

# Introduction to Robotics



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# What is a Robot?

Robot properties:

- Flexibility in Motion
- Mobile robots

daksh ROV: de-mining robot  
20 commissioned in Indian  
army 2011.

100+ more on order  
built by R&D Engineers, Pune

daksh platform derived  
gun mounted robot (GMR)



# Want your personal robot?



Roomba vacuum  
Cleaning robot

By i-robot  
Price: ~ rs. 15-30K

# How to vacuum a space?



Roomba vacuum  
Cleaning robot

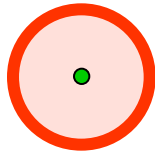
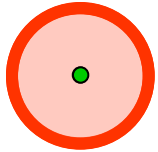
By i-robot  
Price: ~ rs. 30K

<https://www.youtube.com/watch?v=dweVBqei9L>

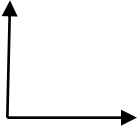
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# Models of Robot Motion

Circular robot

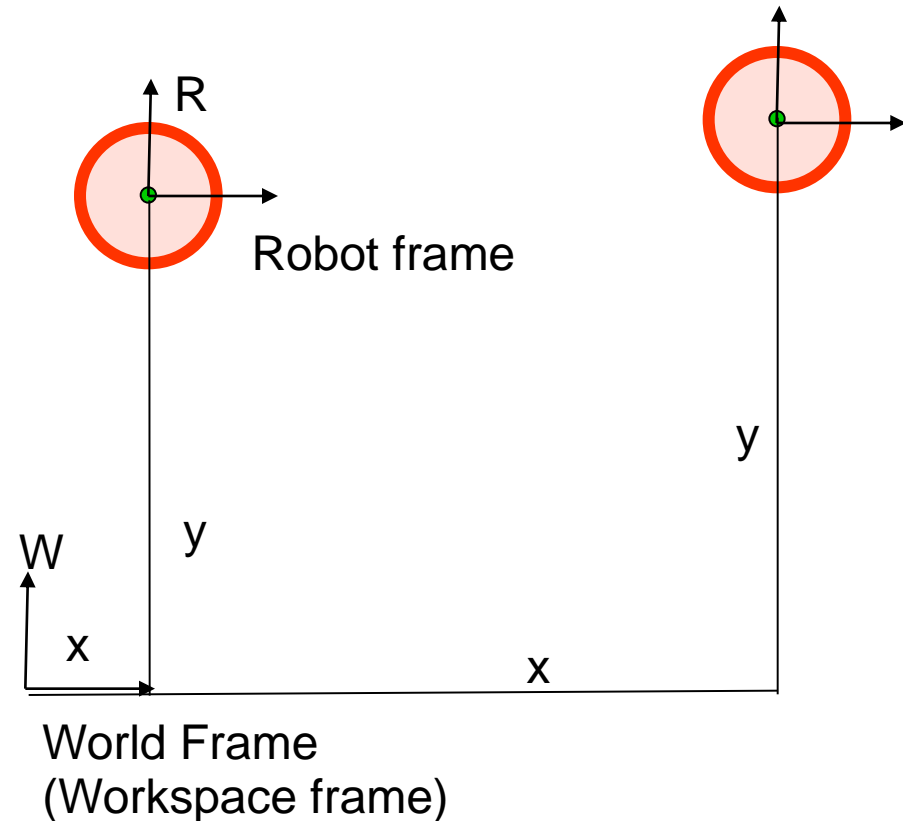


W



World Frame  
(Workspace frame)

# Models of Robot Motion



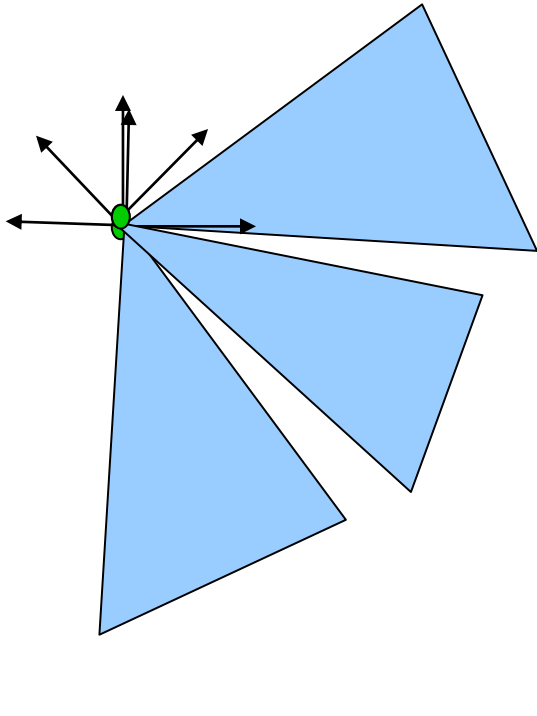
## DEFINITION:

**degrees of freedom:**  
NOTE: Given robot frame R, every point on the robot is known  
**number of parameters needed to fix the robot frame R in the world frame W**

$(x, y) =$  **configuration**  
(vector  $\mathbf{q}$ )

given configuration  $\mathbf{q}$   
for a certain pose of the robot, the set of points on the robot is a function of the configuration: say  $R(\mathbf{q})$

# Non-Circular Robot



DEFINITION:

**degrees of freedom:**

number of parameters needed  
to fix the robot frame  $R$   
in the world frame  $W$

How many parameters needed to fix  
the robot frame if it can only translate?

How many if it can rotate as well?

# Motion in 3-D:

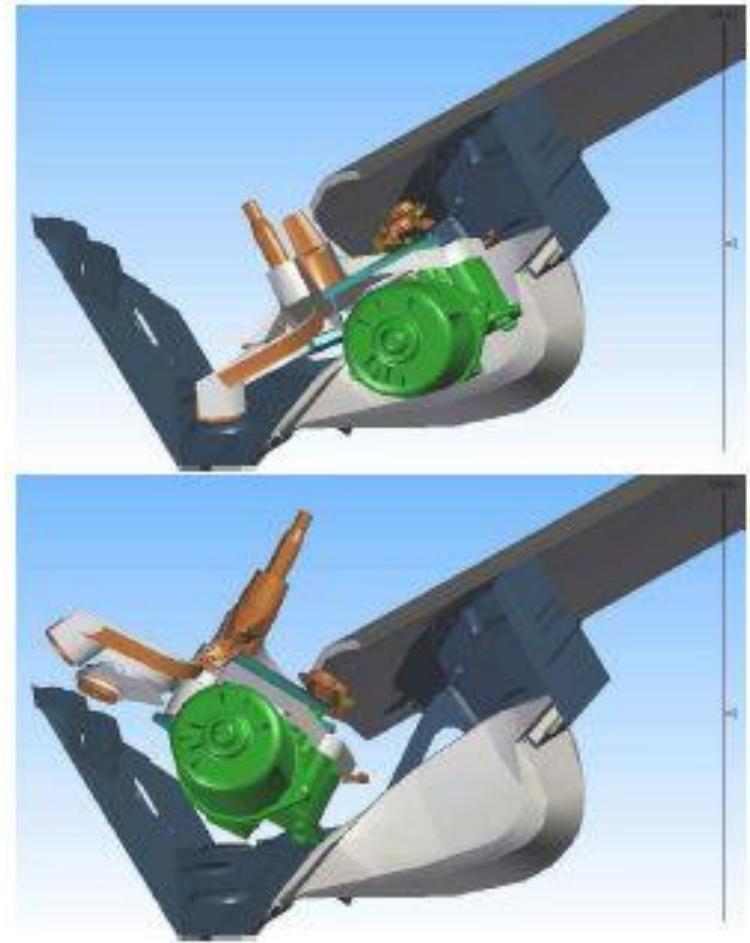
## Piano movers problem

General 3D motion:

How many parameters  
needed to fix the pose?

Can a design be  
assembled?

Test based on CAD models





# Research mobile robot



Turtlebot

Based on i-robot (roomba) platform  
(with kinect RGB-D sensor)

ROS (open-source) software

Price: ~ 75K

Articulated robots

# What is a Robot?

Robots properties:

- Flexibility in Motion
  - Mobile robots
  - Articulated robots

SCARA 4-axis arm  
(4 degrees-of-freedom)

by Systemantics  
Bangalore

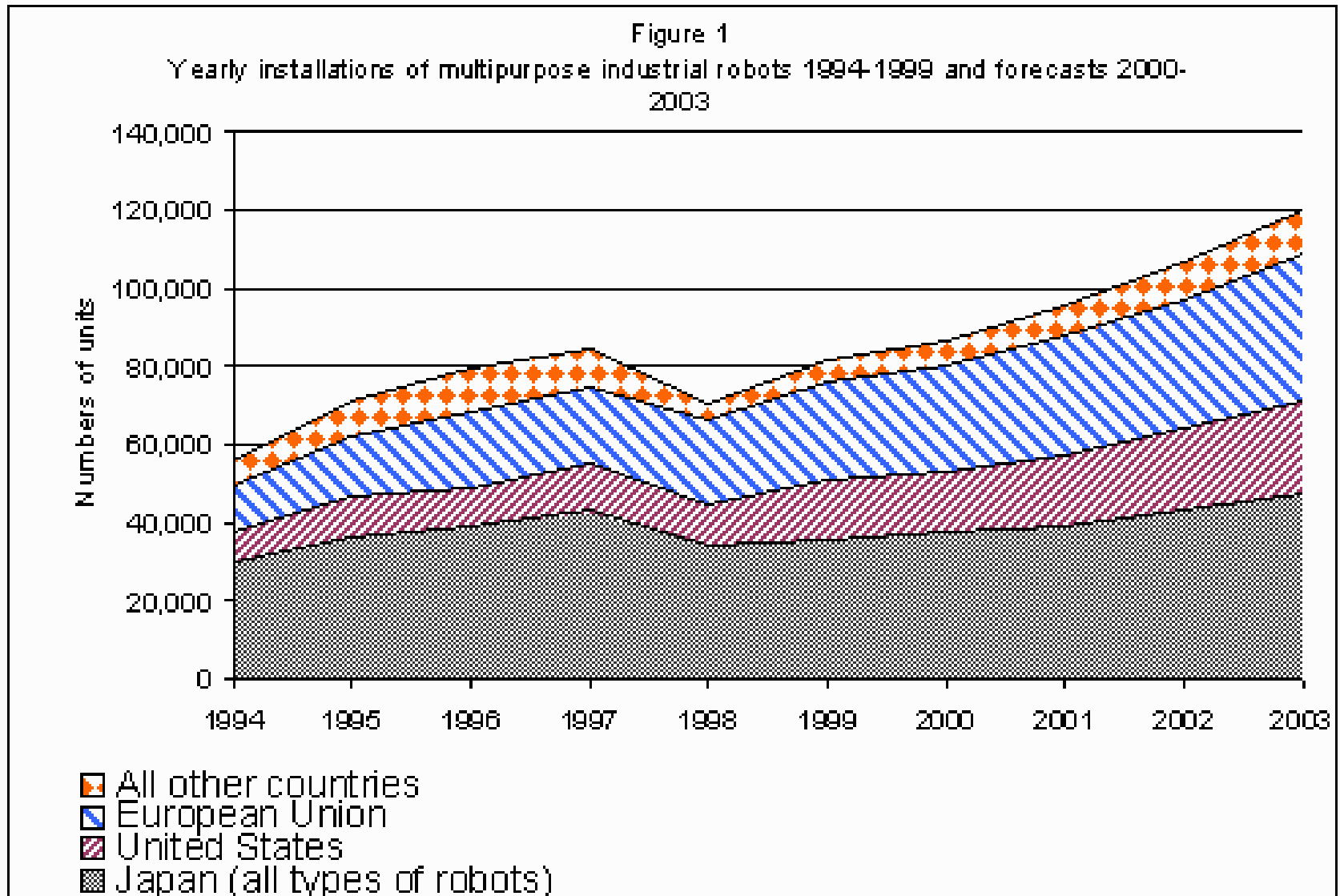




# Industrial Robot

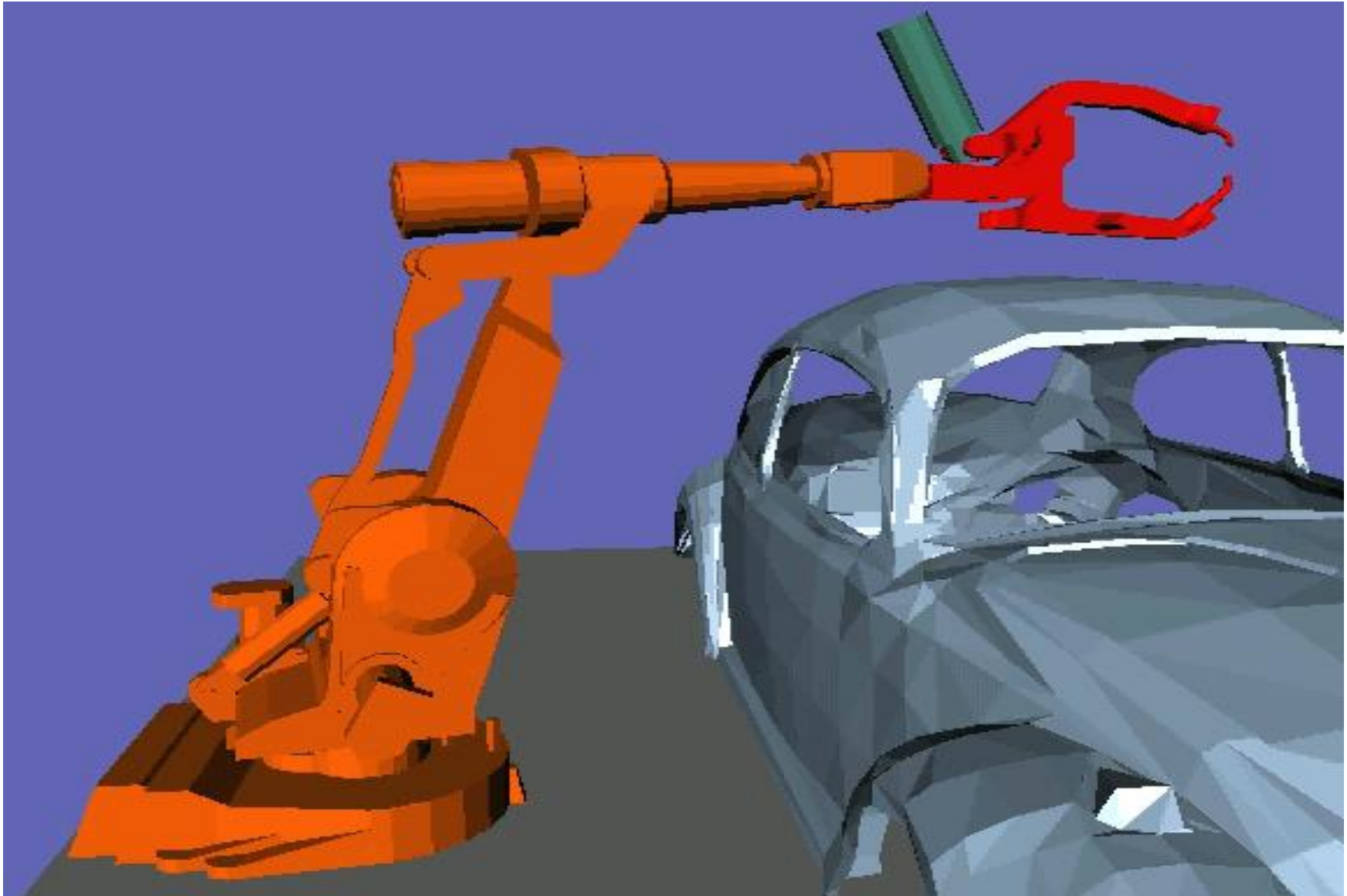


# Industrial Robots





# How to program a welding robot?



# What is a Robot?

Robot properties

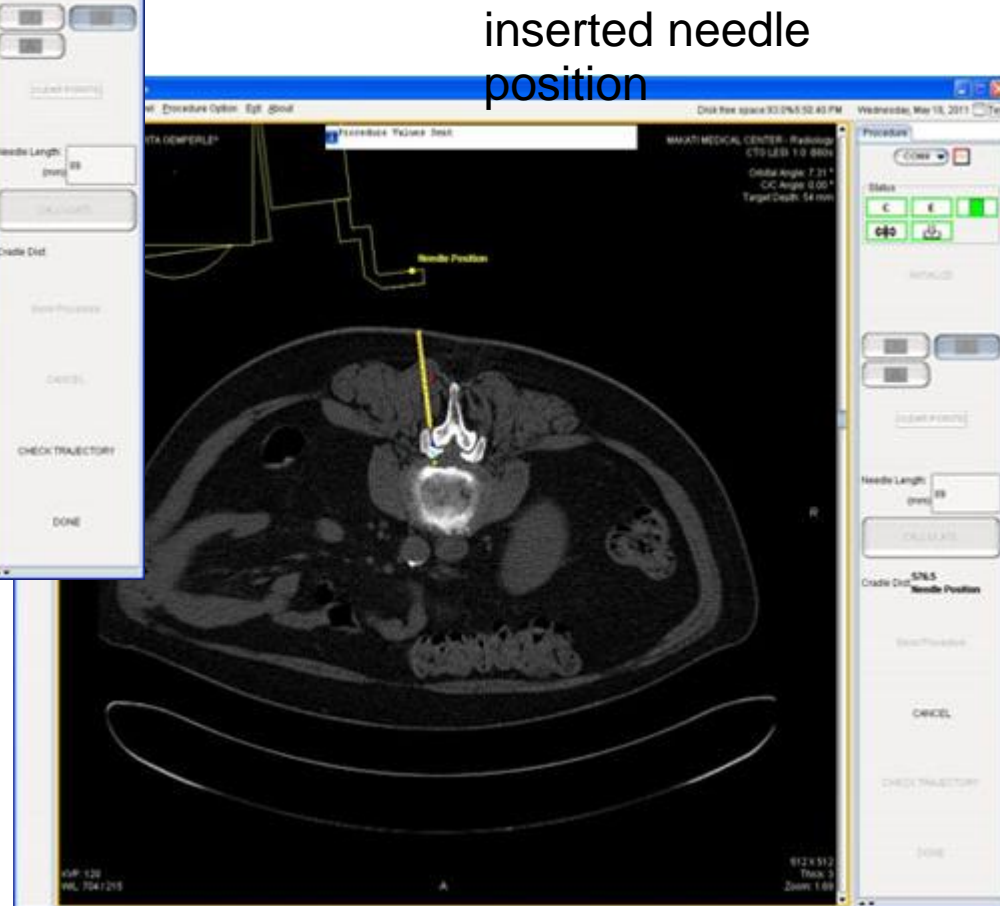
- Flexibility in M
- Mobile robot
- Articulated r
- Industrial r
- Surgical robots



# Surgical Robot : Lumbar biopsy



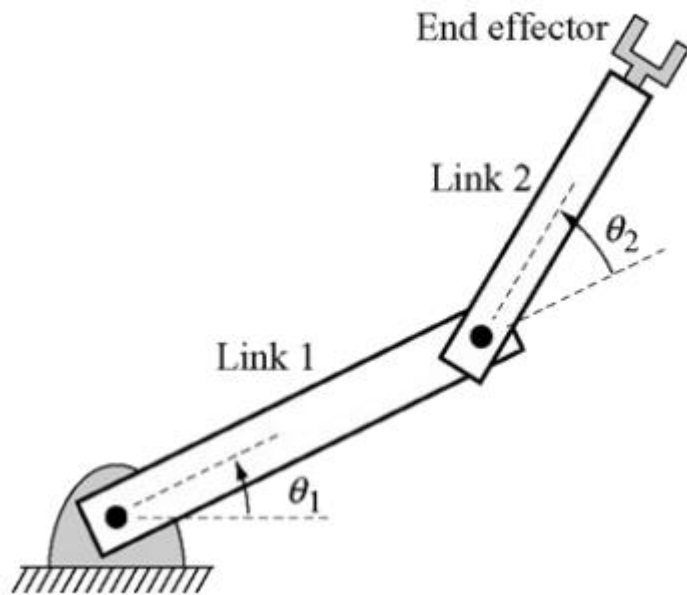
needle path as planned on CAT scan



inserted needle  
position



# Modeling Articulated Robots



## Kinematic chain:

Pose of Link  $n$  depends on the poses of Links  $1 \dots (n-1)$

Transformation between frame of link  $(n-1)$  and link  $n$ , depends on a single motion parameter, say  $\theta_n$

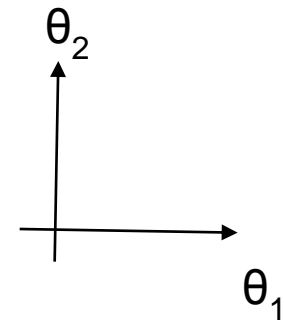
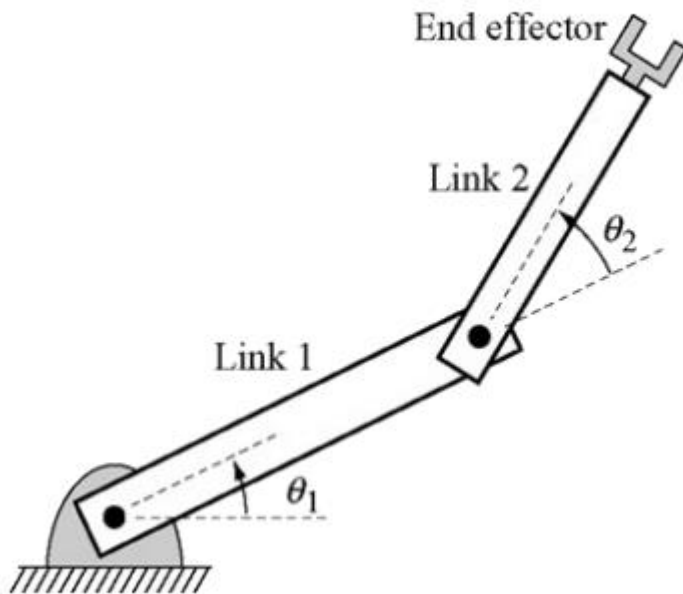
Exercise:

What are the coordinates of the origin of the end-effector center?

