Visibility and Transformation of Optimal Paths in Simple Polygons

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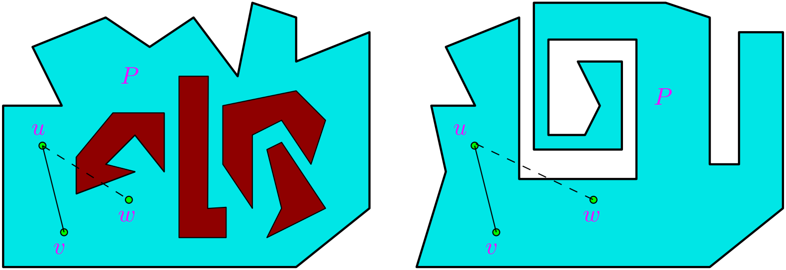
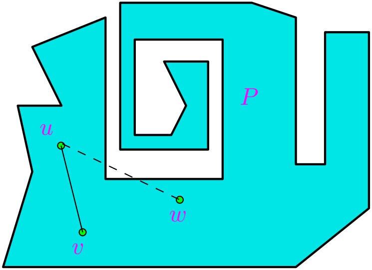
Organization

Outline



- A *polygon* P is defined as a closed region in the plane bounded by a finite set of line segments (called *edges* of P) such that there exists a path between any two points of P which does not intersect any edge of P.



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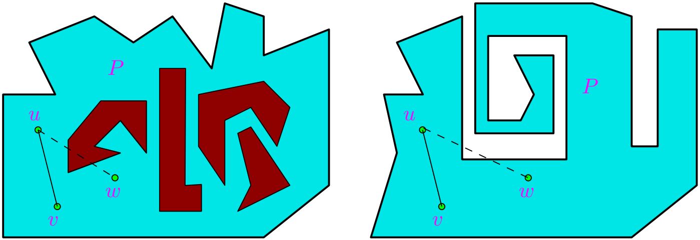
- If the boundary of P consists of two or more cycles, then P is called a *polygon with holes*. Otherwise, P is called a *simple polygon* or a *polygon without holes*.

- Two points u and v in a polygon P are said to be *visible* if the line segment joining u and v lies entirely inside P.



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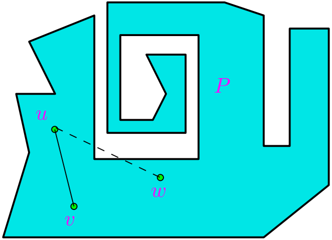
- Using this definition of visibility, a *path* in P can be defined as a sequence of line segments such that the two endpoints of every line segment are mutually visible, i.e., every such line segment lies totally inside P.



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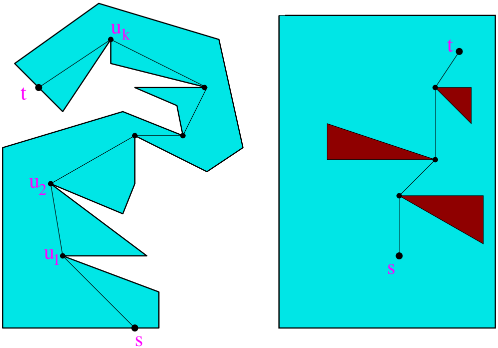
- S. K. Ghosh, *Visibility Algorithms in the Plane*, Cambridge University Press, Cambridge, UK, 2007.

Outline

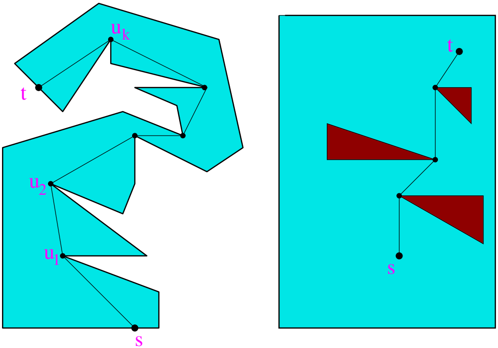
1. Visibility in Polygons
2. Euclidean Shortest Paths
3. Minimum Link Paths
4. Optimal Diffused Reflection Paths

Definitions and properties



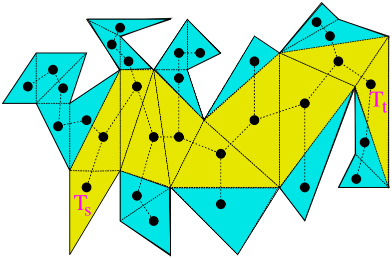
- The *Euclidean shortest path* (denoted as $SP(s,t)$) between two points s and t in a polygon P is the path of smallest length between s and t lying totally inside P.

