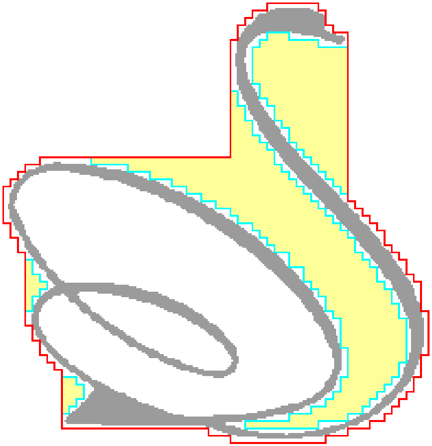
$g = 14$

Orthogonal convex hull



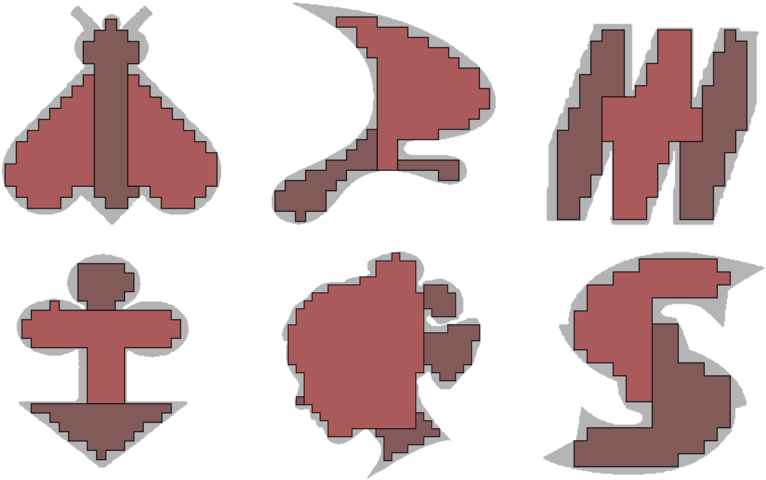
$g = 8$

Orthogonal convex hull

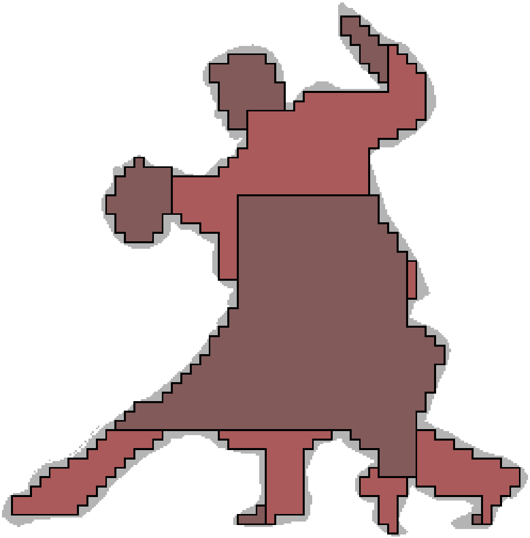


$g = 4$

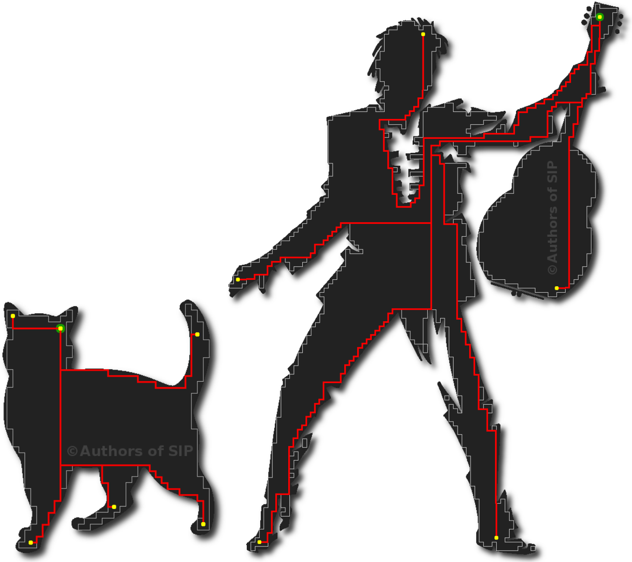
Convex partitioning



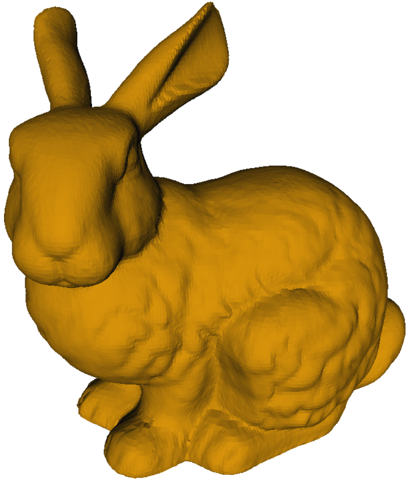
Convex partitioning