# Modeling Articulated Robots

workspace

configuration  
space



Exercise:

Sketch the robot pose for the configuration  $[0, -90]$

# Modeling Articulated Robots

## Forward kinematics

Mapping from configuration  $\mathbf{q}$  to robot pose, i.e.  $R(\mathbf{q})$

Usually,  $R()$  is the product of a sequence of transformations from frame  $i$  to frame  $i+1$ .

Note: Must be very systematic in how frames are attached to each link

## Inverse kinematics

a. Given robot pose, find  $\mathbf{q}$

Or

b. Given end-effector pose, find  $\mathbf{q}$

Q. Is the answer in (b) unique?

# What is a Robot?

## Robots properties

- Flexibility in Motion
  - Mobile robots
  - Articulated robots
  - Digital actors



# Reality: limited functionality



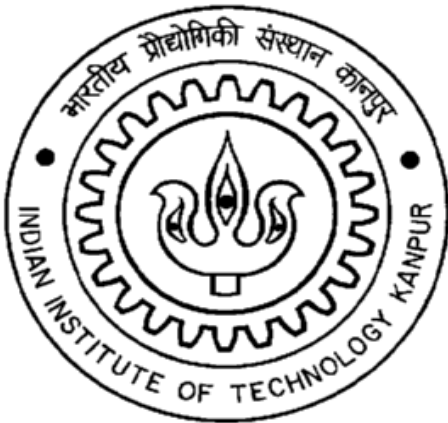
Mobility isnt everything

# What is a Robot?

Robots involve

- Flexibility in Motion
  - Dentists cradle?
  - Washing machine?
- Intentionality
  - Measure : not default probability distribution
    - e.g. Turn-taking (contingent behaviour)
  - Goal : intrinsic or extrinsic

# Robot Motion Planning

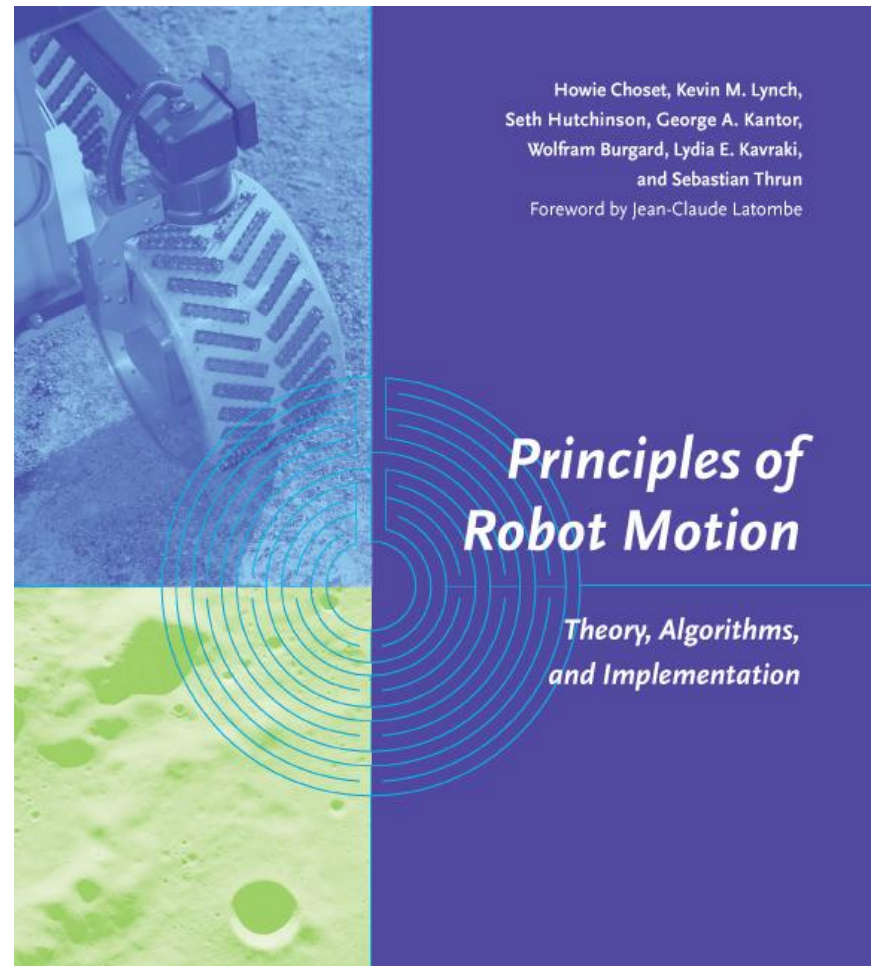


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indian edition  
rs 425

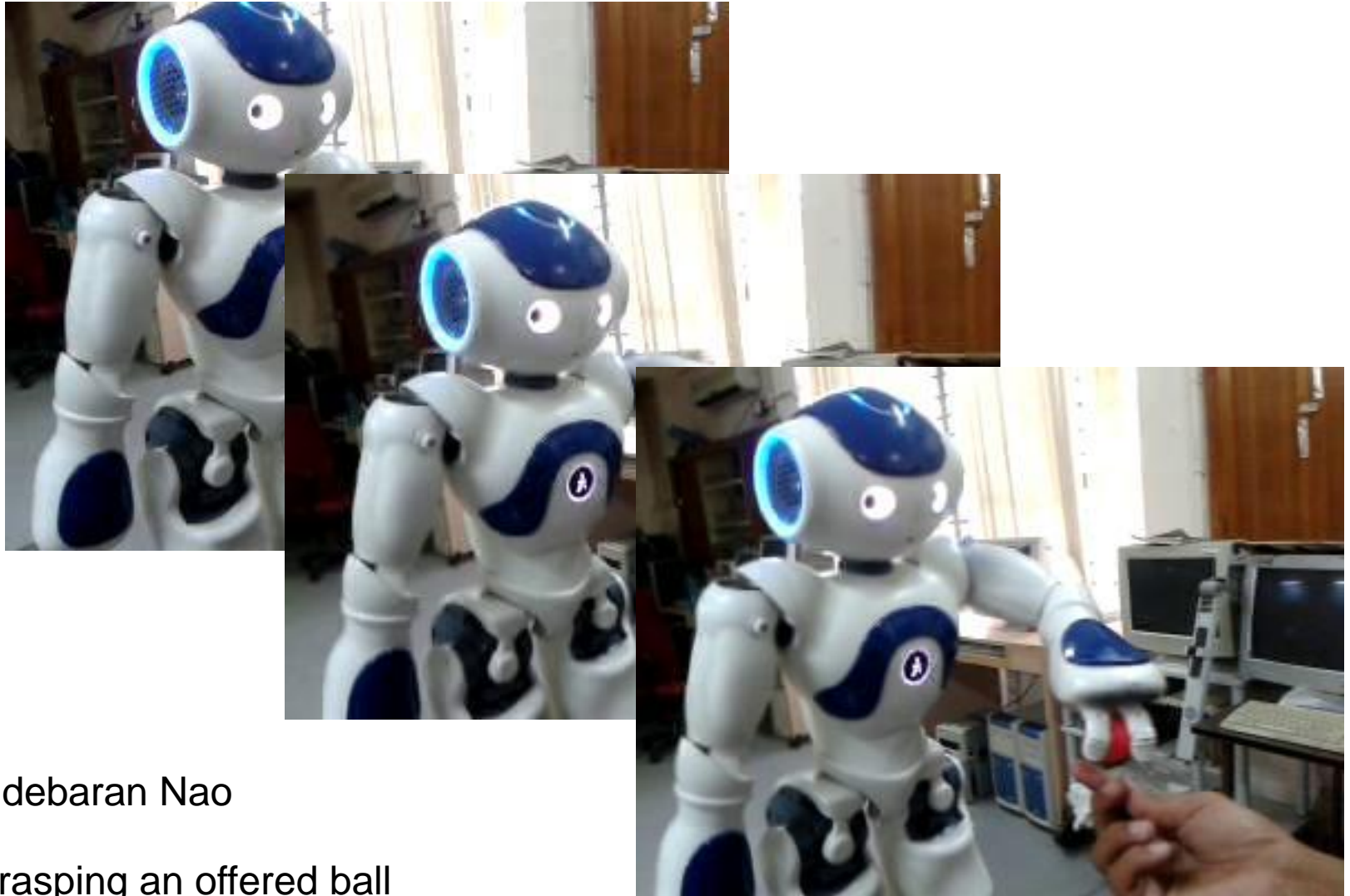


# Sensing and Motion Planning



[bohori venkatesh singh mukerjee 05]  
Bohori/Venkatesh/Singh/Mukerjee:2005

# Programming a robot



Aldebaran Nao

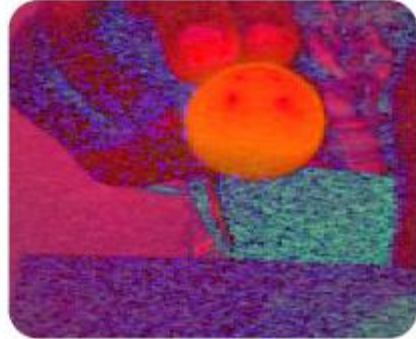
Grasping an offered ball

# Programming a robot

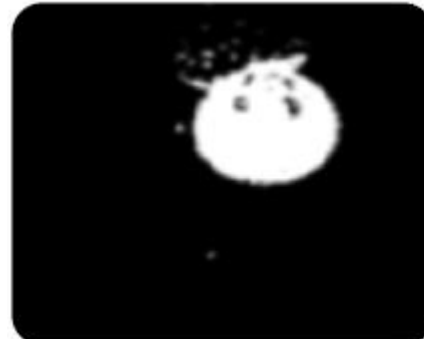
1. detect ball using colour:



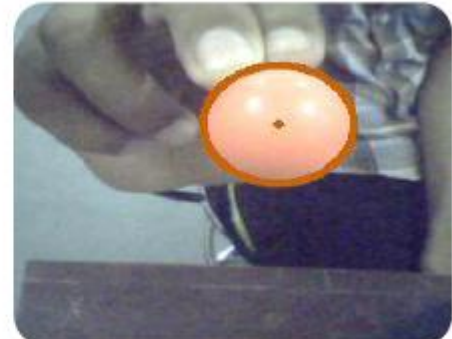
image captured by nao



HSV



binarized



contour detected

2. estimate distance of ball (depth)  
from image size

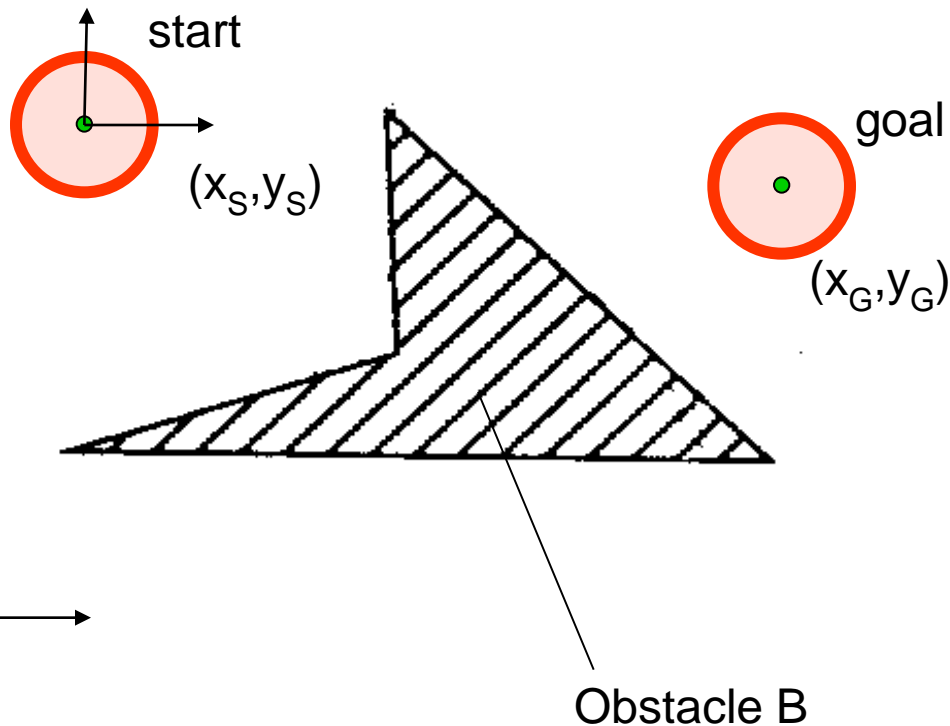
3. Inverse kinematics to grasp ball

Sensing in the workspace

Motion planning in C-space

# Configuration Space

# Robot Motion Planning



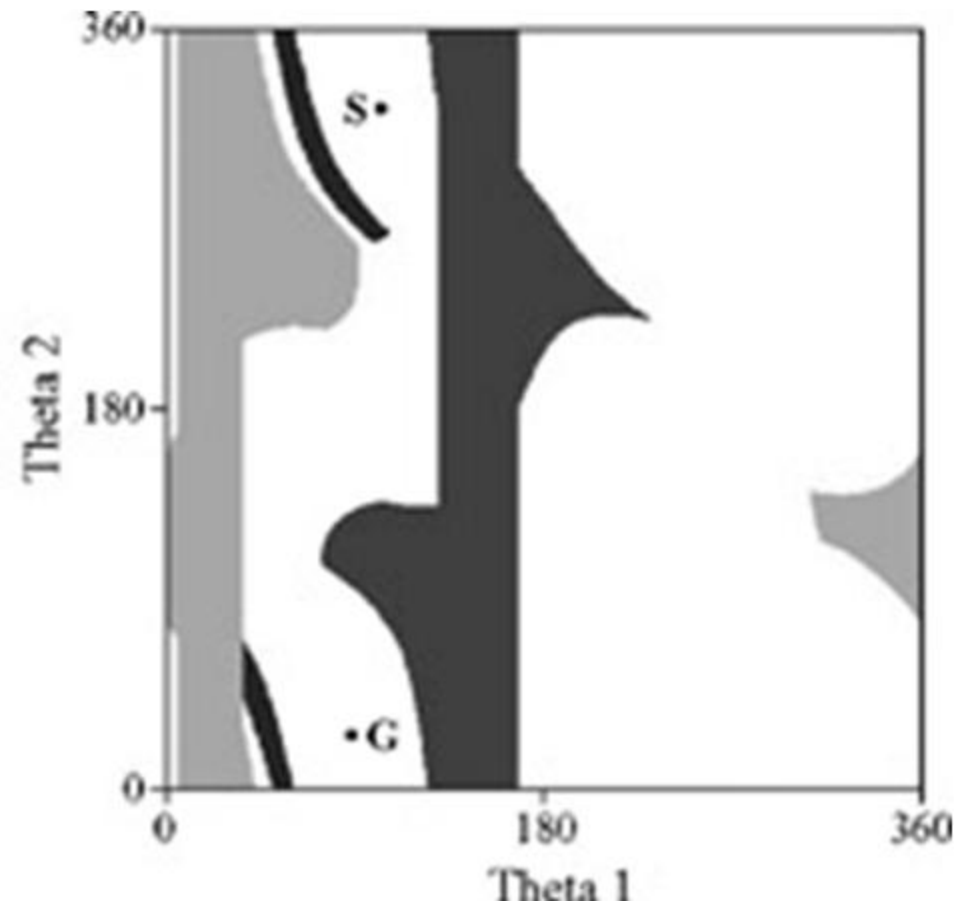
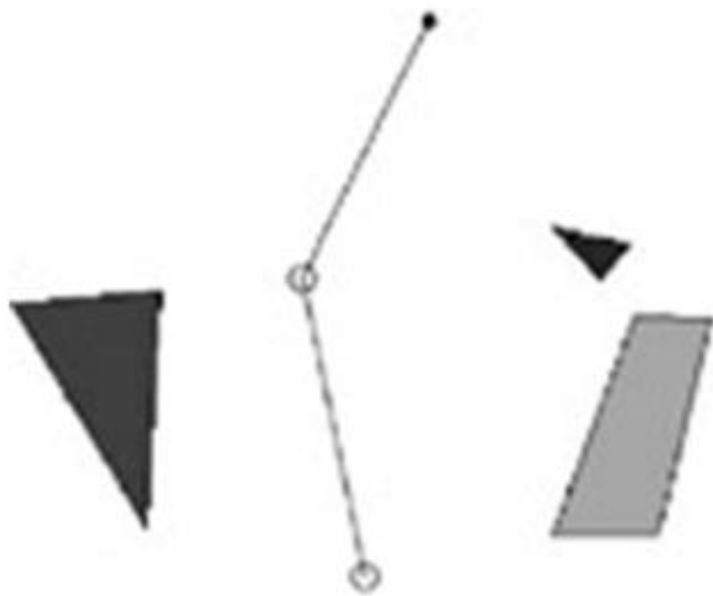
Valid paths will lie among those where the robot does not hit the obstacle

find path  $P$  from start to goal s.t.

$$\text{for all } t, R(t) \cap B = \emptyset$$

How to characterize the set of poses for which the robot does not hit the obstacle B?