Definitions and properties



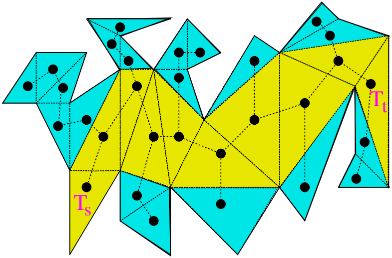
- The *Euclidean shortest path* (denoted as $SP(s,t)$) between two points s and t in a polygon P is the path of smallest length between s and t lying totally inside P.
- Let $SP(s,t) = (s,\ u_1,\ u_2, \ldots,\ u_k,\ t)$. Then, (i) $SP(s,t)$ is a simple path, (ii) $u_1,\ u_2, \ldots,\ u_k$ are vertices of P and (iii) for all i, u_i and u_{i+1} are mutually visible in P. $SP(s,t)$ is *outward convex* at every vertex on the path.

Computing $SP(s, t)$



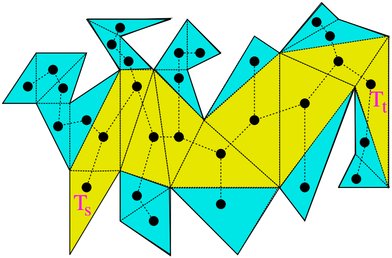
- The dual graph of a triangulation of a simple polygon is a tree.

Computing $SP(s, t)$



- The dual graph of a triangulation of a simple polygon is a tree.
- $SP(s, t)$ passes only through the triangles in the path from T_s and T_t in the dual tree.

Computing $SP(s, t)$



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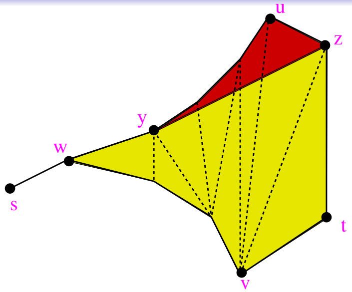
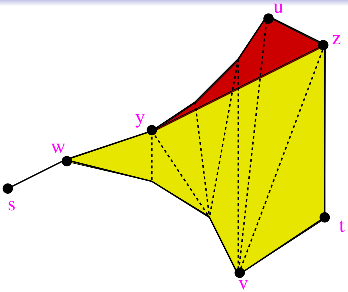
- B. Chazelle. *Triangulating a simple polygon in linear time*, Discrete and Computational Geometry, 6(1991), 485-529. Running time: $O(n)$.



- Let (u, v, z) be a triangle such that $SP(s, u)$ and $SP(s, v)$ have already been computed. Then $SP(s, z)$ can be computed by drawing tangent from z to $SP(s, u)$ or $SP(s, v)$.



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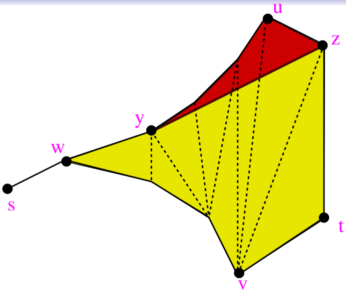
- **Open Problem**: Can $SP(s, t)$ be computed in a simple polygon in $O(n)$ time without triangulation?



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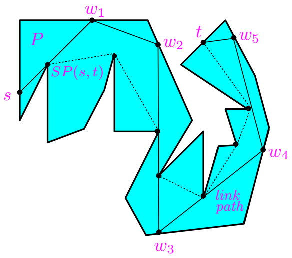
- **Open Problem**: Can $SP(s, t)$ be computed in a simple polygon in $O(n)$ time without triangulation?

- D. Lee and F. Preparata, *Euclidean shortest paths in the presence of rectilinear boundaries*, Networks, 14 (1984), 303-410. Running time: $O(n)$.

Outline

Definitions and properties



- A minimum link path connecting two points s and t inside a polygon P with or without holes (denoted by $MLP(s,t)$) is a polygonal path with the smallest number of turns or links.

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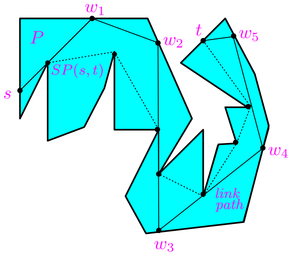

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- S. Suri, *A linear time algorithm for minimum link paths inside a simple polygon*, Computer Graphics, Vision, and Image Processing, 35 (1986), 99-110. Running time: $O(n)$.