Shortest isothetic path



3D cover (outer)



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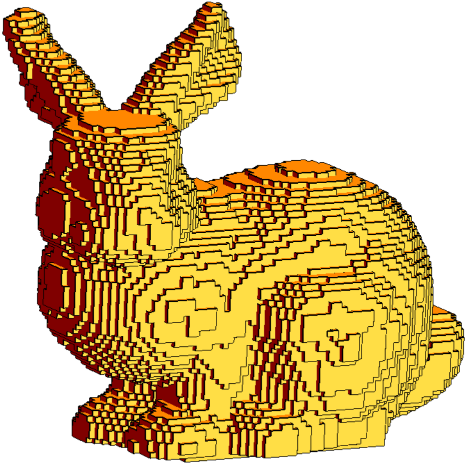
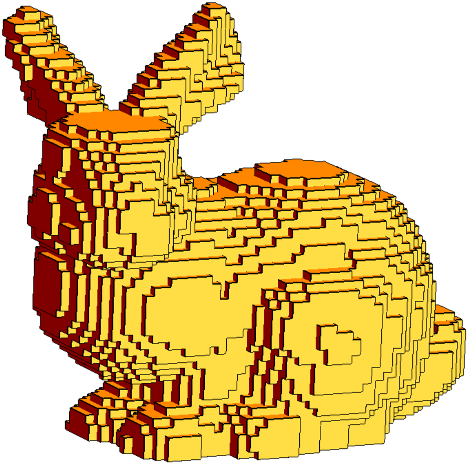


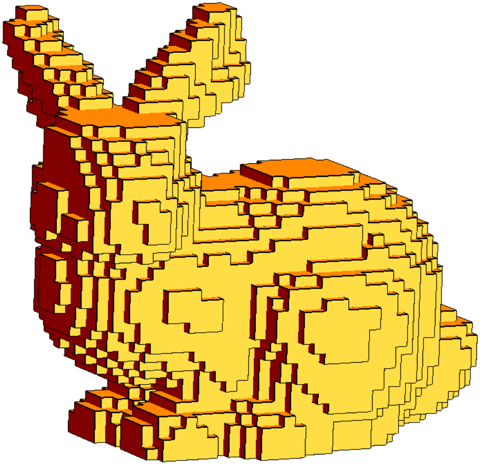
$g = 2$

3D cover (outer)



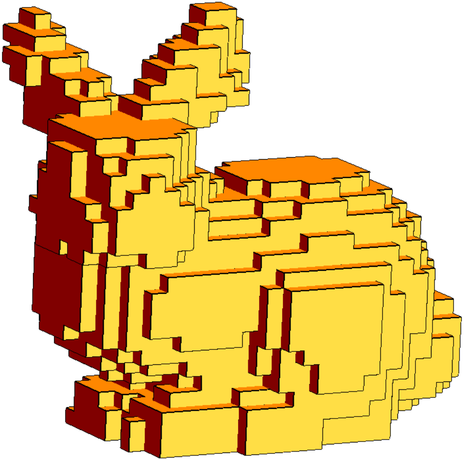
$g = 3$

3D cover (outer)