# Robot Motion Planning



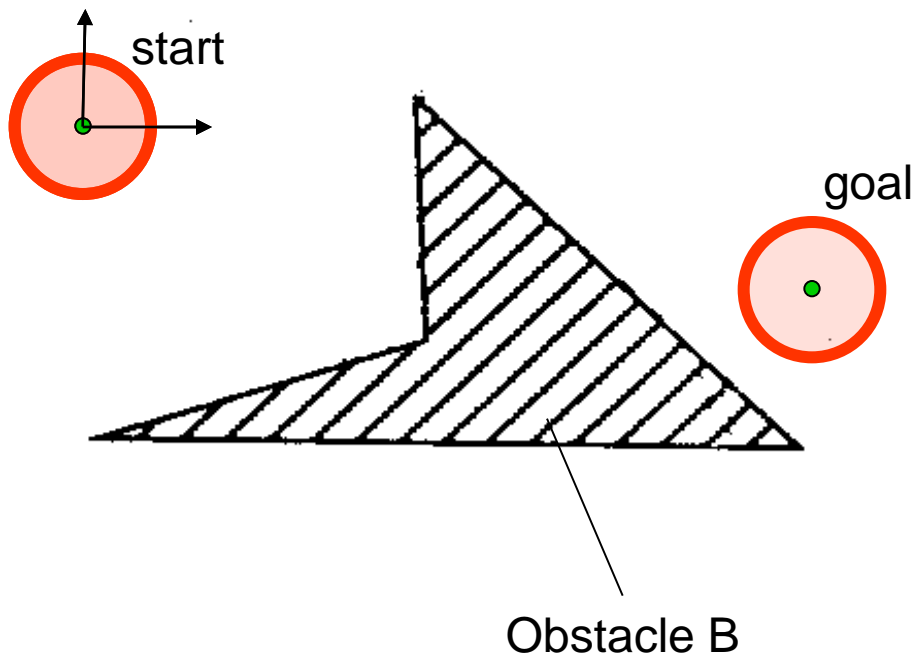
# Continuum approaches vs Discretization

## Two approaches to Robot motion planning:

- **continuum:**  
treat motion space as single continuum  
→ optimization
- **discretization:**  
decompose motion space into regions / segments  
→ graph-search



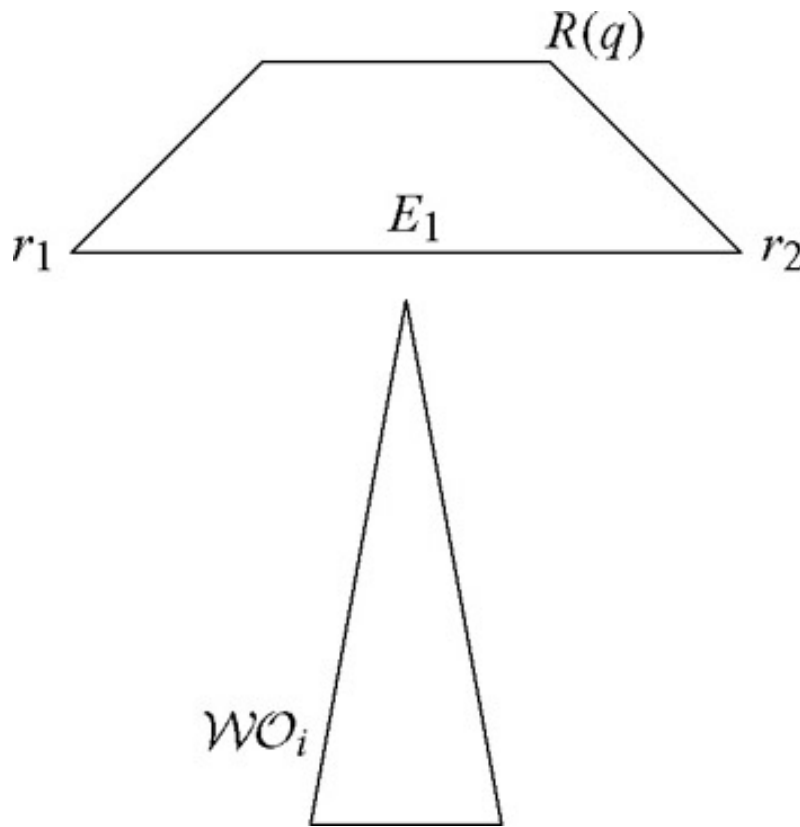
# Potential fields



## Potential fields

1. Goal: negative (attractive) potential  
Obstacles: positive (repulsive) potential
2. Robot moves along gradient
3. Problems:
  - need to integrate the potential over the area of robot
  - problem of local minima

# Finite area robots



Instead of integrating over robot area, restrict to a set of *control* points

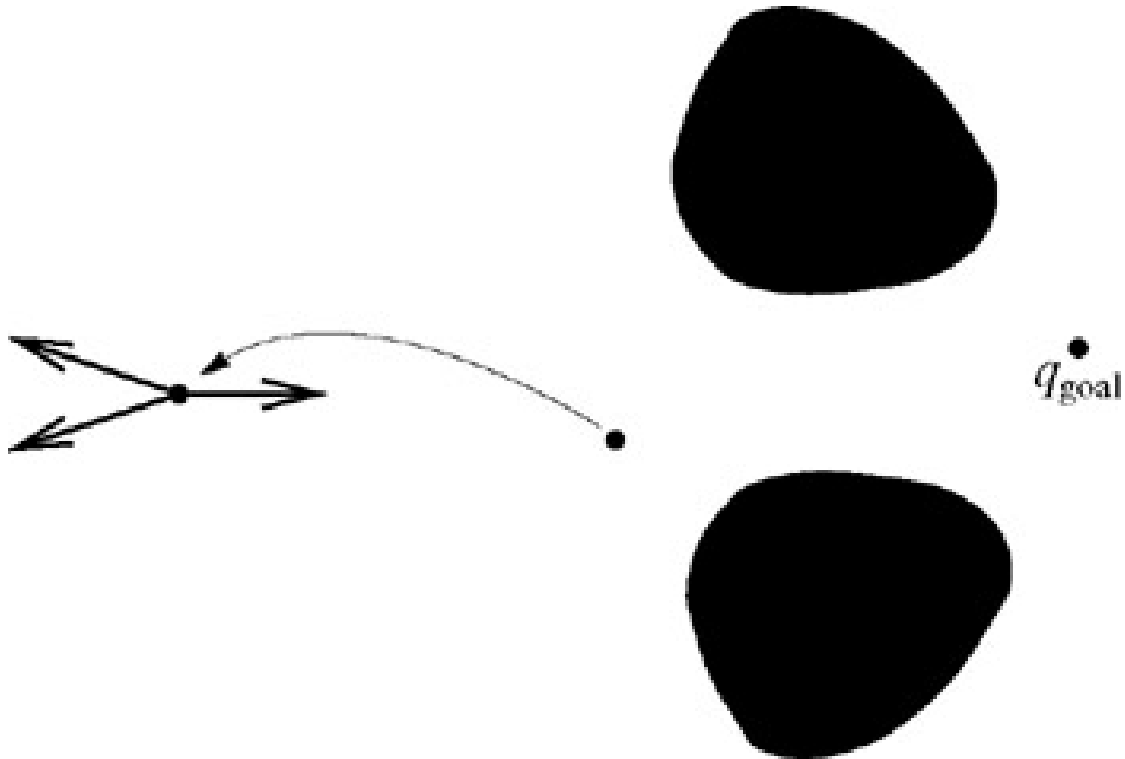
e.g. vertices

Problem:

With control points  $r_1$  and  $r_2$  on robot  $R(q)$ , edge  $E_1$  may still hit Obstacle.

→ Attempt to reduce computation to points

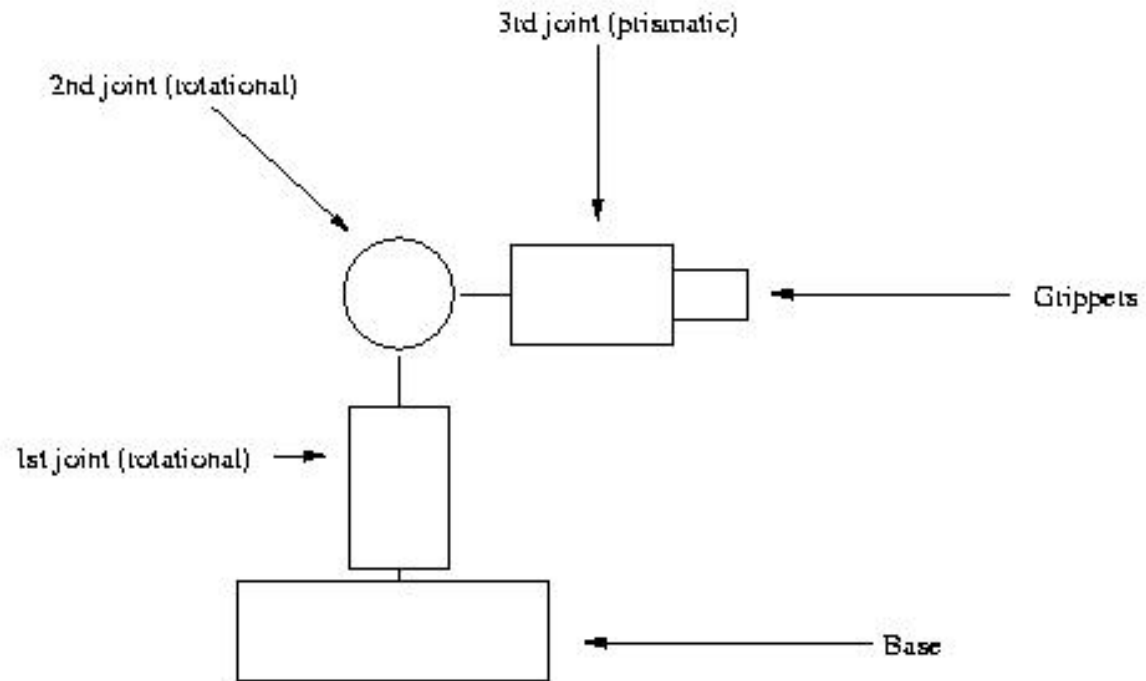
# Local Minima



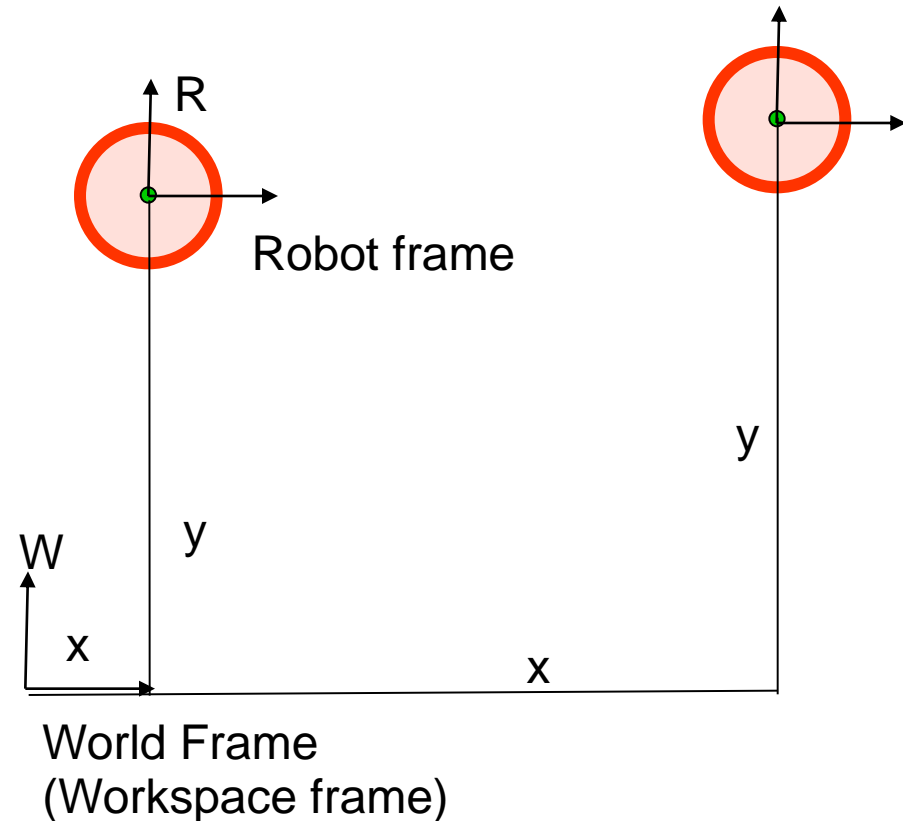
persists even for point robots

# Nature of Configuration spaces

# Robot Model



# Models of Robot Motion



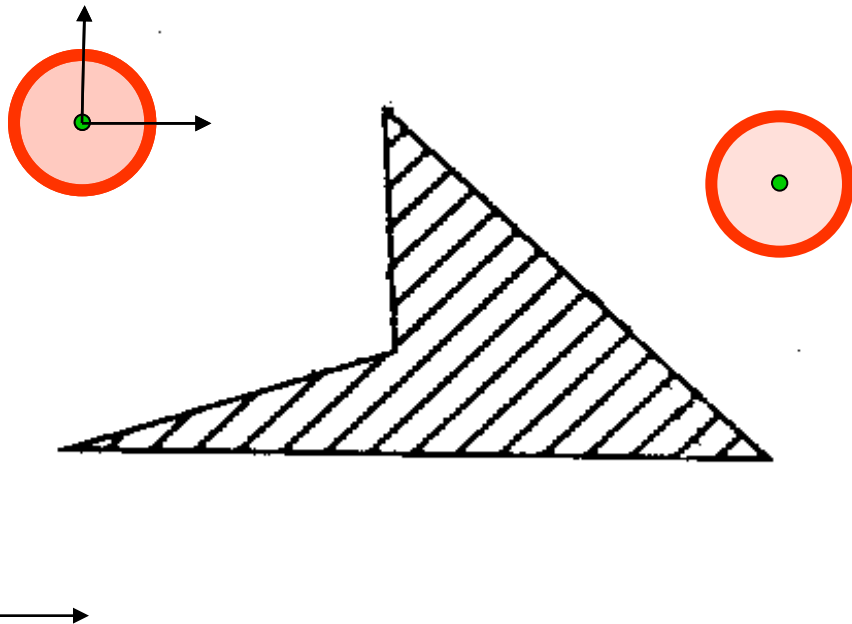
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for a certain pose of the robot, the set of points on the robot is a function of the configuration: say  $R(\mathbf{q})$

# Robot Motion Planning

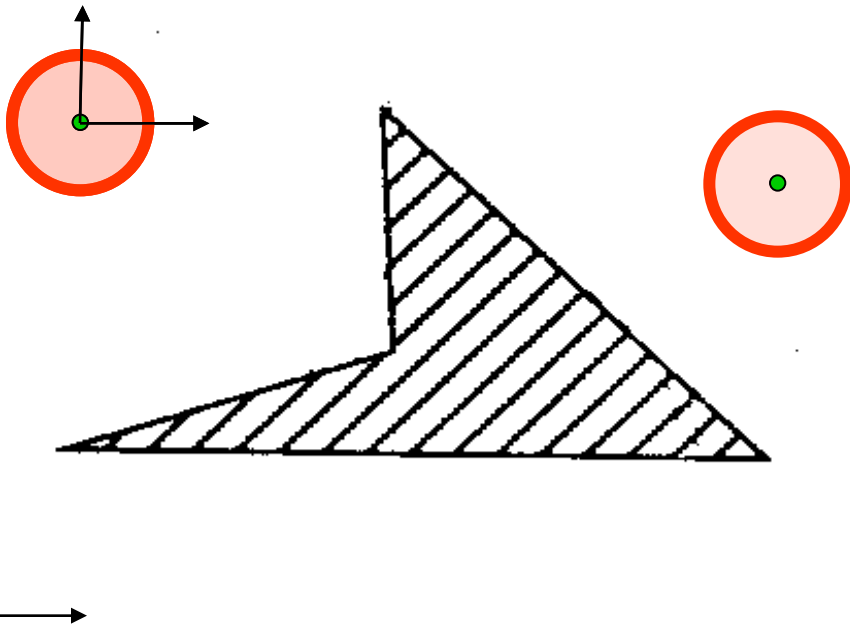


find path  $P$  from  $\mathbf{q}_S$  to  $\mathbf{q}_G$  s.t. for all  $\mathbf{q} \in P$ ,  $R(\mathbf{q}) \cap B = \emptyset$

? generate paths and check each point on every path?

Would it be easier to identify  $Q_{free}$  first?

# Robot Motion Planning



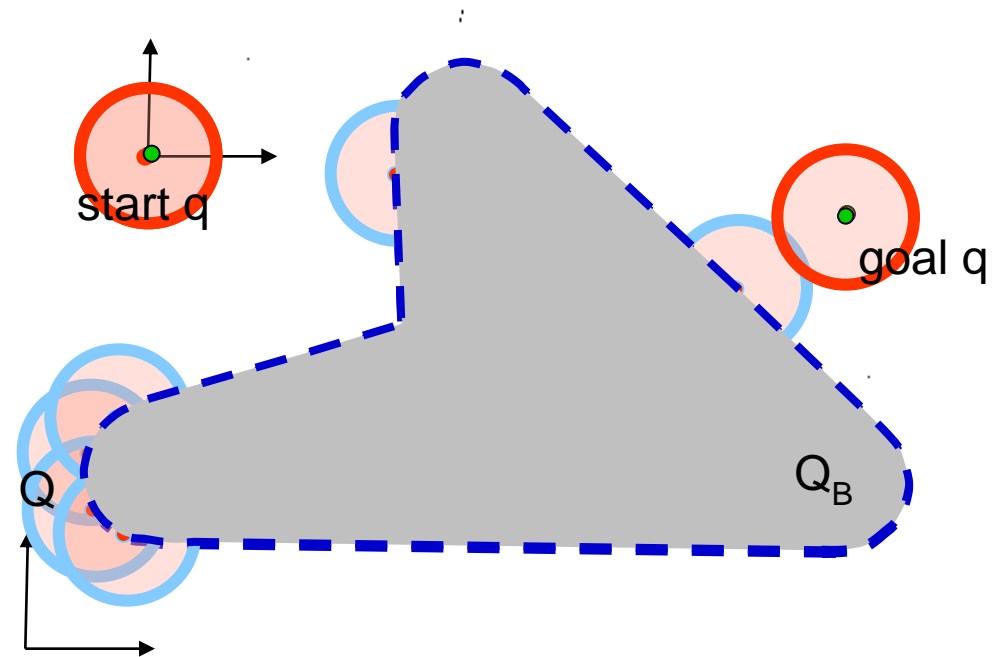
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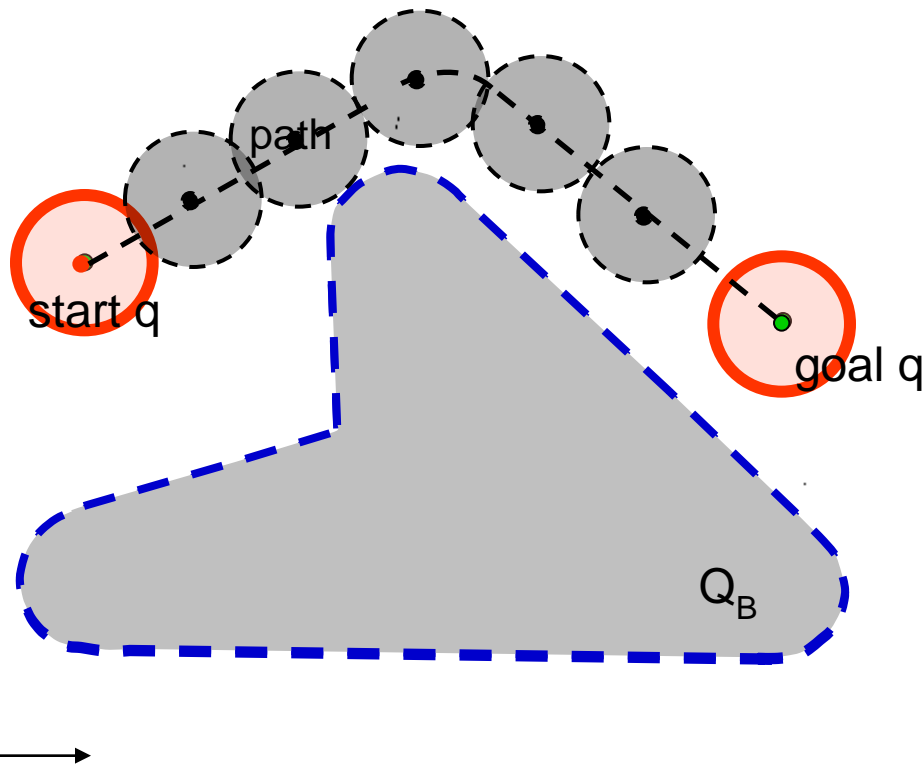