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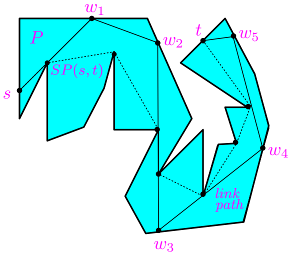

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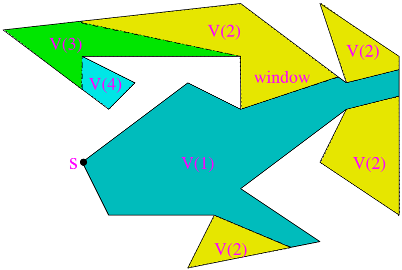
- S. K. Ghosh, *Computing the visibility polygon from a convex set and related problems*, Journal of Algorithms, 12 (1991), 75-95. Running time: $O(n)$.

Computing $MLP(s, t)$: Suri's algorithm



- $V(1)$ is the visibility polygon of s.

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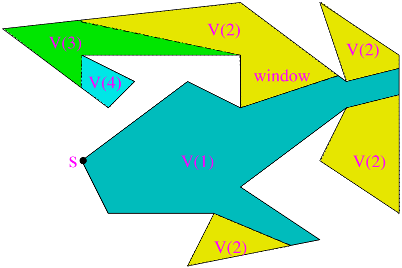
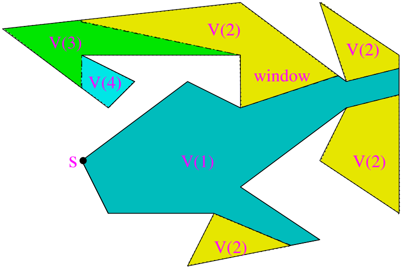


- $V(1)$ is the visibility polygon of s.
- For $i > 1$, $V(i)$ is the set of points of P that are visible from some point on a window of $V(i-1)$.

Computing $MLP(s, t)$: Suri's algorithm



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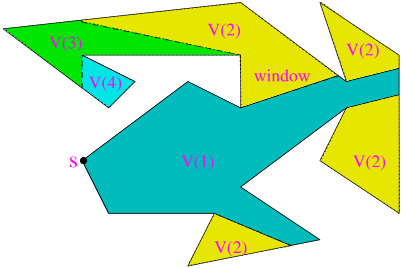
- So, number of links (called *link distance*) required from s to any point of $V(i)$ is i.

Computing $MLP(s,t)$: Suri's algorithm



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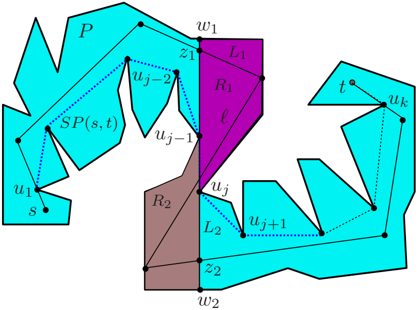
- So, number of links (called *link distance*) required from s to any point of $V(i)$ is i.
- The turing points of MLP(s,t) are on the windows of $V(i)$ for all i.

Computing $MLP(s,t)$: Ghosh's algorithm

- Ghosh's algorithm transforms $SP(s,t)$ into $MLP(s,t)$ in $O(n)$ time.



- Let $SP(s,t) = (s, u_1, ..., u_k, t)$.

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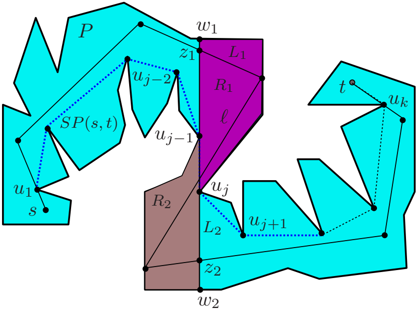
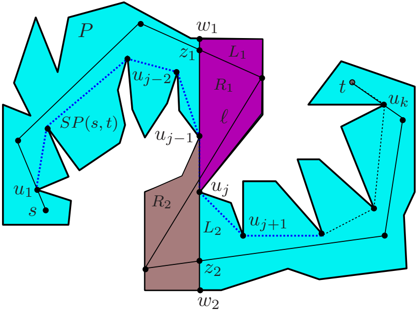


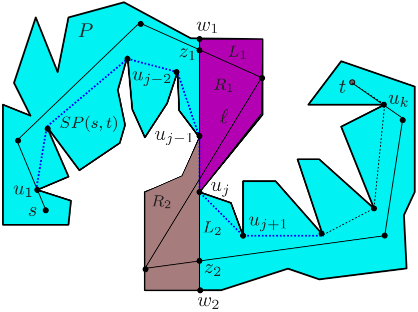
- Let $SP(s,t) = (s, u_1, ..., u_k, t)$.
- An edge $u_j u_{j-1}$ of $SP(s,t)$ is called *eave* if u_{j-2} and u_{j+1} lie on the opposite sides of the line passing through u_j and u_{j-1}.