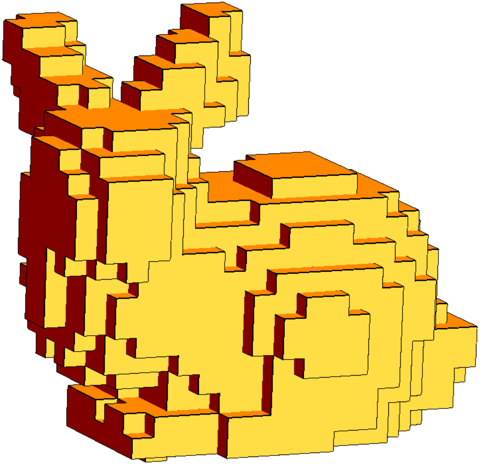
$g = 4$

3D cover (outer)



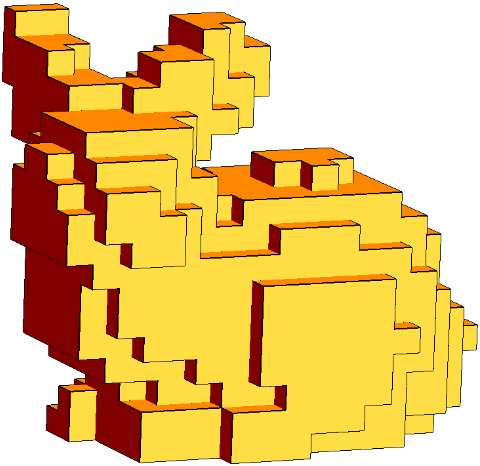
$g = 6$

3D cover (outer)



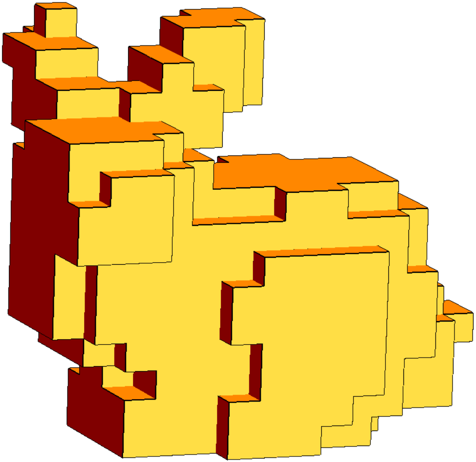
$g = 8$

3D cover (outer)



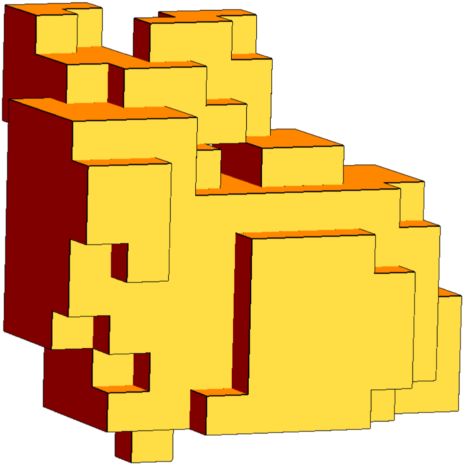
$g = 10$

3D cover (outer)



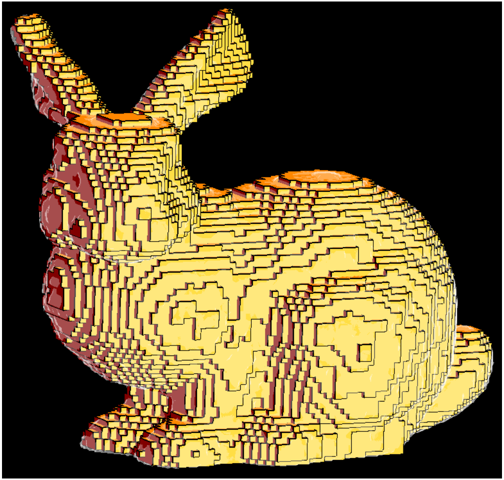
$g = 12$

3D cover (outer)