# Robot Motion Planning



$$Q_B = [ \mathbf{q} \mid R(\mathbf{q}) \cap B \neq \emptyset ]$$

# Motion Planning in C-space

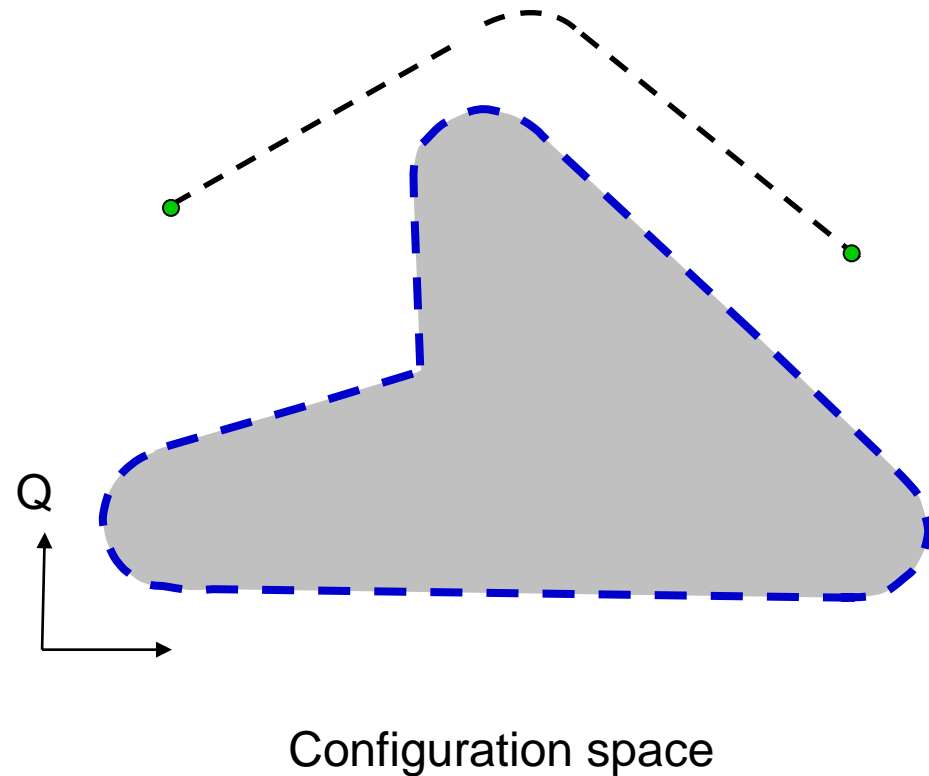
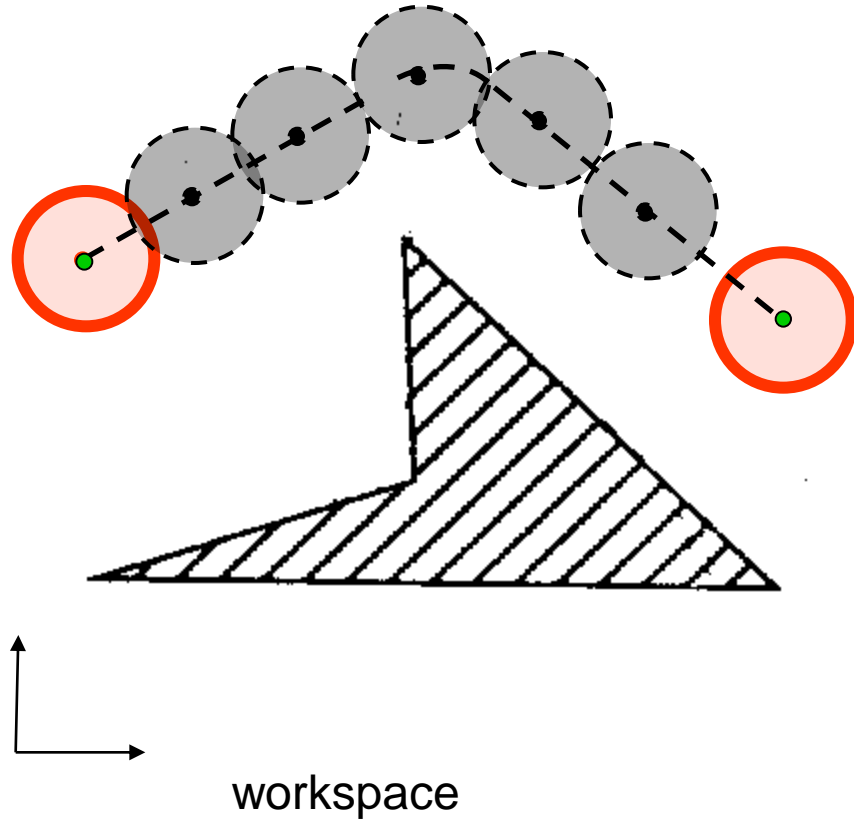


configurations are points in C-space

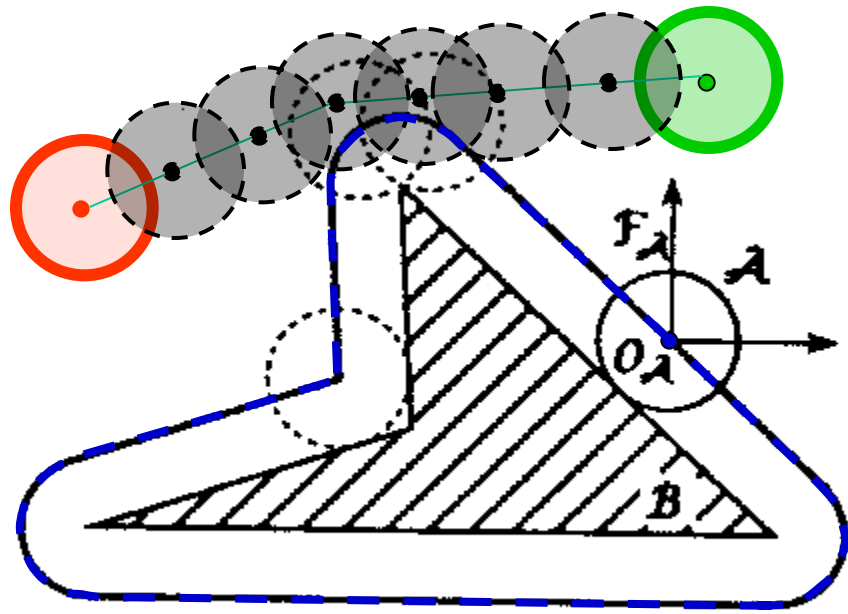
path  $P$  is a line

if  $P \cap Q_B = \emptyset$ , then path is in  $Q_{\text{free}}$

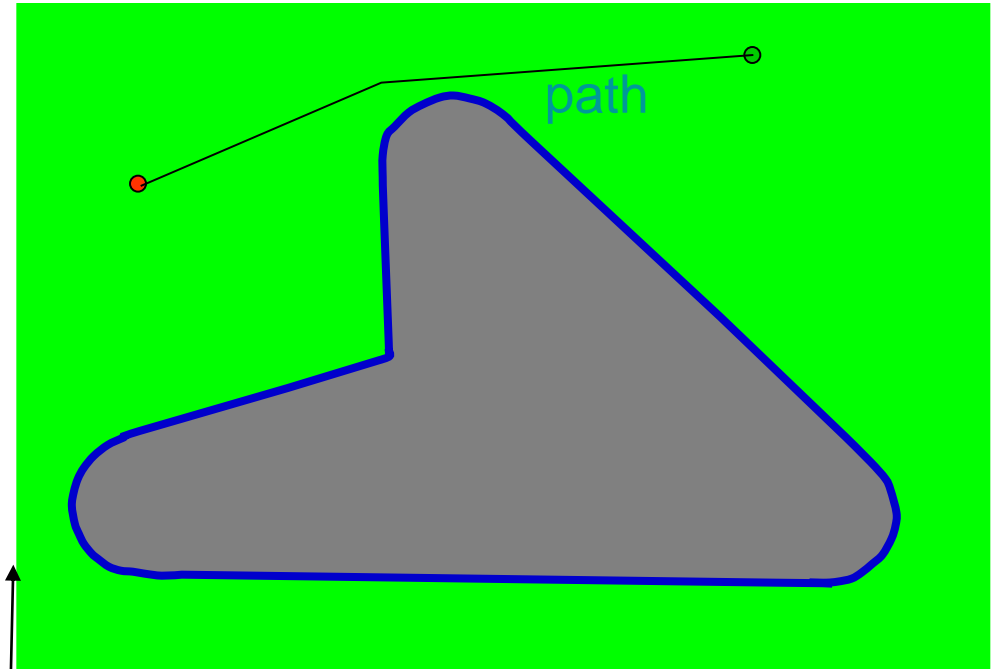
# Motion Planning in C-space



# Robot Motion Planning



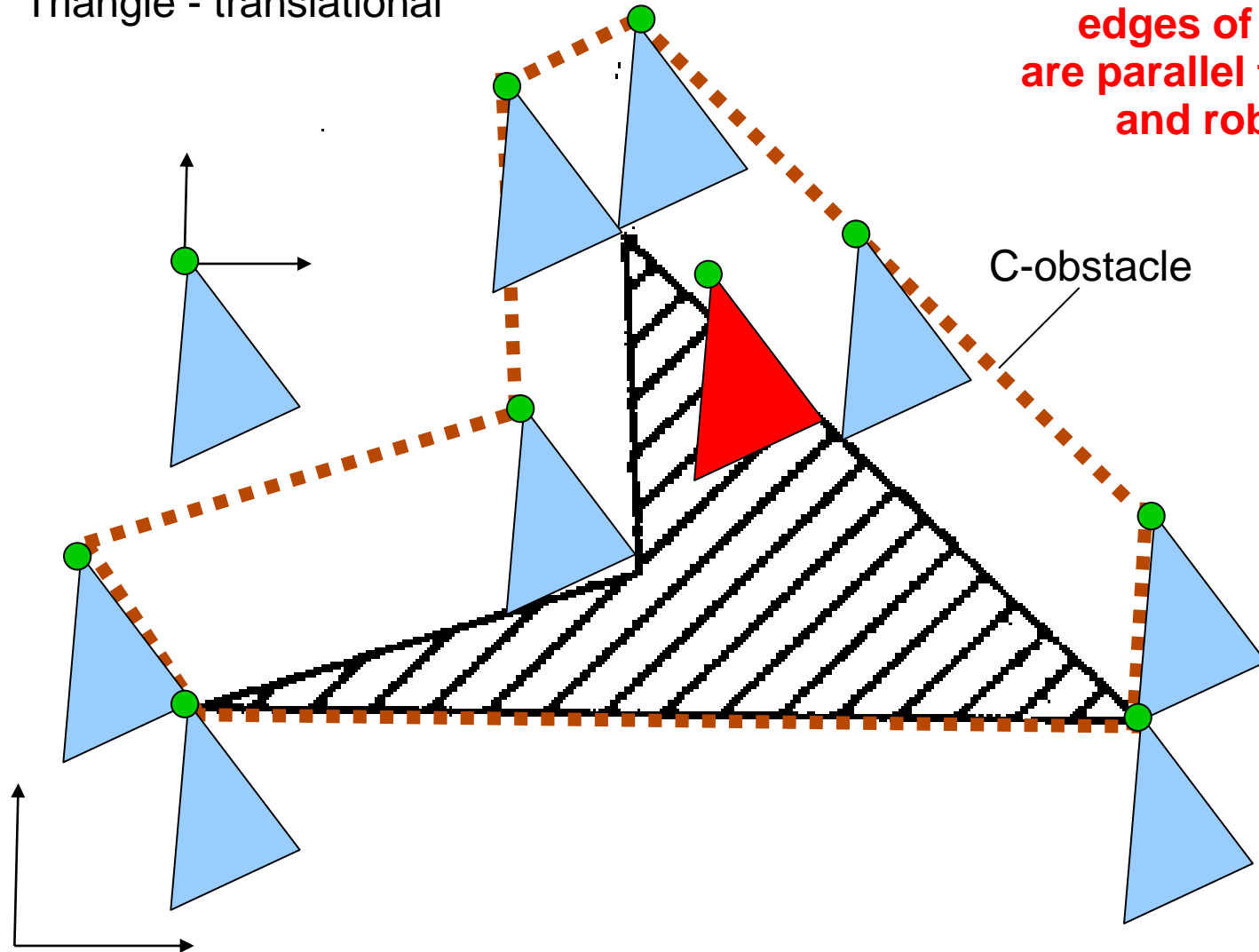
workspace  
 $W$



configuration space  
 $C$

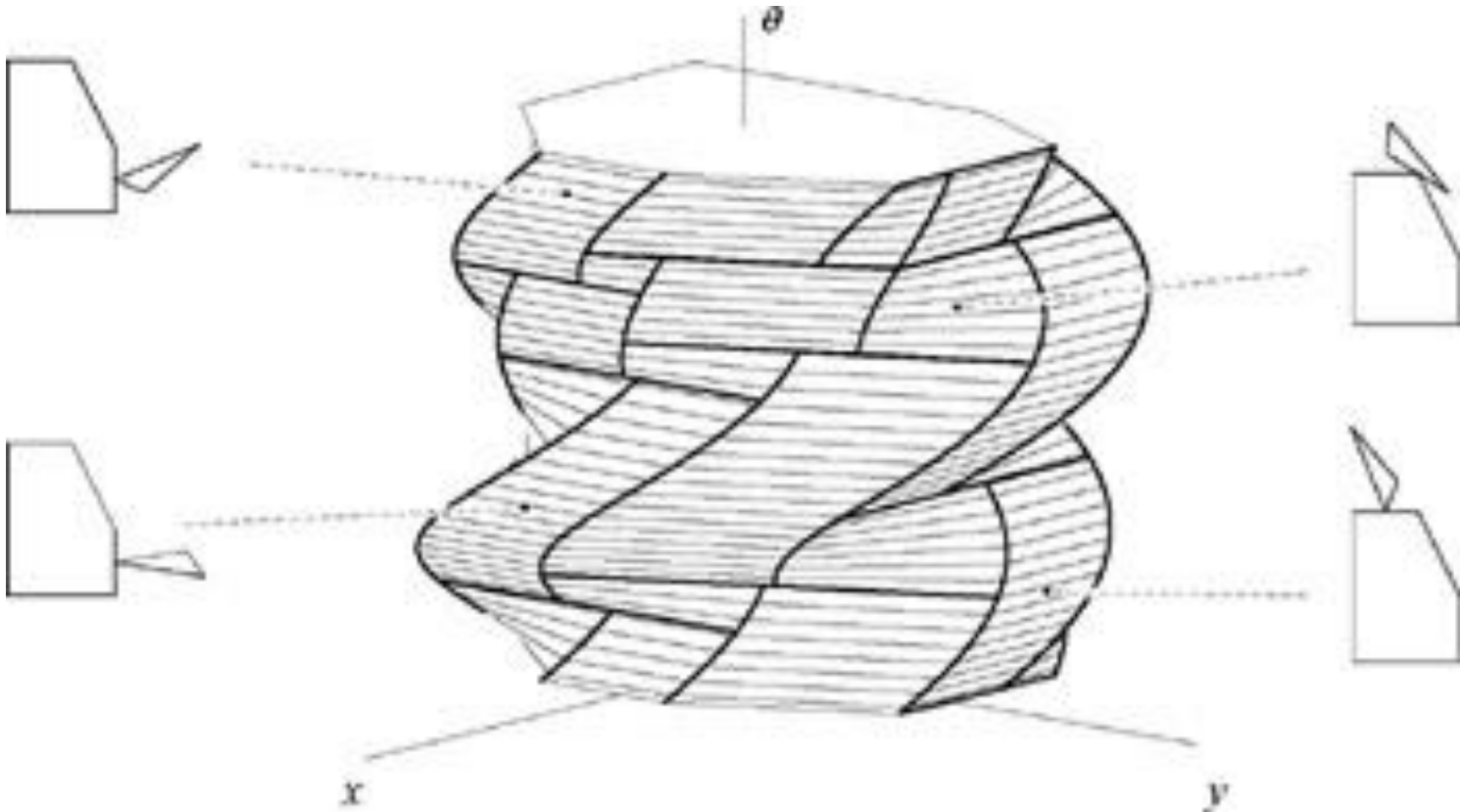
# Non-circular mobile robots

Triangle - translational

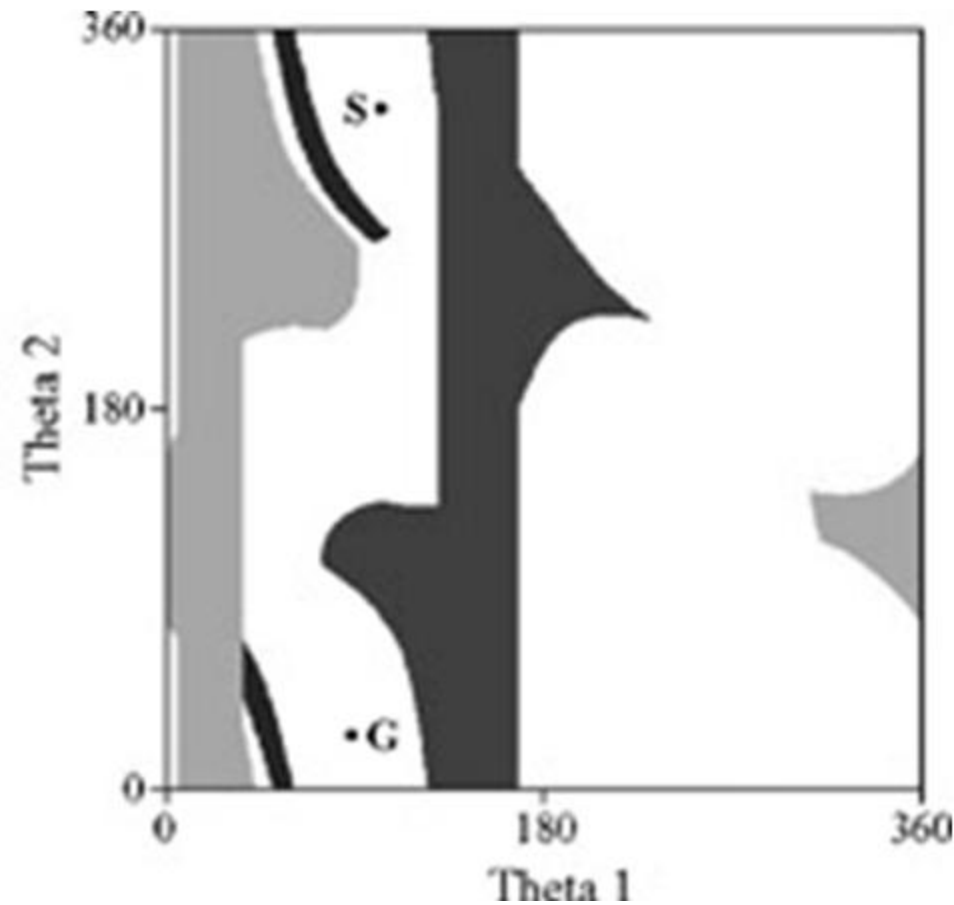
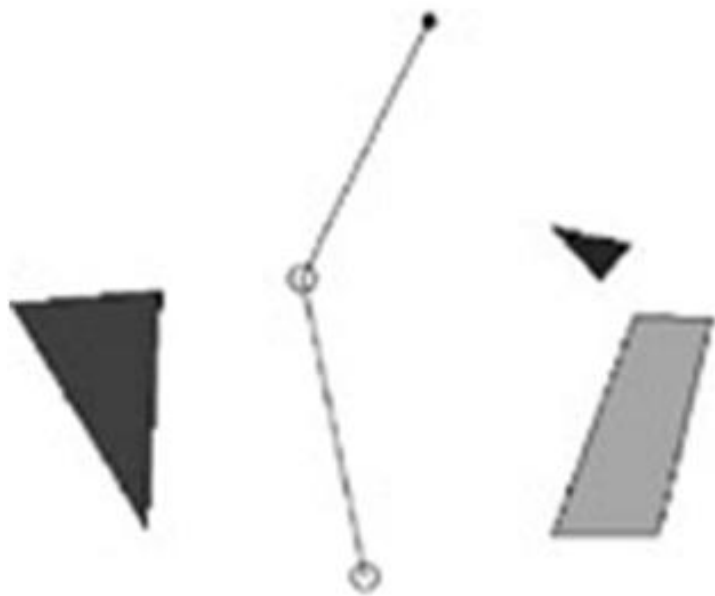


# Non-circular mobile robots

C-space with rotation  $\theta$  (polygonal obstacle)



# Configuration Space for Articulated Robots



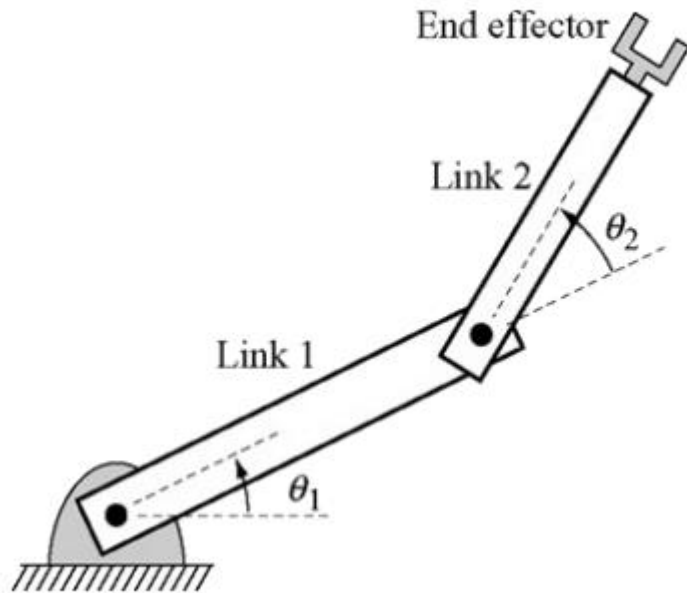
# Configuration Space Analysis

Basic steps (for ANY constrained motion system):

1. determine degrees of freedom (DOF)
2. assign a set of configuration parameters  $\mathbf{q}$   
e.g. for mobile robots, fix a frame on the robot
3. identify the mapping  $R : Q \rightarrow W$ , i.e.  $R(\mathbf{q})$  is the set of points occupied by the robot in configuration  $\mathbf{q}$
4. For any  $\mathbf{q}$  and given obstacle  $B$ , can determine if  $R(\mathbf{q}) \cap B = \emptyset$ .  $\rightarrow$  can identify  $Q_{\text{free}}$   
Main benefit: The search can be done for a point
5. However, computation of C-spaces is not needed in practice; primarily a conceptual tool.