Ghosh's algorithm



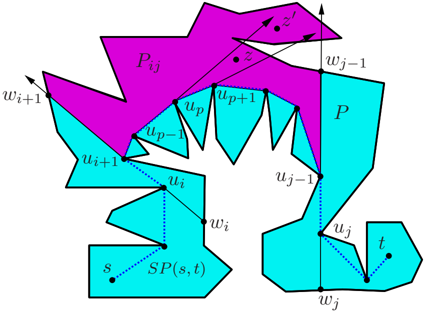
- If an edge $u_k u_{k+1}$ of $SP(s,t)$ is a sub-segment of a link in a link path, we say that the link path contains $u_k u_{k+1}$.

Ghosh's algorithm



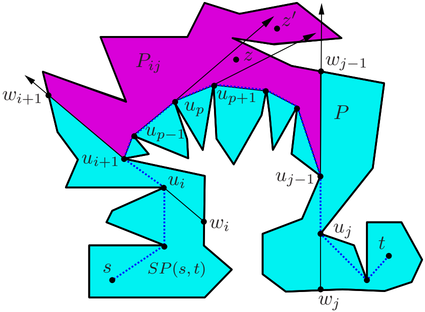
- If an edge $u_k u_{k+1}$ of $SP(s,t)$ is a sub-segment of a link in a link path, we say that the link path contains $u_k u_{k+1}$.
- There exists a minimum link path between s and t that contains all eaves of $SP(s,t)$.

Ghosh's algorithm



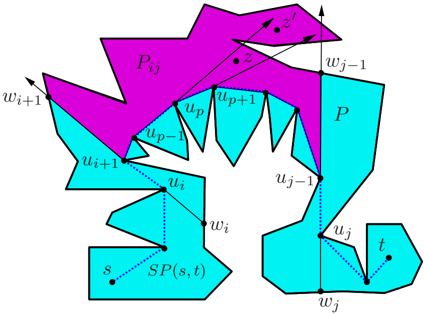
- Decompose P into sub-polygons by extending each eave from both ends to the boundary of P.

Ghosh's algorithm



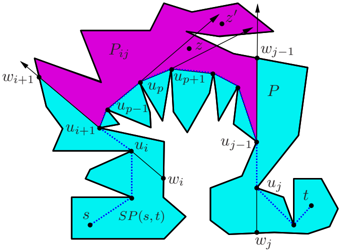
- Decompose P into sub-polygons by extending each eave from both ends to the boundary of P.
- If two consecutive extensions intersect at a point z, then z is a turning point of $MLP(s,t)$.

Ghosh's algorithm



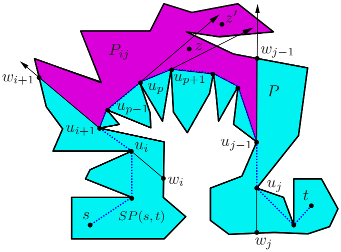
- Decompose P into sub-polygons by extending each eave from both ends to the boundary of P.
- If two consecutive extensions intersect at a point z, then z is a turning point of $MLP(s,t)$.
- Construct minimum link paths connecting the extensions of every pair of consecutive eaves on $SP(s,t)$ to form $MLP(s,t)$.

Ghosh's algorithm



- Consider one such sub-polygon (say, P_{ij}) between the non-intersecting extensions of two consecutive eaves $u_i u_{i+1}$ and $u_{j-1} u_j$ of $SP(s,t)$.

Ghosh's algorithm



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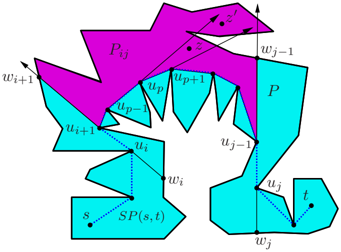

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- A link path is called *convex* if it makes only left or only right turns at every turning point in the path.

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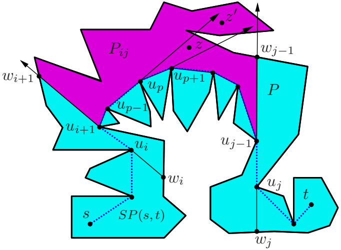

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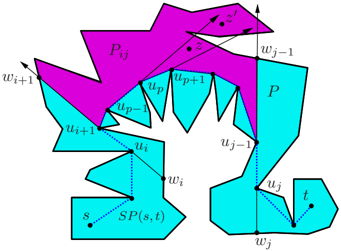
- A minimum link path L_{ij} is a convex path inside P_{ij}.