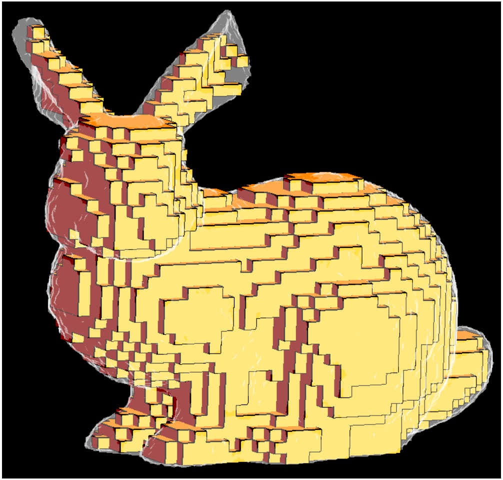
$g = 16$

3D cover (inner)



$g = 2$

3D cover (inner)

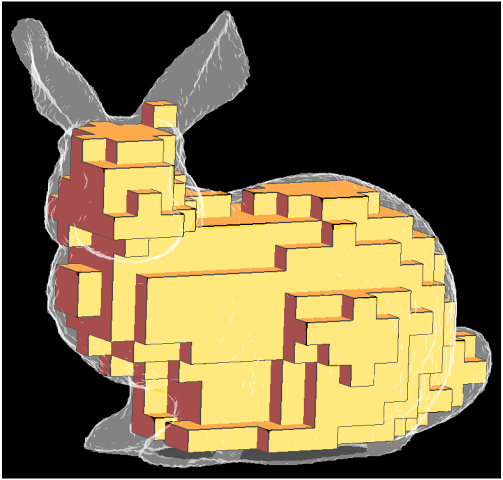


$g = 4$

3D cover (inner)

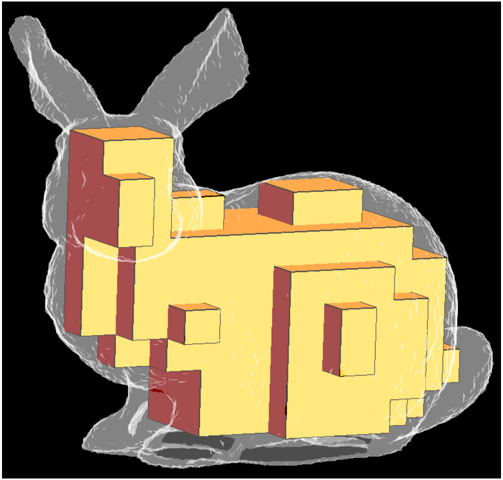
$g = 6$

3D cover (inner)



$g = 8$

3D cover (inner)



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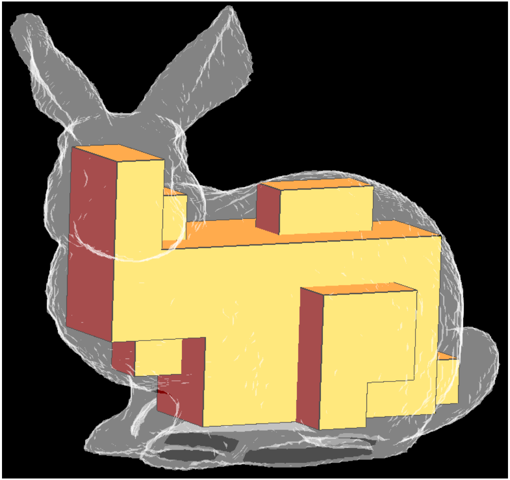
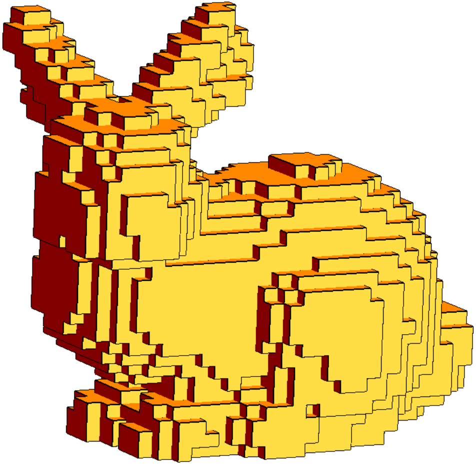
3D cover (inner)