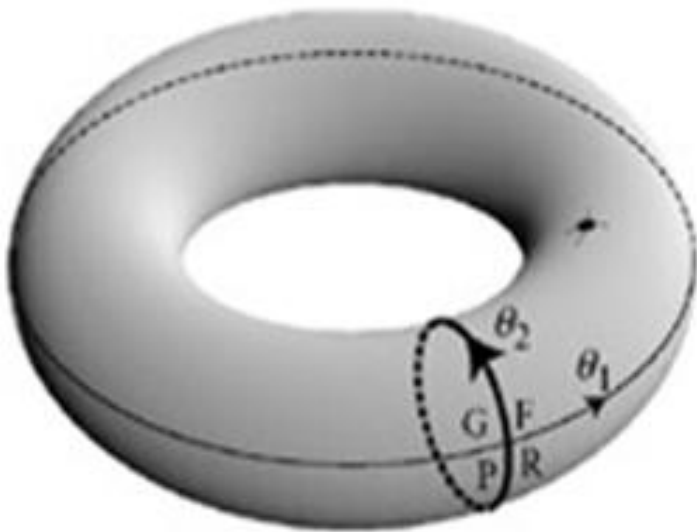
# Articulated Robot C-space



How many parameters needed to fix the robot pose ?

What may be one assignment for the configuration parameters?

# C-space as manifolds



Topology of C-space: Torus ( $S^1 \times S^1$ )

Choset, H et al 2007, Principles of robot motion: Theory, algorithms, and implementations, chapter 3

# C-space as manifolds

- **manifold**: generalization of curves / surfaces

every point on manifold has a neighbourhood homeomorphic to an open set in  $\mathbb{R}^n$

- Mapping  $\Phi : S \rightarrow T$  is bijective (covers all of  $T$  and mapping is unique)

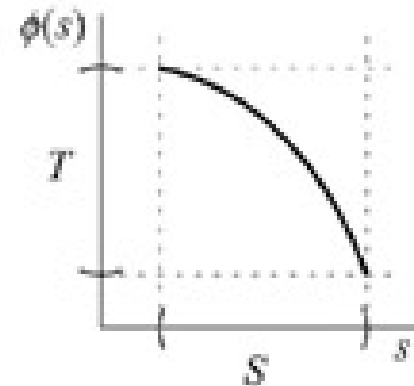
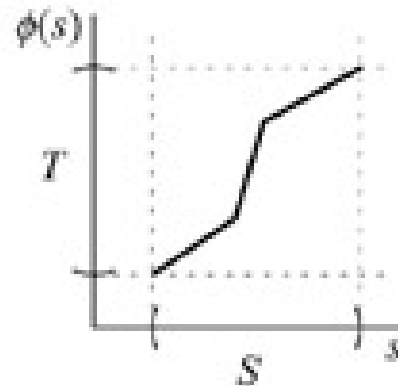
$\Phi$  is

*homeomorphic*:

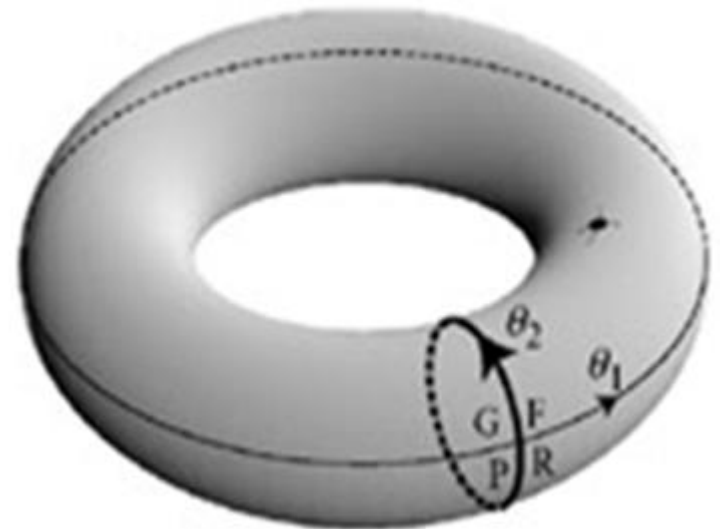
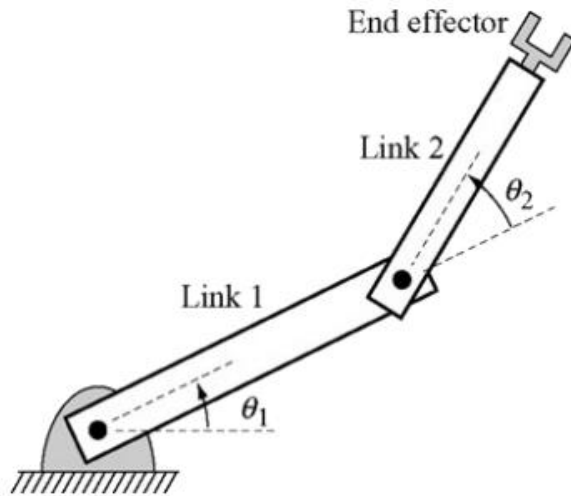
( $f / f^{-1}$  are continuous)

*diffeomorphic* :

( $f / f^{-1}$  are  $C^\infty$  smooth)



# C-space as manifolds



Neighbourhood of  $q$  is mappable to  $\mathbb{R}^2$

global topology is not  $\mathbb{R}^2$  but  $S^1 \times S^1$  (torus)

# Map from C-space to W

Given configuration  $\mathbf{q}$ , determine volume occupied by  $R(\mathbf{q})$  in workspace

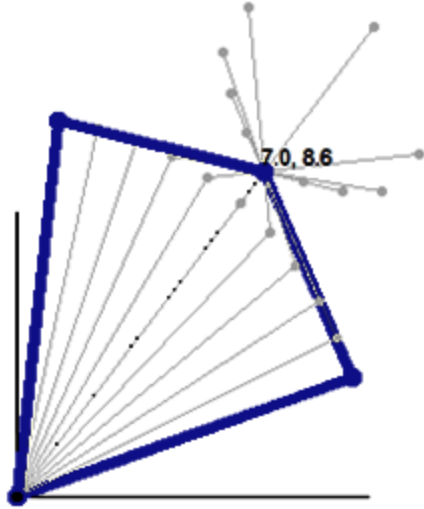
For multi-link manipulators, spatial pose of link  $(n+1)$  depends on joint configuration  $\mathbf{q}$  for joints 1, 2, ...,  $n$ .

→ **Forward Kinematics**

Map from W to C-space: given pose in workspace, find  $\mathbf{q}$

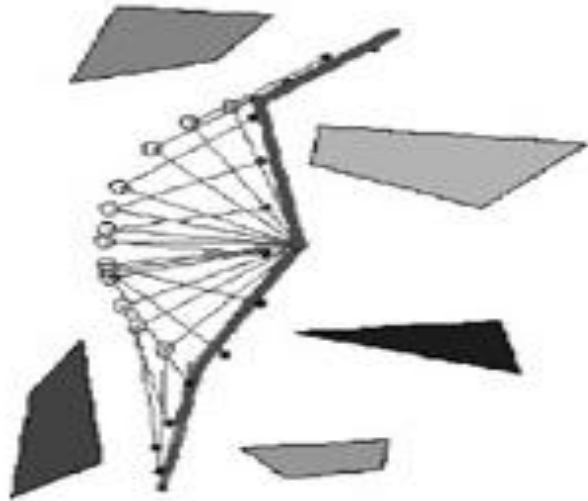
→ **Inverse Kinematics**

# Mapping obstacles

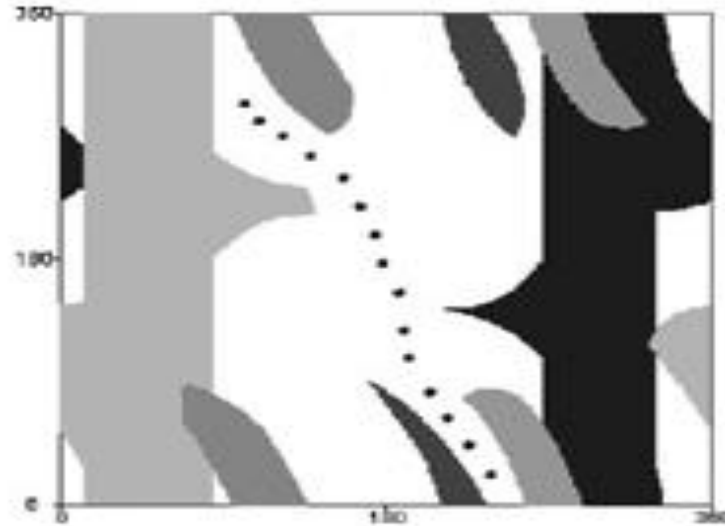


Point obstacle in  
workspace

# Articulated Robot C-space



Path in workspace

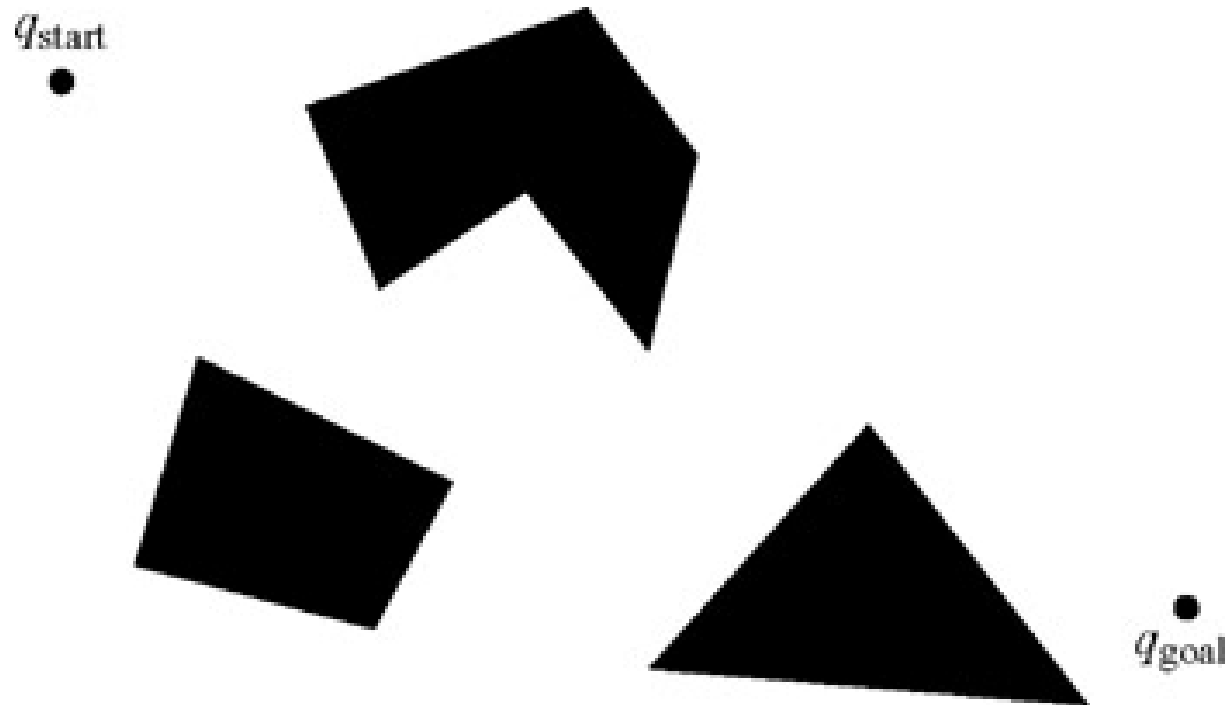


Path in Configuration Space

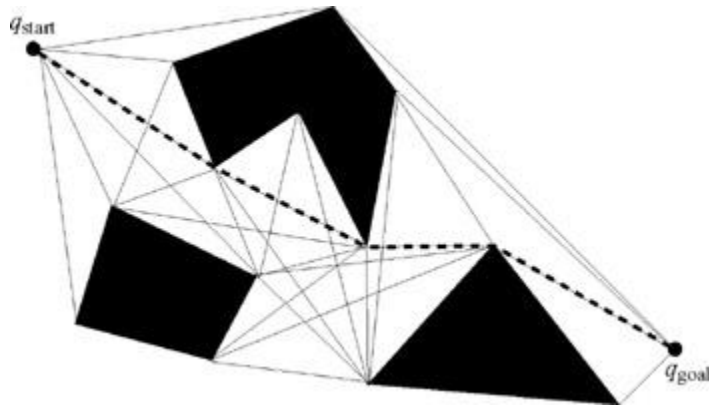
Graph-based approaches



# Visibility Graph methods



# Visibility Graph methods



Construct edges between visible vertices

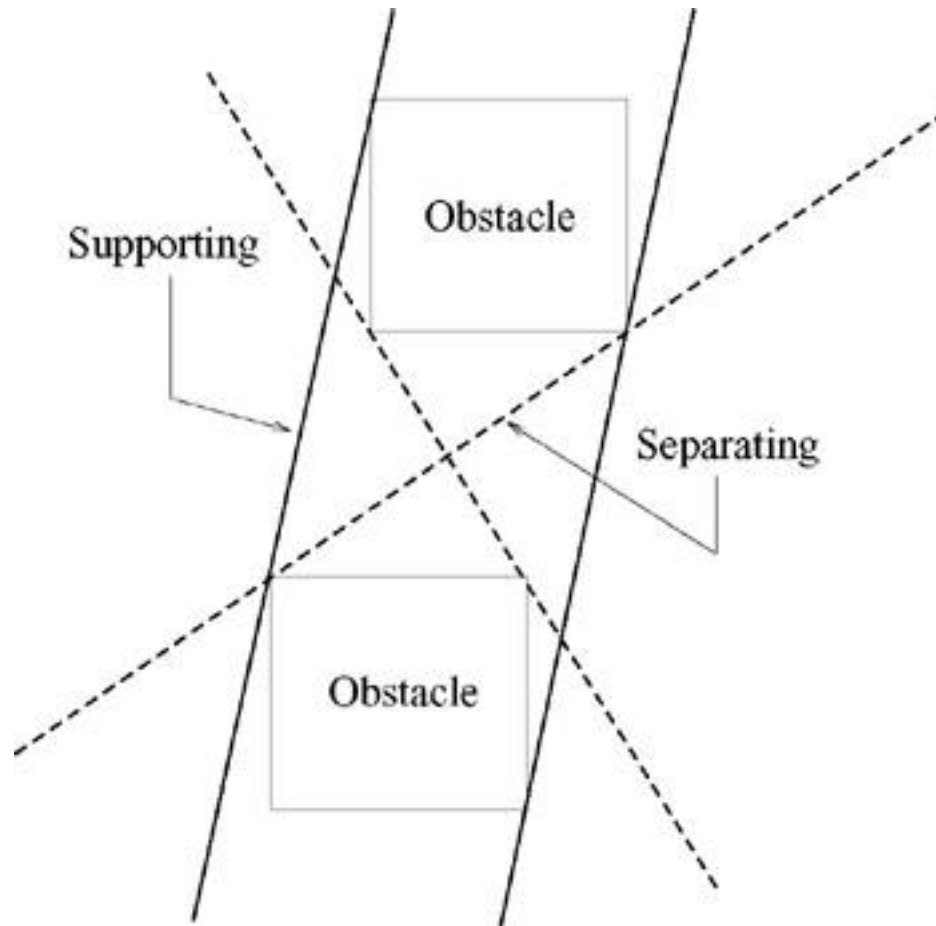
Sufficient to use only **supporting** and **separating** tangents

Complexity:

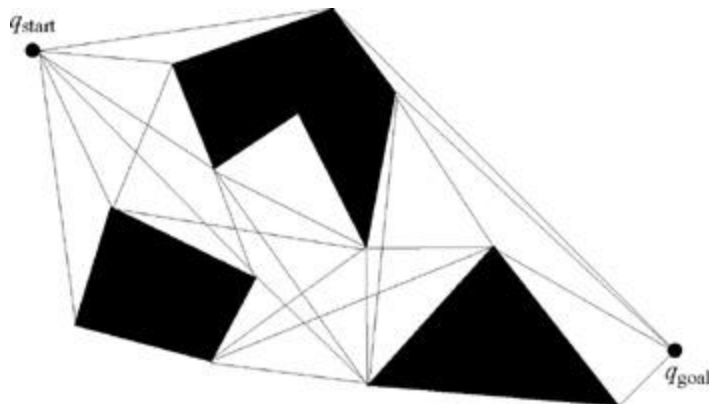
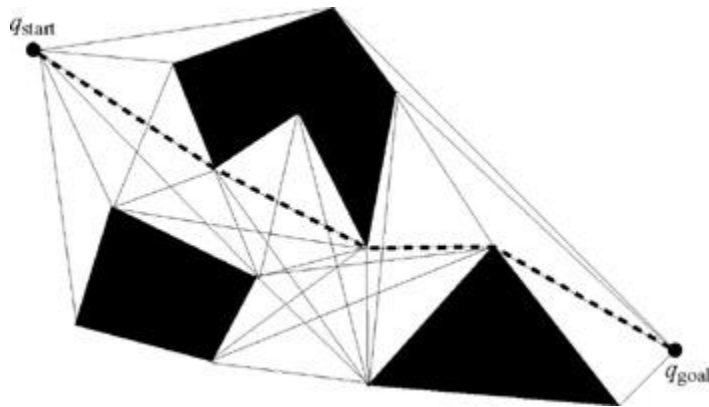
Direct visibility test:  $O(n^3)$   
(tests for each vtx:  $O(n)$  emanations  
x  $O(n)$  obst edges)

Plane sweep algorithm:  $O(n^2 \log n)$

# Visibility Graph methods



# Visibility Graph methods



Sufficient to use only **supporting** and **separating** tangents

Finds “shortest” path – but too close to obstacles

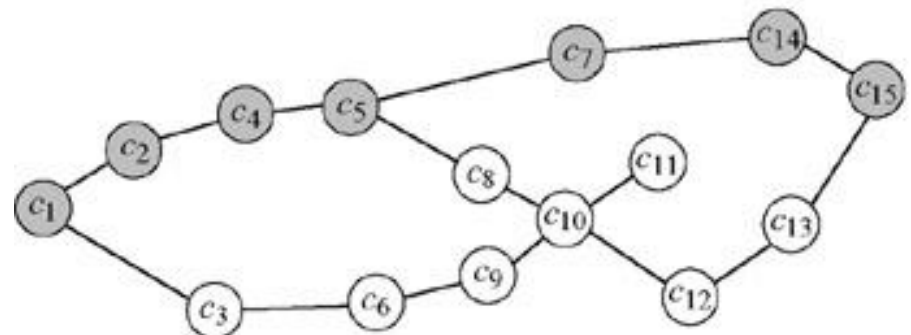
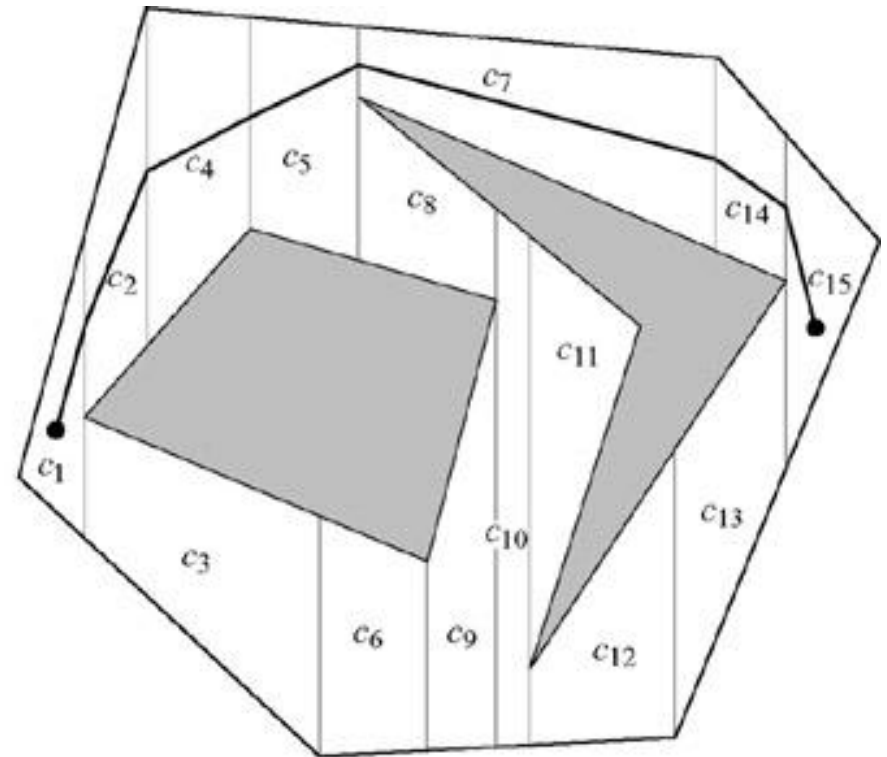
# Cell decomposition methods

Trapezoidal decomposition:  
Each cell is convex.