Ghosh's algorithm



- The segment zu_p is called the *left tangent* (or *right tangent*) of z at the vertex $u_p \in SP(u_{i+1}, u_{j-1})$ if zu_p lies inside P_{ij} and z lies to the right of $\overrightarrow{u_{p-1}u_p}$ (respectively, $\overrightarrow{u_p u_{p-1}}$) and to the left of $\overrightarrow{u_p u_{p+1}}$ (respectively, $\overrightarrow{u_{p+1}u_p}$).

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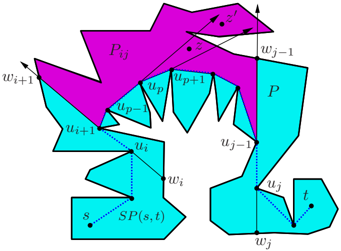

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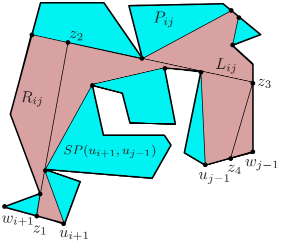
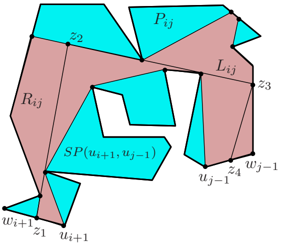
- If a point $z \in P_{ij}$ is on a convex path between $u_{i+1}w_{i+1}$ and $u_{j-1}w_{j-1}$ inside P_{ij}, then z has both left and right tangents.

Ghosh's algorithm



- Let R_{ij} denote the set of all points of P_{ij} such that every point of R_{ij} has both left and right tangents to $SP(u_i, u_j)$.

Ghosh's algorithm



- Let R_{ij} denote the set of all points of P_{ij} such that every point of R_{ij} has both left and right tangents to $SP(u_i, u_j)$.
- R_{ij} is called *complete visibile region* of P_{ij} and it can be computed in $O(n)$ time.

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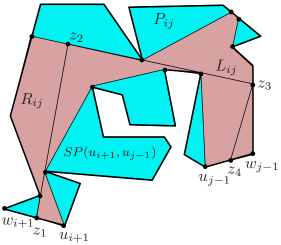

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- Choose an appropriate point z_1 on $u_{i+1}w_{i+1}$.

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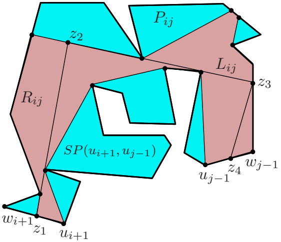

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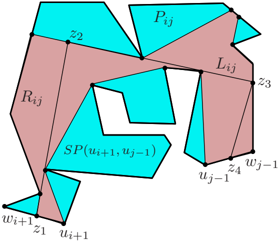
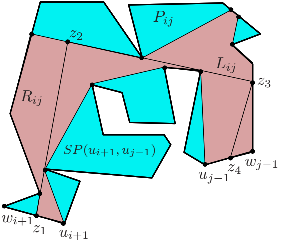
- R_{ij} is called *complete visibile region* of P_{ij} and it can be computed in $O(n)$ time.
- Choose an appropriate point z_1 on $u_{i+1}w_{i+1}$.
- Draw the right tangent from z_1 to $SP(u_{i+1}, u_{j-1})$ and extend the tangent till it meets the boundary of R_{ij} at some point z_2.

Ghosh's algorithm



- Again, draw the right tangent from z_2 to $SP(u_{i+1}, u_{j-1})$ and extend the tangent till it meets the boundary of R_{ij} at some point z_3.

Ghosh's algorithm



- Again, draw the right tangent from z_2 to $SP(u_{i+1}, u_{j-1})$ and extend the tangent till it meets the boundary of R_{ij} at some point z_3.
- Repeat this process of construction till a point z_q is found on $u_{j-1}w_{j-1}$.

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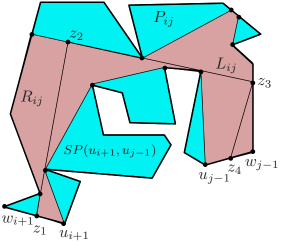

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- Repeat this process of construction till a point z_q is found on $u_{j-1}w_{j-1}$.
- Thus, the greedy path z_1z_2, z_2z_3,...,$z_{q-1}z_q$ is constructed between $u_{i+1}w_{i+1}$ and $u_{j-1}w_{j-1}$.