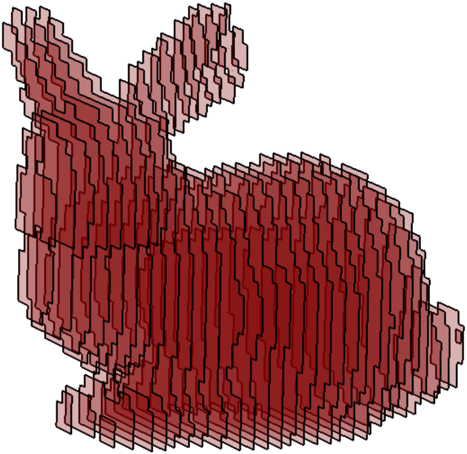
$g = 16$

3D slicing



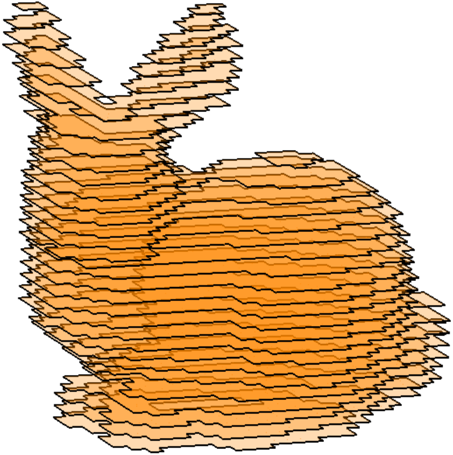
high resolution

3D slicing



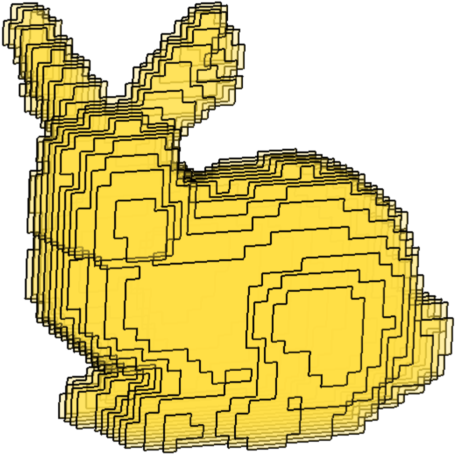
along x-axis

3D slicing



along y-axis

3D slicing



along z-axis

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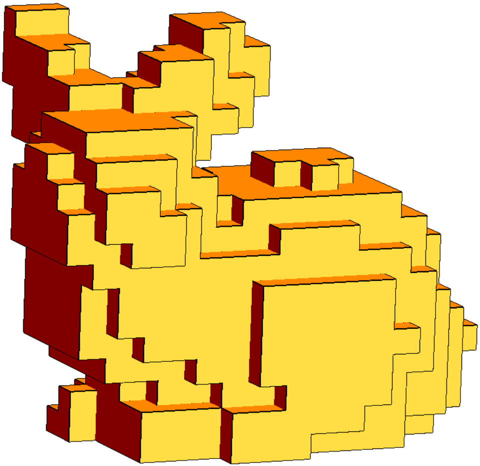
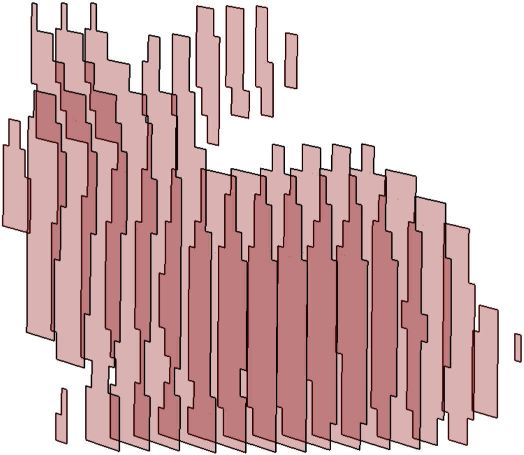