Sweep line construction:  
 $O(n \log n)$

Graphsearch:  $O(n \log n)$

Path: avoids obstacle  
boundary but has high  
curvature bends



# Roadmap methods

# Roadmaps

any roadmap RM must have three properties:

*Connectivity:*

path exists between any  $q'_{START}$  and  $q'_{GOAL}$  in RM

*Accessibility:*

exists a path from any  $q_{START} \in Q_{free}$  to some  $q'_{START} \in RM$

*Departability:*

exists a path from some  $q'_{GOAL} \in RM$  to any  $q_{GOAL} \in Q_{free}$

# Staying away from Obstacles: Generalized Voronoi Graphs



Voronoi Region of obstacle  $i$  :

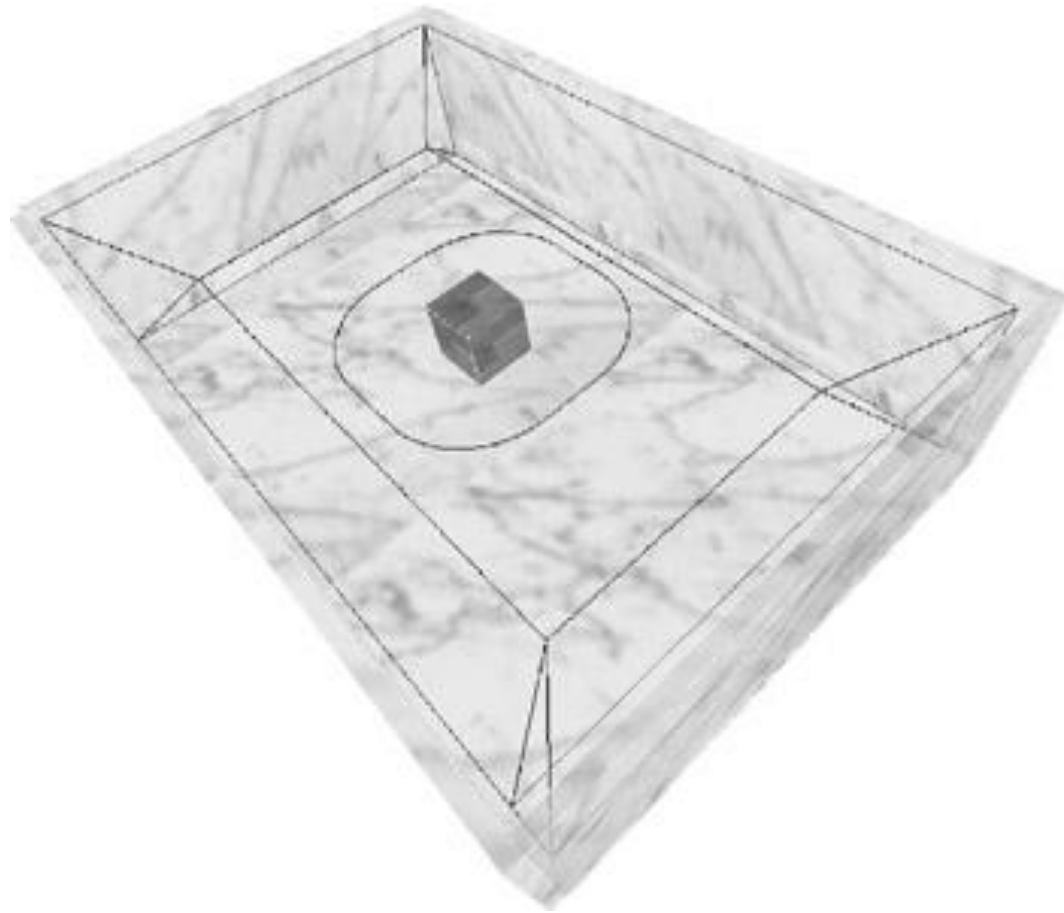
$$\mathcal{F}_i = \{q \in \mathcal{Q}_{\text{free}} \mid d_i(q) \leq d_h(q) \quad \forall h \neq i\},$$

Voronoi diagram:

set of  $q$  equidistant from at least two obstacles



# Generalized Voronoi Graphs



# GVG Roadmaps

## Accessibility / Deparability:

Gradient descent on distance from dominant obstacle :

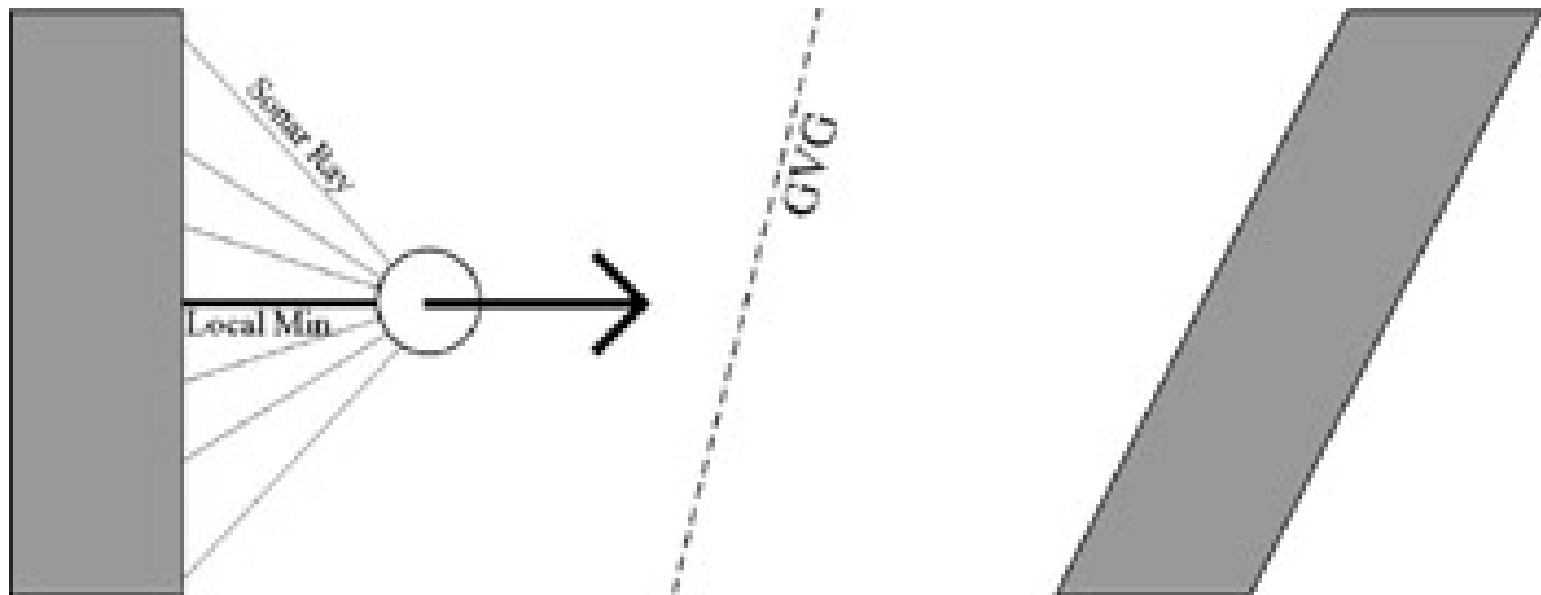
→ guaranteed to reach from any  $q_{START} \in Q_{free}$  to some  $q'_{START} \in RM$

→ motion is along a “retract” or brushfire trajectory

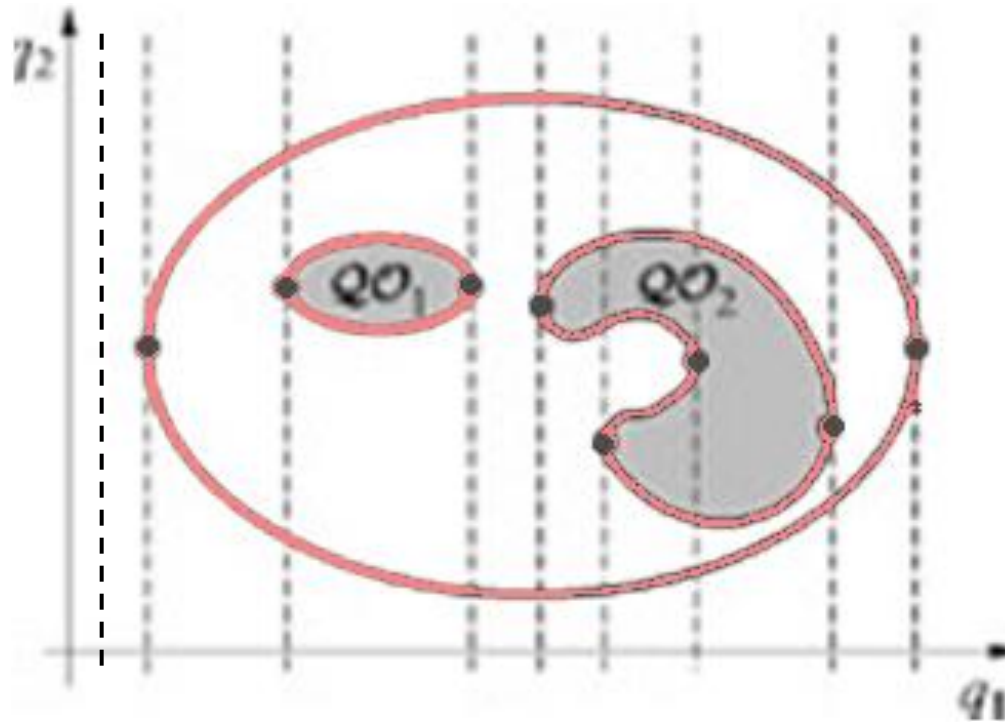
## Connectivity:

GVG is Connected if path exists

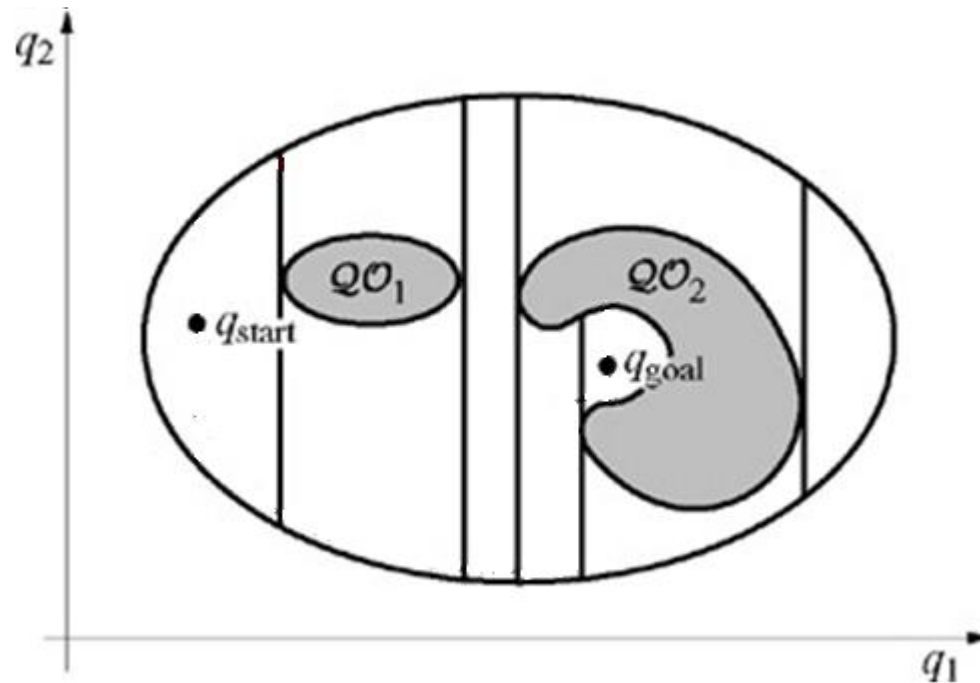
# Sensor based Voronoi roadmap construction



# Canny's Silhouette roadmap



# Canny's Silhouette roadmap



# Canny's Complexity Analysis

$n$ : = degrees of freedom of robot (dim of C-space)

obstacles C-space boundaries represented as  $p$  polynomials of maximum degree  $w$

Complexity:

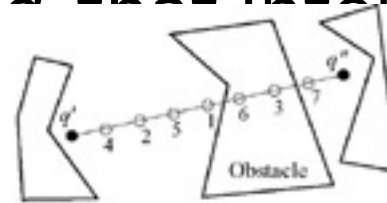
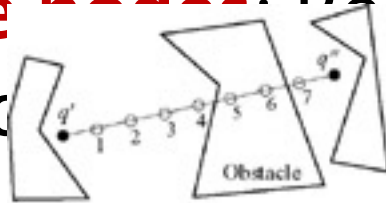
any navigation path-planning problem can be solved in  $p^n(\log p)w^{O(n^4)}$  time

# Probabilistic Roadmap (PRM)

# Probabilistic Roadmap

Sample  $n$  poses  $q_1 \dots q_n$  in the WORKSPACE

**Free space nodes:** Points that intersect with an obstacle in  $Q_{free}$



**Local planner:** If path  $\langle q_i, q_j \rangle$  collision-free, add edge to graph

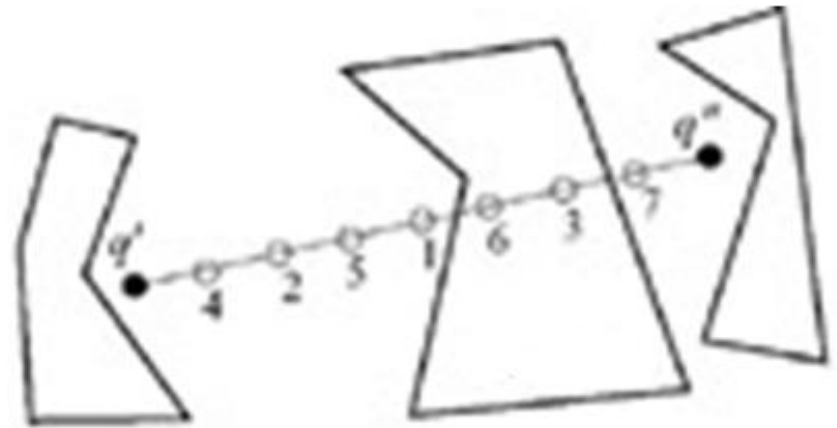
Resulting graph = *Probabilistic Roadmap*



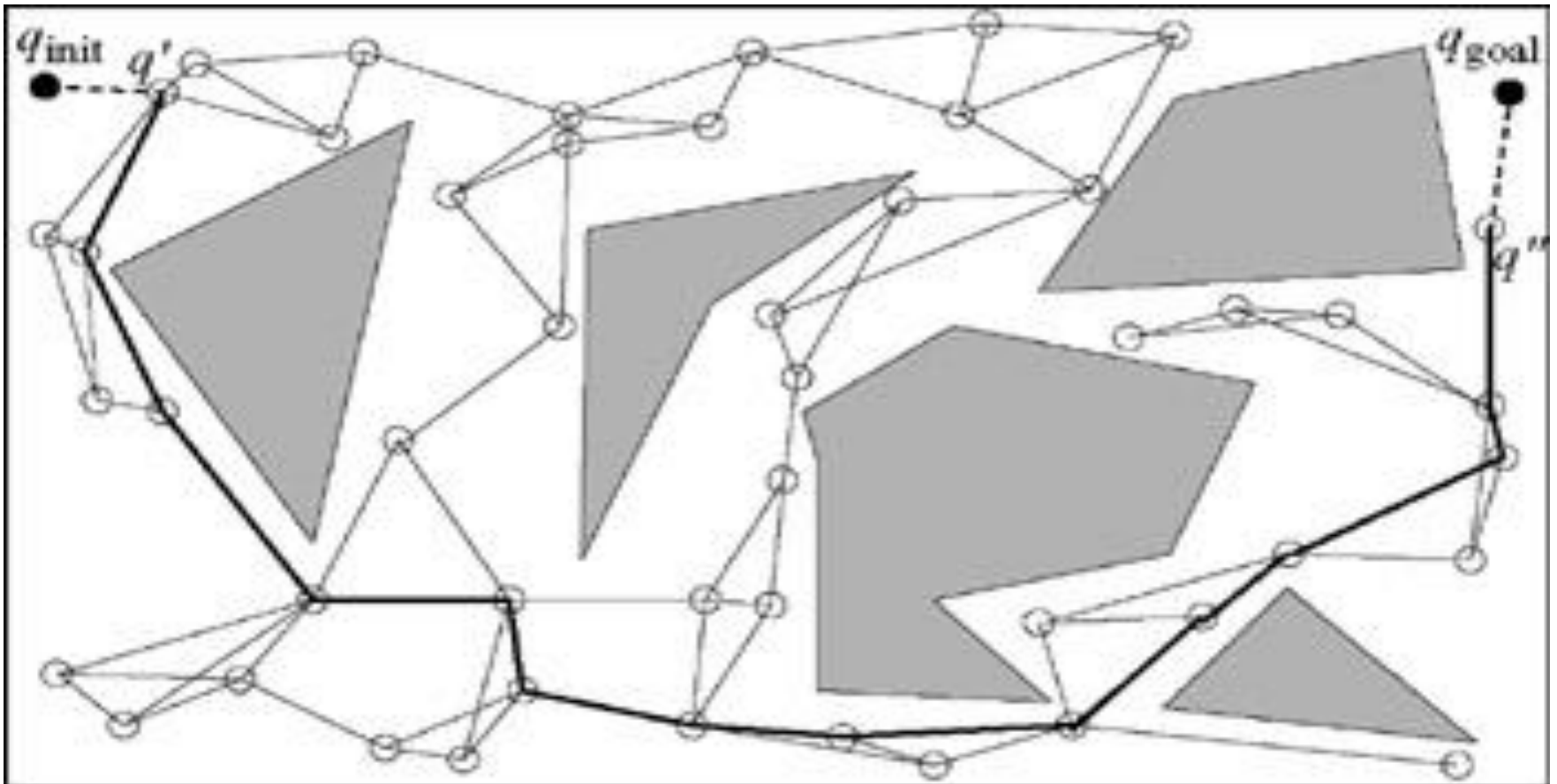
# Local Planner

Objective: Test if path  
 $\langle q_i, q_j \rangle$  is collision-free

Linear Subdivision  
algorithm: start at  
midpoint( $q_i, q_j$ ) ;  
subdivide  
recursively until  
desired precision