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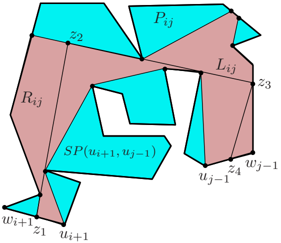

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- Repeat this process of construction till a point z_q is found on $u_{j-1}w_{j-1}$.
- Thus, the greedy path z_1z_2, z_2z_3,...,$z_{q-1}z_q$ is constructed between $u_{i+1}w_{i+1}$ and $u_{j-1}w_{j-1}$.
- The greedy path $z_1z_2, z_2z_3, \ldots, z_{q-1}z_q$ is a minimum link path inside P_{ij}.

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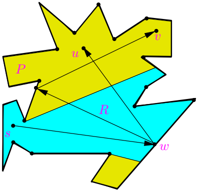
Outline

Indirect visibility



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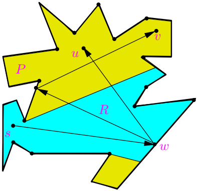

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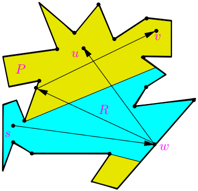

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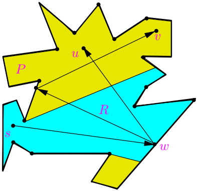

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- There is another type of reflection of light called *diffuse* reflection, where a light ray incident at a point is reflected in all possible interior directions.

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- Status: There is no polynomial time algorithm known for the above problem. On the other hand, the problem is also not known to be NP-hard.

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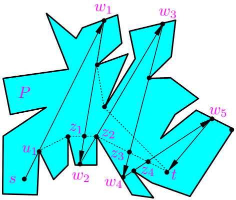
Greedy method



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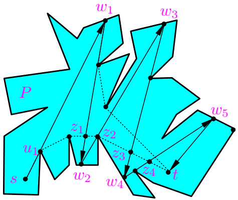

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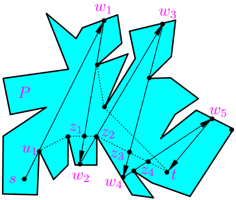

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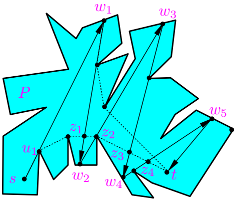

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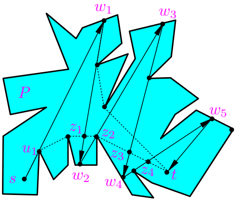

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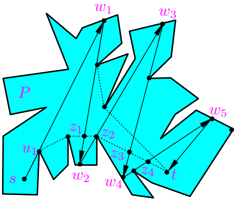

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- The greedy path can be computed in $O(n^2)$ time.

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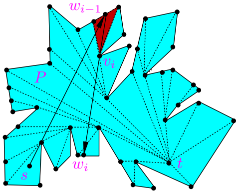
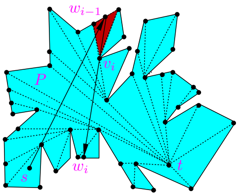


- Instead of computing shortest paths repeatedly, the algorithm computes the shortest path tree rooted at t, and then constructs the shortest path map by extending the edges of the tree.

Greedy method



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- Observe that the next vertex (say, v_i) of w_{i-1} in the shortest path from w_{i-1} to t is the vertex of the triangle in the shortest path map which contains w_{i-1}.

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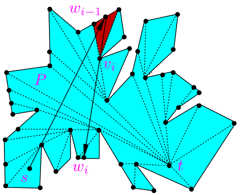

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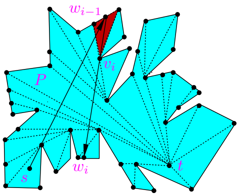

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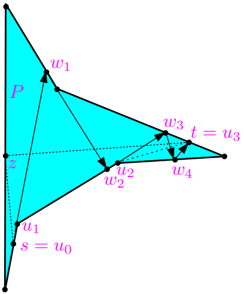
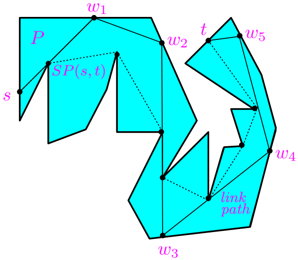
- J. Hershberger, *Finding the visibility graph of a polygon in time proportional to its size*, Algorithmica 4:141-155, 1989.