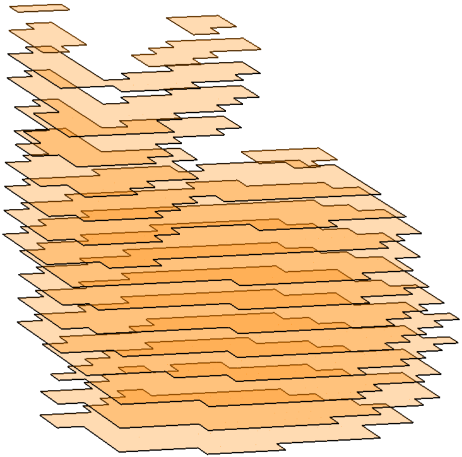
low resolution

3D slicing



along x-axis

3D slicing



along y-axis

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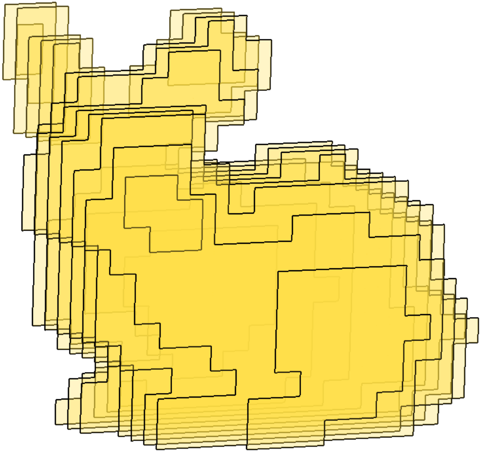

along z-axis

Further reading I

A. Biswas, P. Bhowmick, M. Sarkar, and B. B. Bhattacharya, A Linear-time Combinatorial Algorithm to Find the Orthogonal Hull of an Object on the Digital Plane, *Information Sciences*, **216**: 176–195, 2012.

A. Biswas, P. Bhowmick, and B. B. Bhattacharya. Construction of Isothetic Covers of a Digital Object: A Combinatorial Approach, *Journal of Visual Communication and Image Representation*, **21**(4): 295–310, 2010.

M. Dutt, A. Biswas, and P. Bhowmick, ACCORD: With Approximate Covering of Convex Orthogonal Decomposition, *DGCI 2011: 16th IAPR International Conference on Discrete Geometry for Computer Imagery*, LNCS **6607**: 489–500, 2011.

M. Dutt, A. Biswas, P. Bhowmick, and B. B. Bhattacharya, On Finding Shortest Isothetic Path inside a Digital Object, *15th International Workshop on Combinatorial Image Analysis: IWCIA'12*, 2012 [To appear in LNCS, Springer]

Further reading II

N. Karmakar, A. Biswas, P. Bhowmick, and B.B. Bhattacharya, A Combinatorial Algorithm to Construct 3D Isothetic Covers, *International Journal of Computer Mathematics*, 2012 (in press).

N. Karmakar, A. Biswas, and P. Bhowmick, Fast Slicing of Orthogonal Covers Using DCEL, *15th International Workshop on Combinatorial Image Analysis: IWCIA'12*, 2012 [To appear in LNCS, Springer]

N. Karmakar, A. Biswas, P. Bhowmick, and B.B. Bhattacharya, Construction of 3D Orthogonal Cover of a Digital Object, *14th International Workshop on Combinatorial Image Analysis: IWCIA'11*, LNCS **6636**: 70–83, 2011.

R. Klette and A. Rosenfeld, *Digital Geometry: Geometric Methods for Digital Picture Analysis*, Morgan Kaufmann, San Francisco, 2004.

Thank You