# Probabilistic Roadmaps (PRM)



# Sampling-based motion planning

Sample  $n$  poses  $q_1 \dots q_n$  in the workspace

Reject  $q_n$  that overlap with an obstacle,  
remaining poses are in  $Q_{\text{free}}$

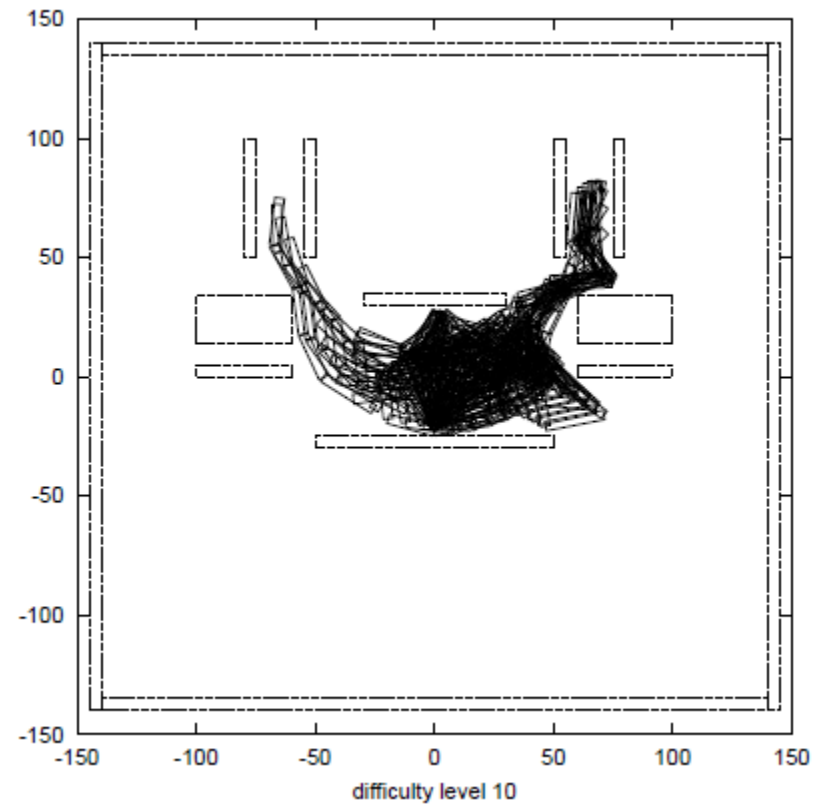
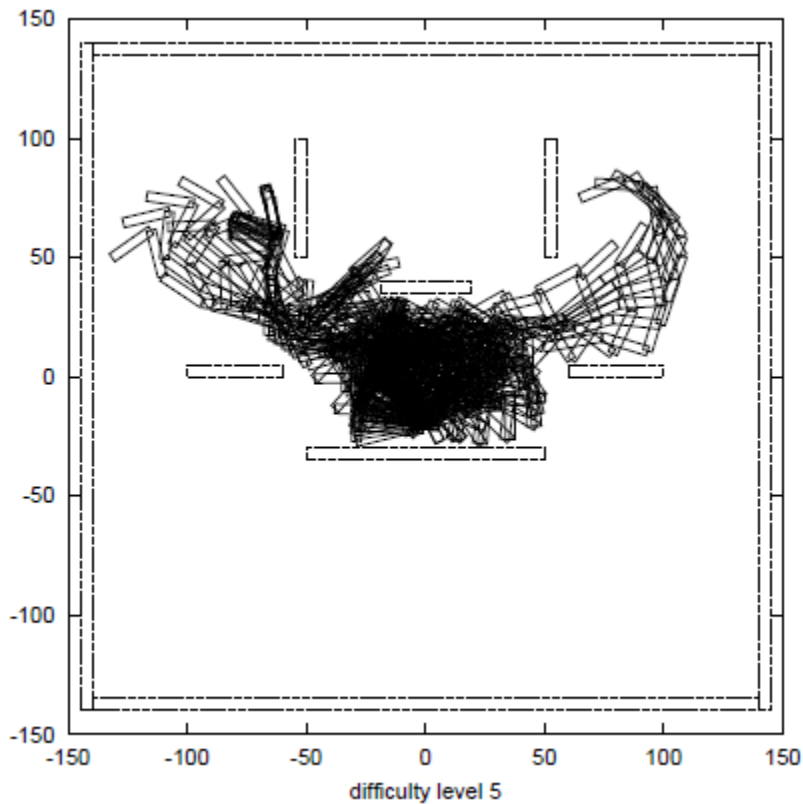
Use local planning to determine if a path  
exists between neighbours  $q_i$  and  $q_j$ .

Resulting graph = *Probabilistic Roadmap*

Probabilistically complete:

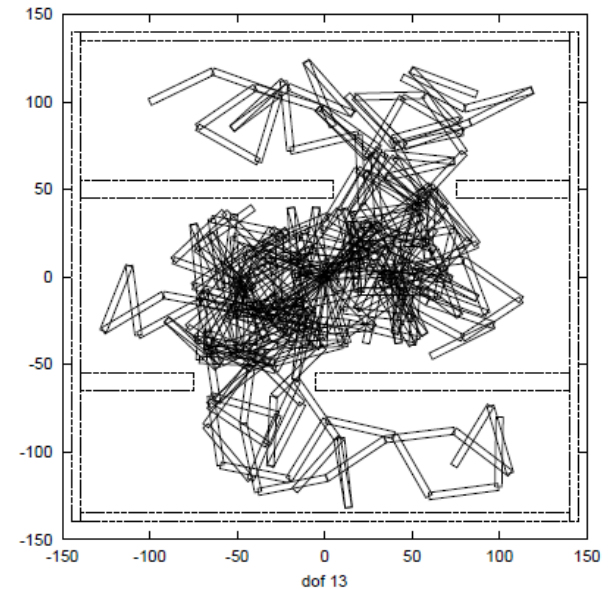
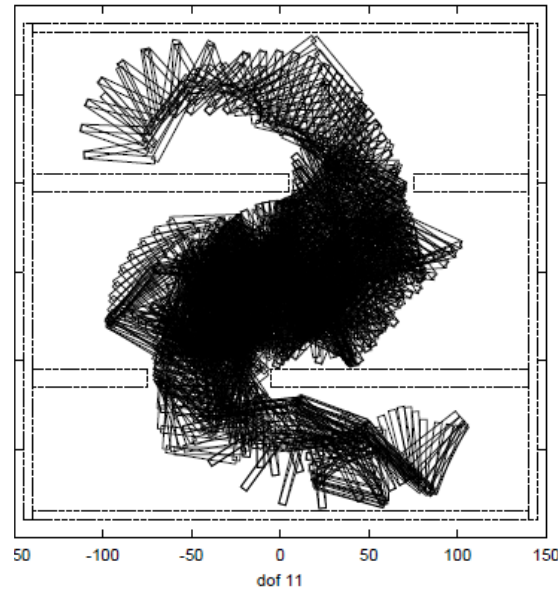
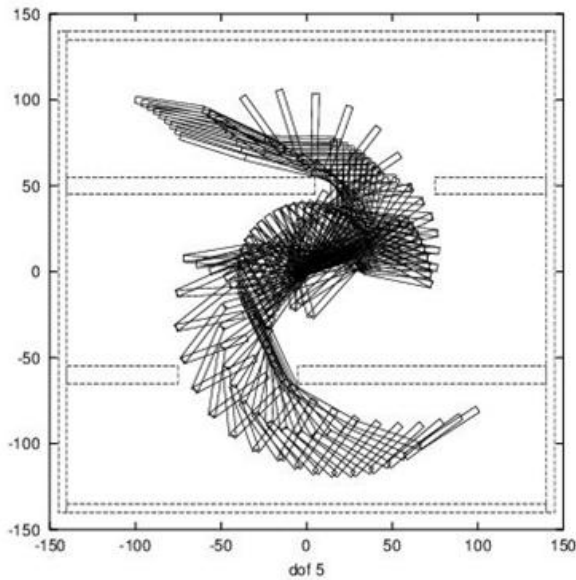
As #samples  $n \rightarrow \infty$ , Prob (success)  $\rightarrow 1$

# Hyper-redundant robot motion planning

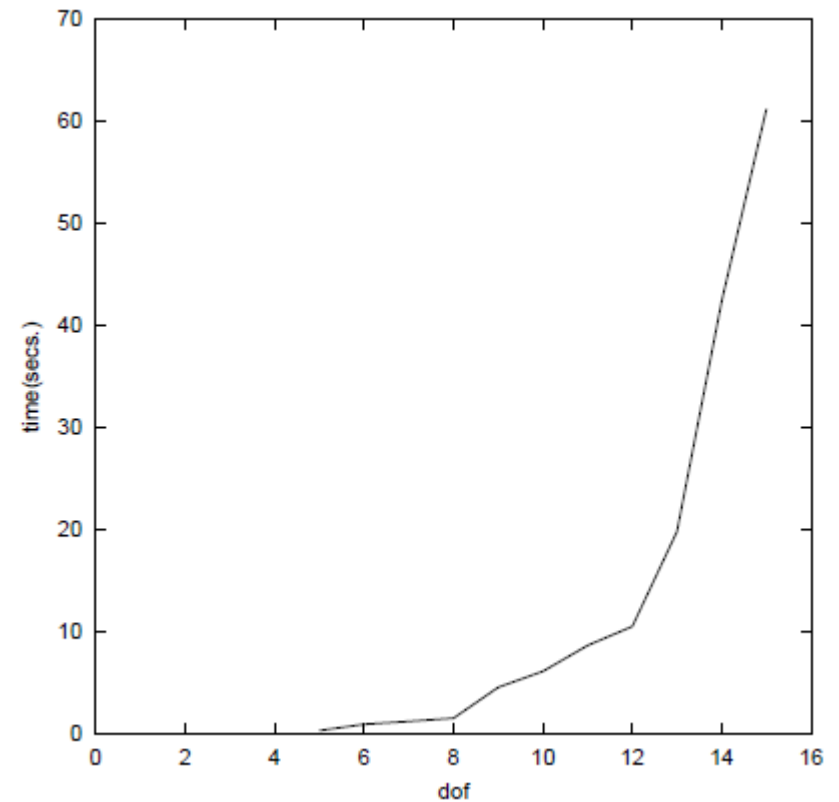


[sinha mukerjee dasgupta 02]

# Hyper-redundant robot motion planning



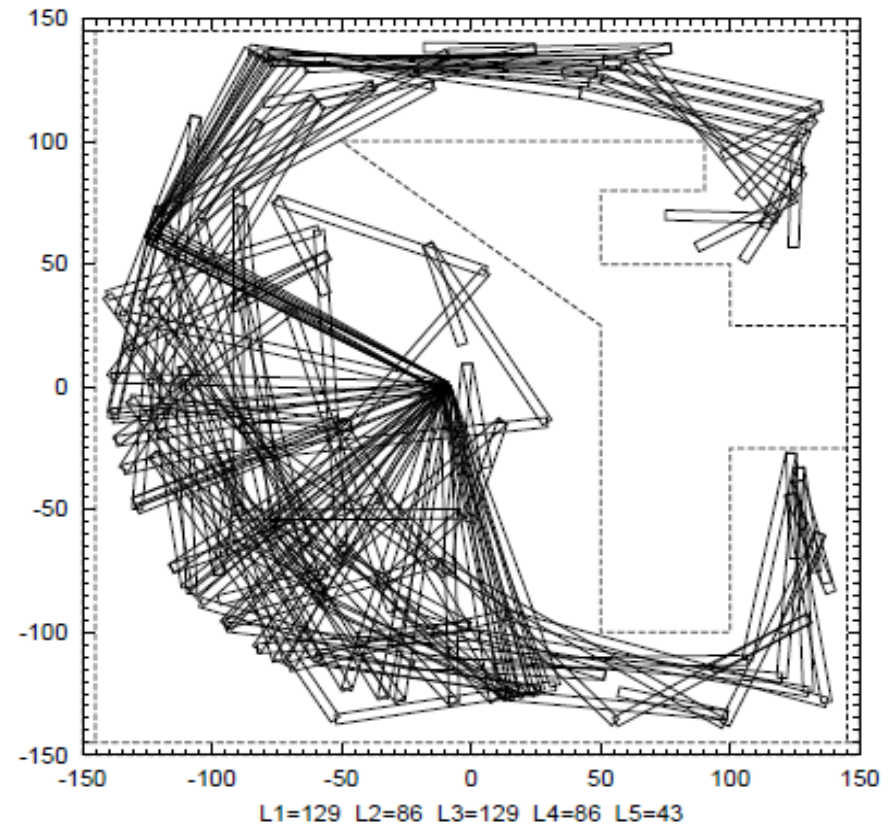
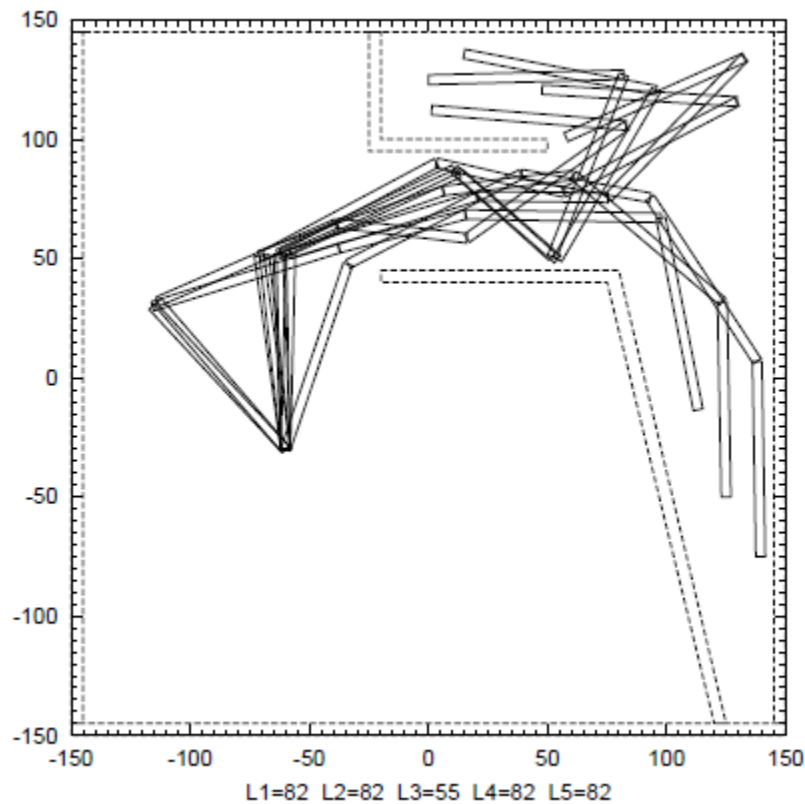
# Hyper-redundant motion planning



Time:  
Exponential in DOFs

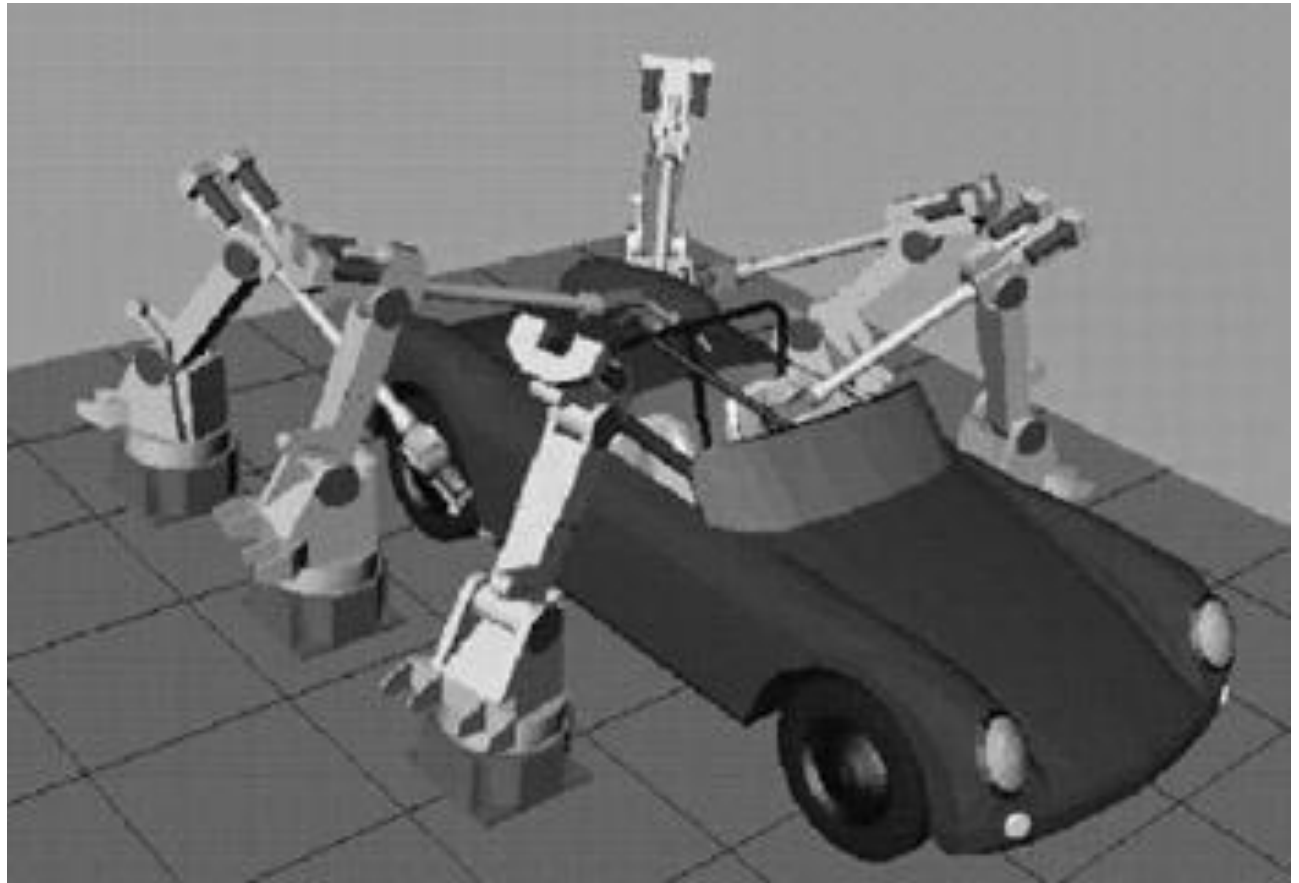
[sinha mukerjee dasgupta 02]

# Design for manipulability



[sinha mukerjee dasgupta 02]