Worst case ratio



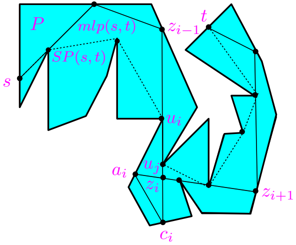
- The number of links in the greedy path can be at most $(n-1)/2$ times that of an optimal diffuse reflection path.

Transforming a $MLP(s, t)$



- If all turning points of a $MLP(s, t)$ lie on edges of P, then the path is an optimal diffuse reflection path.

Transforming a $MLP(s, t)$



- For every turning point z_i not lying on any edge of P, extend $z_i z_{i+1}$ from z_i to the boundary of P meeting it at a point a_i.

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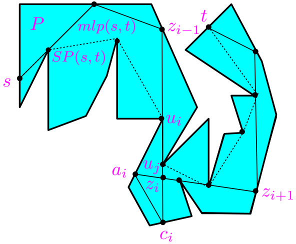

- For every turning point z_i not lying on any edge of P, extend $z_i z_{i+1}$ from z_i to the boundary of P meeting it at a point a_i.
- Similarly, extend $z_i z_{i-1}$ from z_i to the boundary of P meeting it at a point c_i.

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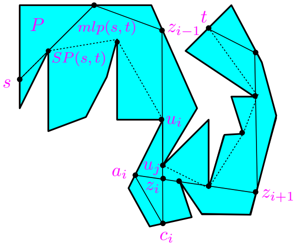

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- Similarly, extend $z_i z_{i-1}$ from z_i to the boundary of P meeting it at a point c_i.
- If the segment $a_i c_i$ lies inside P, then the diffuse reflection path is $(sz_1, z_1 z_2, \ldots, z_{i-1} c_i, c_i a_i, a_i z_{i+1}, \ldots, z_{m-1} z_m, z_m t)$.

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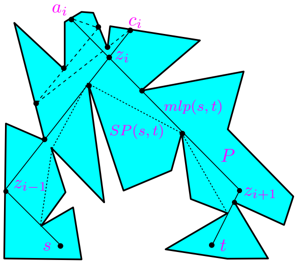
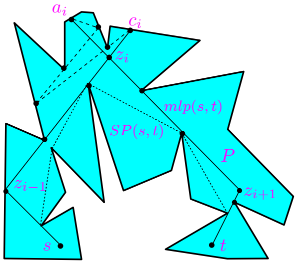


- Otherwise, a_i and c_i are connected by a greedy path to construct a diffuse reflection path.

Transforming a $MLP(s,t)$



- Otherwise, a_i and c_i are connected by a greedy path to construct a diffuse reflection path.
- The greedy paths connecting every pair of a_i and c_i lie in disjoint regions of P.

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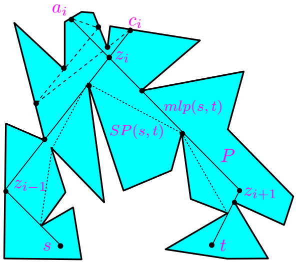
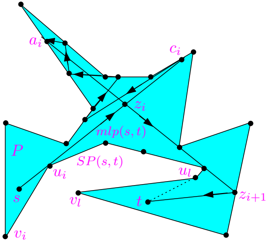


- Otherwise, a_i and c_i are connected by a greedy path to construct a diffuse reflection path.
- The greedy paths connecting every pair of a_i and c_i lie in disjoint regions of P.
- Therefore, a $MLP(s, t)$ can be transformed into a diffuse reflection path from s to t in $O(n + k \log n)$ time.

Worst case ratio



- Let m' be the number of turning points of the $MLP(s,t)$ not lying on the boundary of P.

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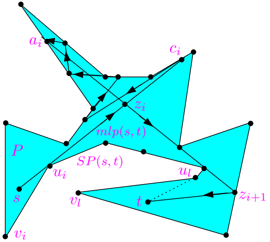

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- So, the number of links in the greedy path can be at most $n - 2m' + 2m' - m - 1$ because

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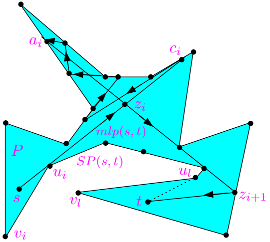

- Let m' be the number of turning points of the $MLP(s,t)$ not lying on the boundary of P.
- So, the number of links in the greedy path can be at most $n - 2m' + 2m' - m - 1$ because
  - the greedy link path from c_i to a_i does not pass through one vertex of the edge containing c_i and another vertex of the edge containing a_i,

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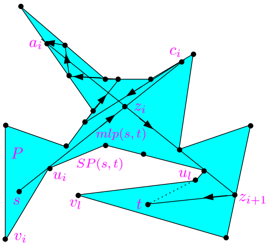

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  - the last two links for each of the m' greedy paths do not pass through vertices of P, and

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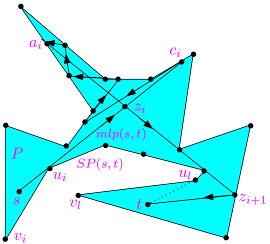
- 

  - the last two links for each of the m' greedy paths do not pass through vertices of P, and
  - the number of vertices in the clockwise boundary of P from v_i to v_l (including v_i and v_l) must be at least $2 + m - 1$ as $m - 1$ links of the minimum link path pass through distinct vertices the shortest path.

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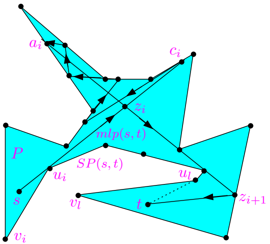

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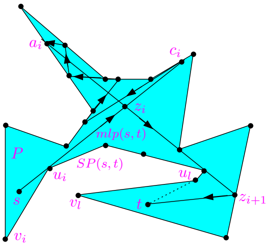
- 

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- Therefore, the diffuse reflection path has at most $n - m - 1 + m = n - 1$ links.
- So, the number of links in the transformed path is at most $(n-1)/m$ times that of an optimal diffuse reflection path.

Combinatorial approach

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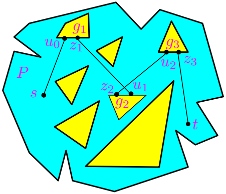
- J. ORourke and I. Streinu, *The vertex edge visibility graph of a polygon*, Computational Geometry, 10: 105-120, 1998.

Combinatorial approach



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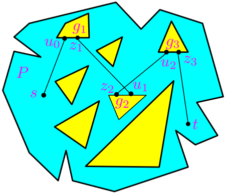

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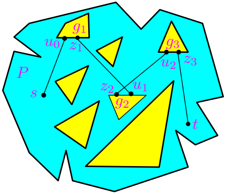

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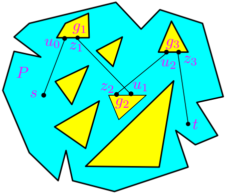

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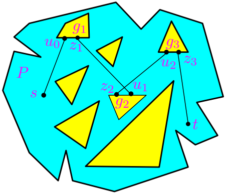

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- So, a sequence of links $su_0, z_1u_1, \ldots, z_{k-2}u_{k_2}, z_{k-1}t$ can be constructed.

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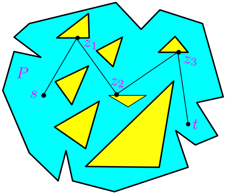
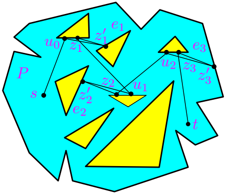


- If $z_i = u_{i-1}$, for all i, then we have a diffuse reflection path $sz_1, z_1z_2, \ldots, z_{k-1}t$ with the minimum number of reflections.

Combinatorial approach



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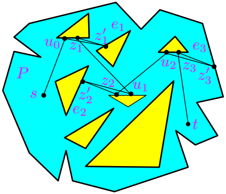

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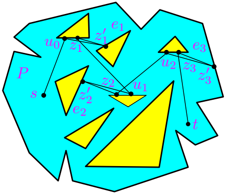

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- Hence, $(su_0, u_0z_1', z_1'z_1, z_1u_1, \ldots, u_{k-2}z_{k-1}', z_{k-1}'z_{k-1}, z_{k-1}t)$ becomes a diffuse reflection path between s and t.

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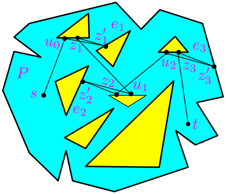

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- This path can be at most three times that of an optimal diffusion reflection path.

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- All pairs of weakly visible edges of P can be located in $O(n \log n + E)$ time (i) by computing the visibility graph of vertices of P and (ii) by traversing the visibility graph using *funnel sequences* with polygons edges as bases of the funnels.

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- S. K. Ghosh and D. M. Mount, *An output-sensitive algorithm for computing visibility graphs*, SIAM Journal on Computing, vol. 20, pp. 888-910, 1991.

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- It will be interesting to design an algorithm, possibly by transforming a $MLP(s, t)$), giving a simple and structured path as well as giving a good bound.
- Finally, it is open whether an optimal path can be computed for this problem in a low-order polynomial time.

Concluding remarks

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