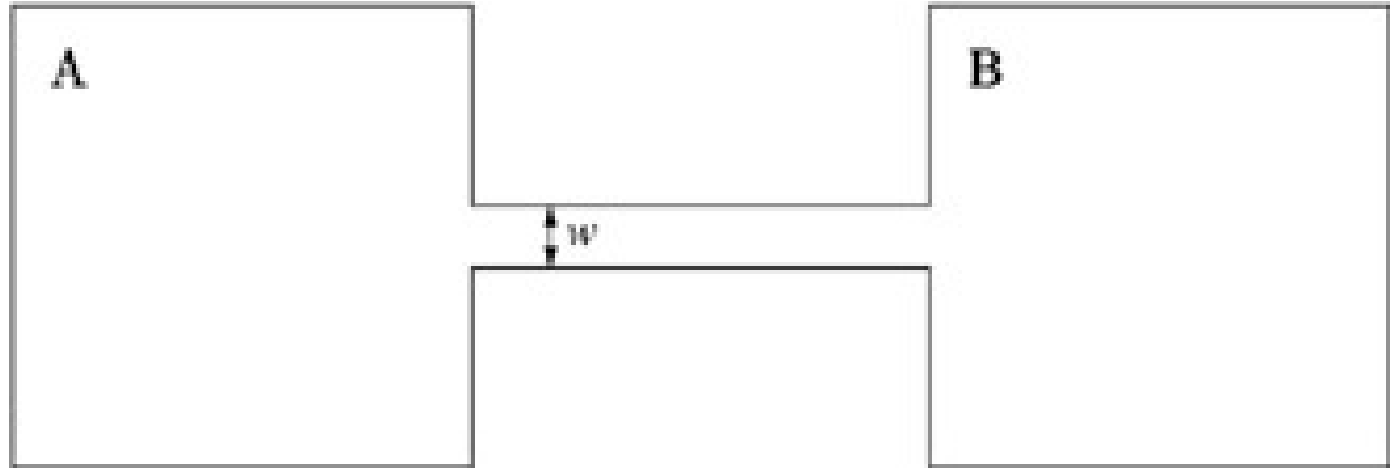
# PRM applications



42 DOFs: [Sánchez and J. C. Latombe 02]

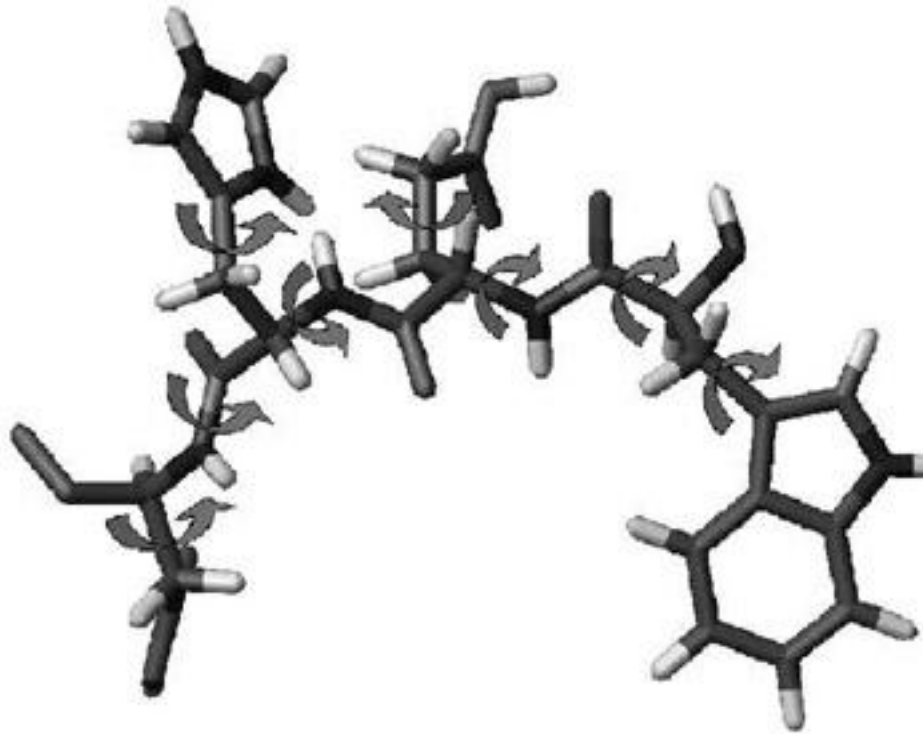


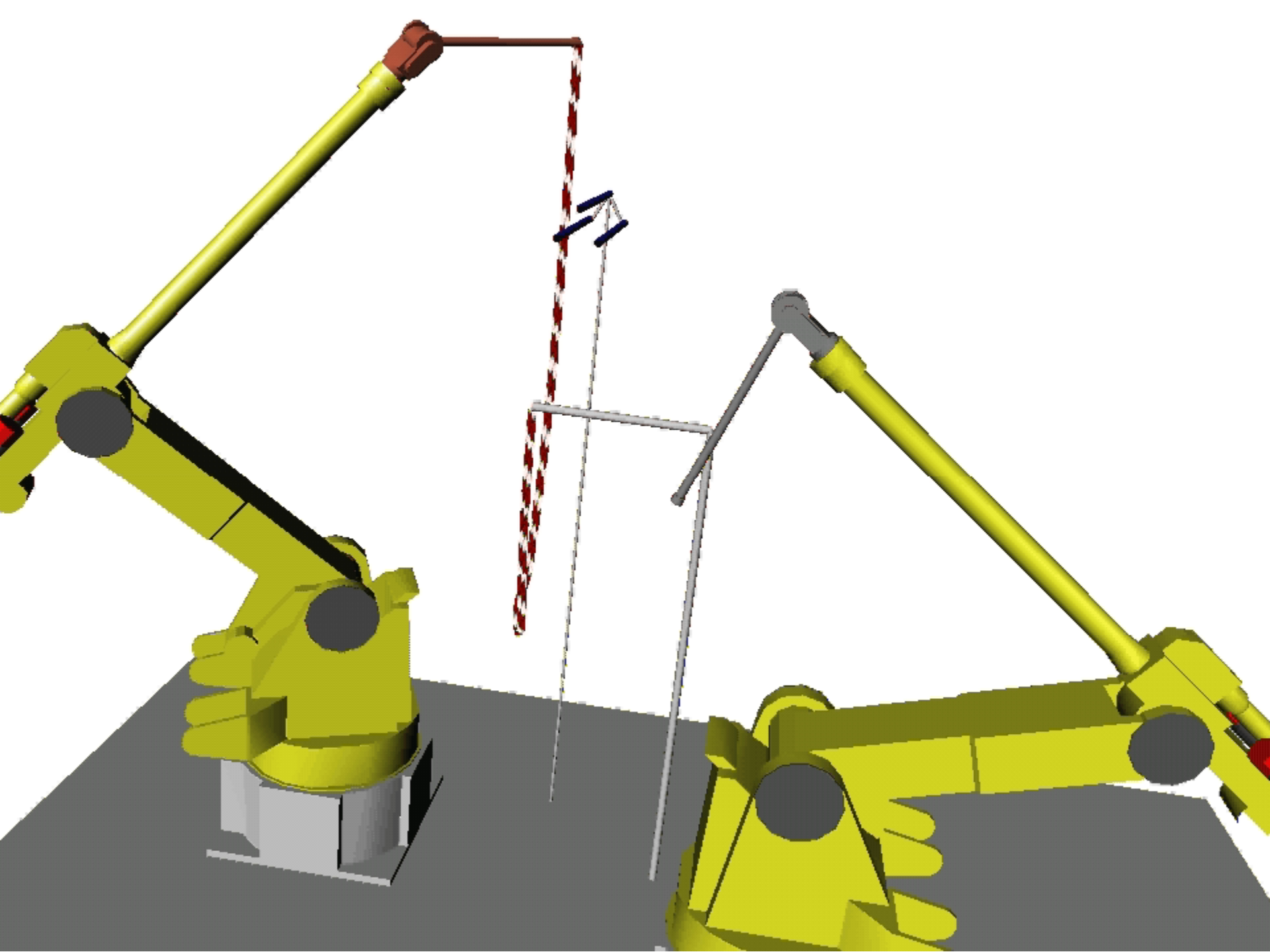
# Narrow corridor problem



- Solution: generate more samples near boundary
- bias the sample towards boundary region
  - if midpoint between two obstacle nodes is free, add

# PRM applications : Protein folding





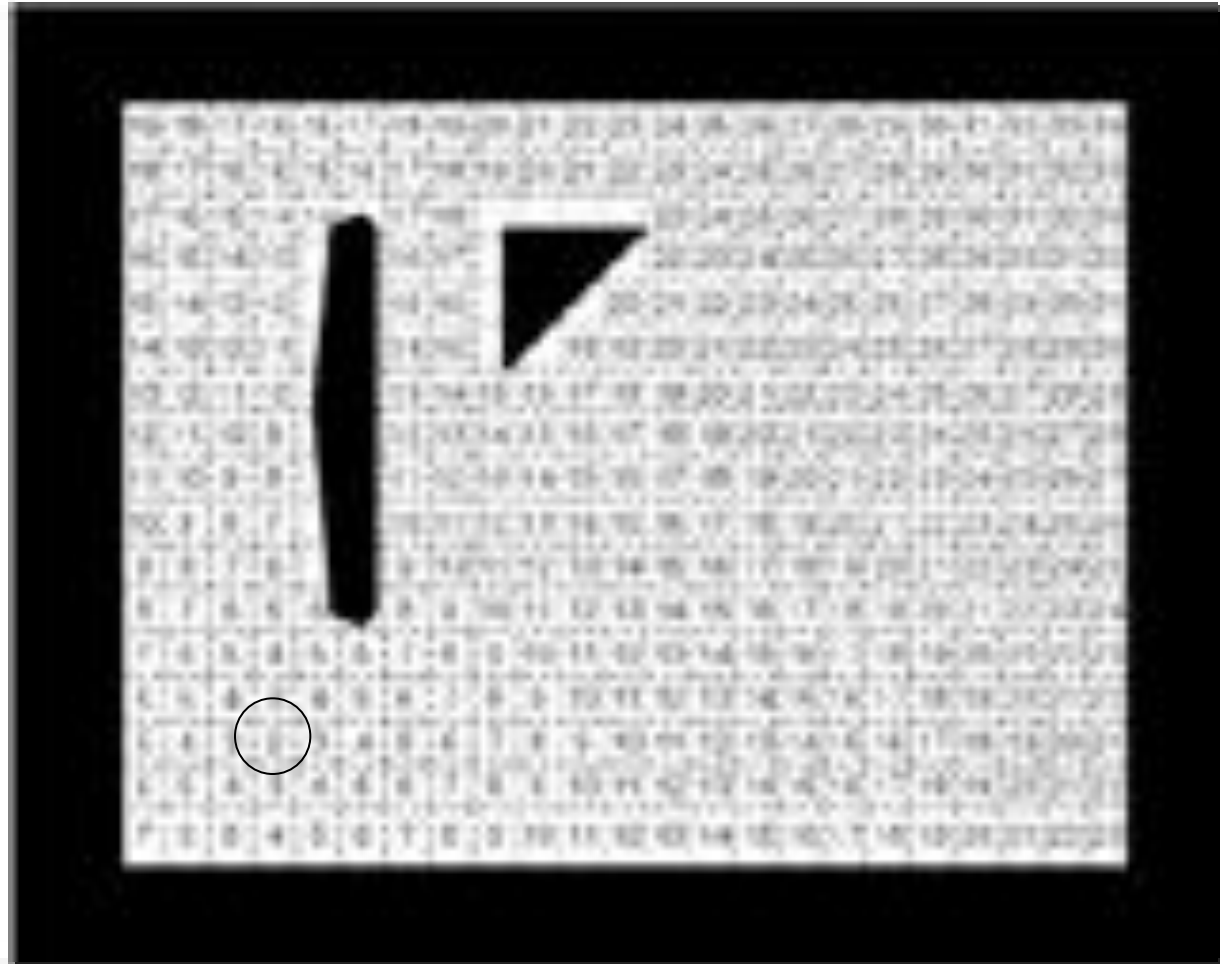
# Continuum methods: Overcoming Local minima

# Grid-based: Wave-front

- Grid-based model
- given a start grid cell  $\mathbf{q}_s$  assign it the value “2”
  - Every neighbour gridcell gets +1
  - Until grid is filled
- Given a goal cell  $\mathbf{q}_G$  use greedy search to find path back to goal

# Grid-based: Wave-front

$O(k^d)$  space /  
time

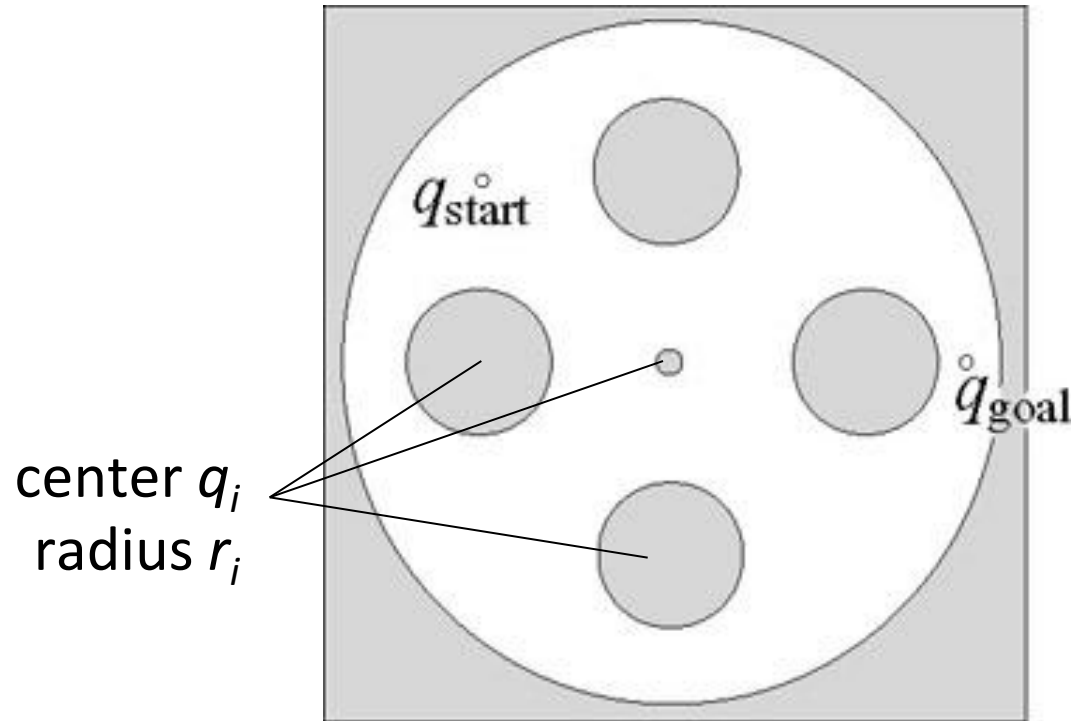


# Navigation Function : Sphere space

- Spherical wall ( $r_0$ ), with spherical obstacles inside

- Obstacle distance  
 $\mathcal{QO}_i = \{q \mid \beta_i(q) \leq 0\}$   
 $\beta_0(q) = -d^2(q, q_0) + r_0^2, \quad \text{— wall}$   
 $\beta_i(q) = d^2(q, q_i) - r_i^2, \quad \text{— obstacles}$

# Sphere space

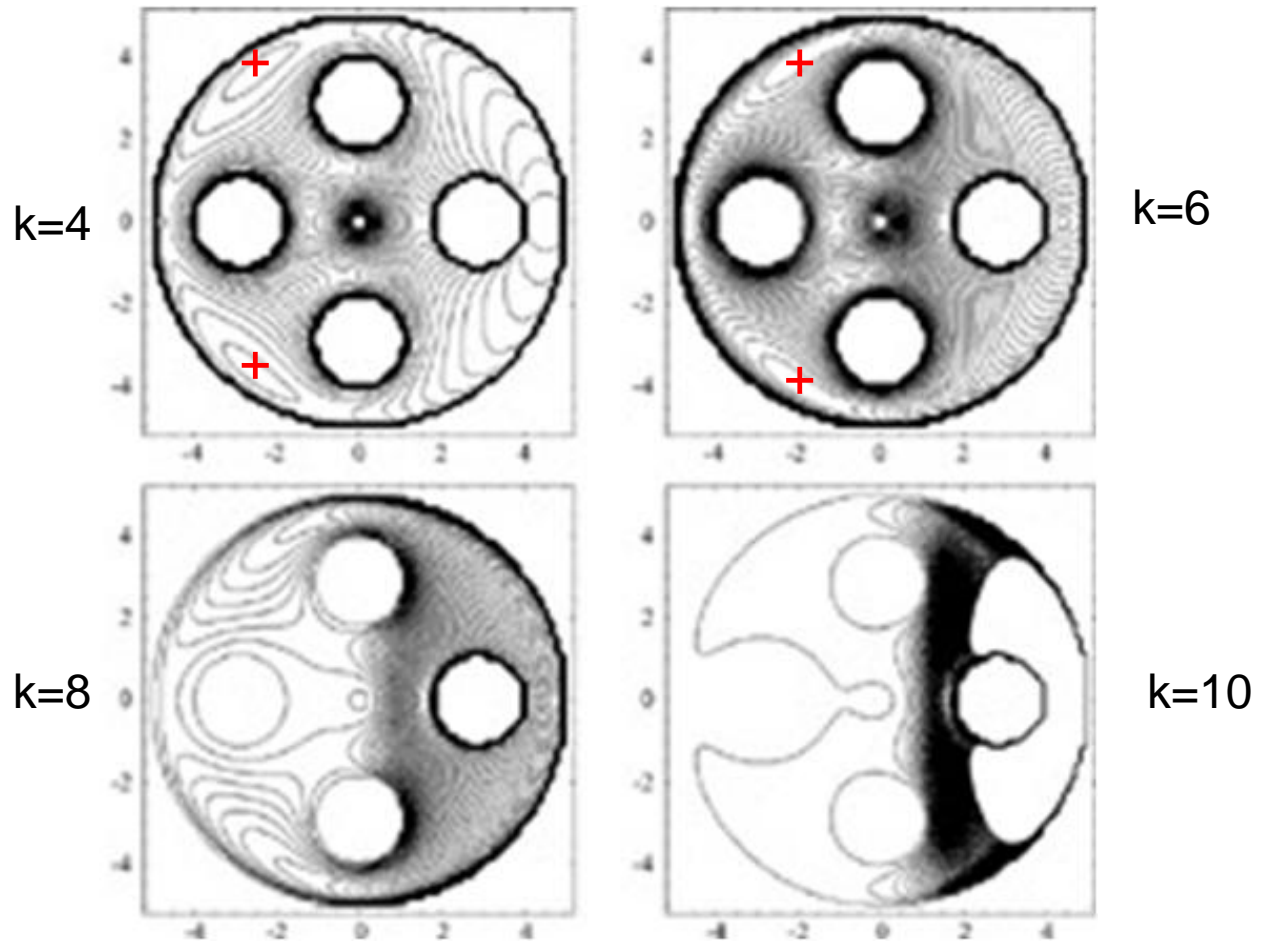




# Navigation Function : Sphere space

- Spherical wall ( $r_0$ ), with spherical obstacles inside
- Obstacle distance  $\beta_0(q) = -d^2(q, q_0) + r_0^2$ , — wall  
 $\mathcal{QO}_i = \{q \mid \beta_i(q) \leq 0\}$   $\beta_i(q) = d^2(q, q_i) - r_i^2$ , — obstacles
- Goal potential with high exponent  $\gamma_\kappa(q) = (d(q, q_{\text{goal}}))^{2\kappa}$
- Instead of sum, use product to combine obstacle potentials  $\beta(q) = \prod_{i=0}^n \beta_i(q)$ .
- For high  $k$ ,  $\frac{\gamma_\kappa}{\beta}(q)$  has unique minima at goal

# Navigation Function

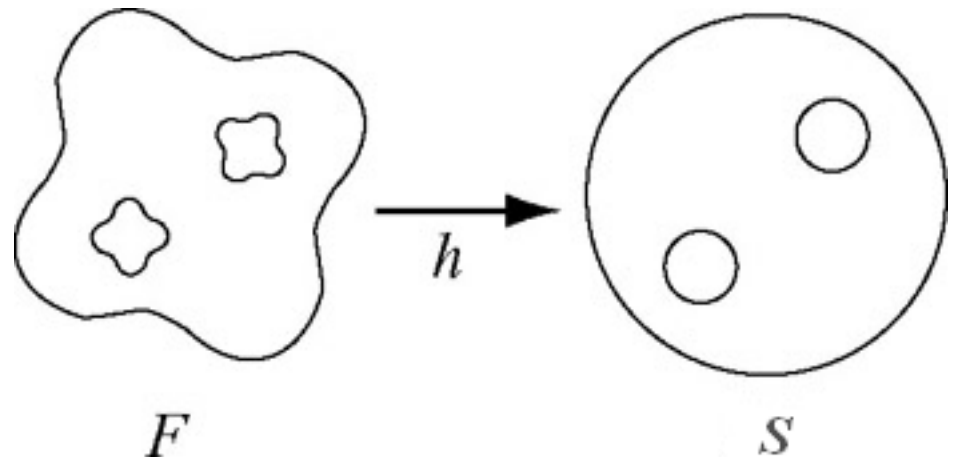


# Navigation Function

$\varphi : S \rightarrow [0, 1]$  :  
navigation function on  
sphere space  $S$ .

For any space  $F$  if exists  
diffeomorphic  
mapping  $h : F \rightarrow S$   
(i.e.  $h$  is smooth, bijective, and  
has a smooth inverse),

then  $\varphi = \varphi \circ h$  is a  
navigation function on  $F$



Conclusion

# Beyond Geometry

- Geometry is not everything!!
- Real robots have limitations on acceleration owing to torque / inertia → **Dynamics**
- **Learning** to plan motions?
  - Babies learn to move arms
  - Learn low-dimensional representations of motion
- **Grasping** / Assembly : Motions along obstacle boundary

# Humans and Robots

