Introduction to Computational Geometry

Sasanka Roy
Chennai Mathematical Institute

Organization of the Talk

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1. Preliminaries, Generic definition and Literature

2. Some technical details of easy versions

3. Conclusion

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What are we going to talk about?

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We have some data

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Geometric Data

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What do I mean ????

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I mean: we have

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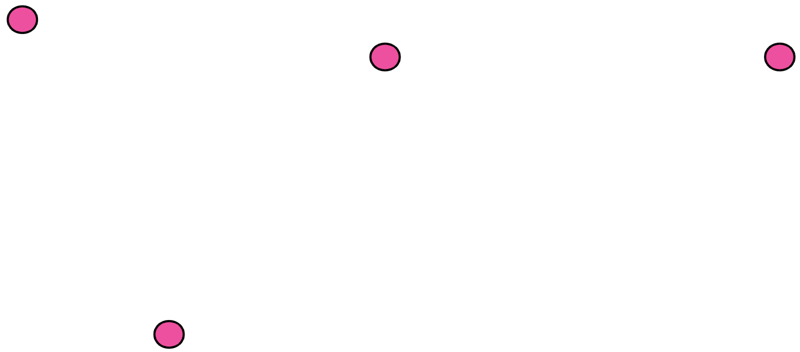
I mean: we have points,



What are we going to talk about?

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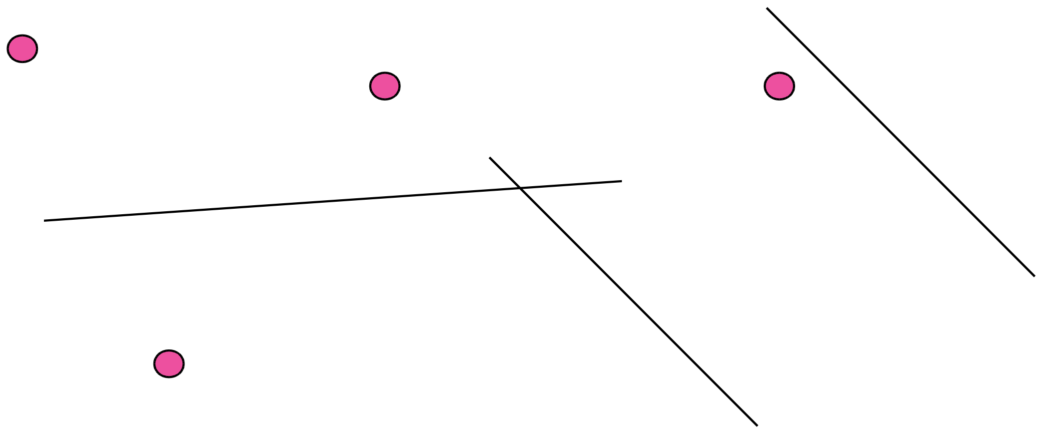
Geometric Data

Geometric Data ????

What do I mean ????

I mean: we have points, line segments,



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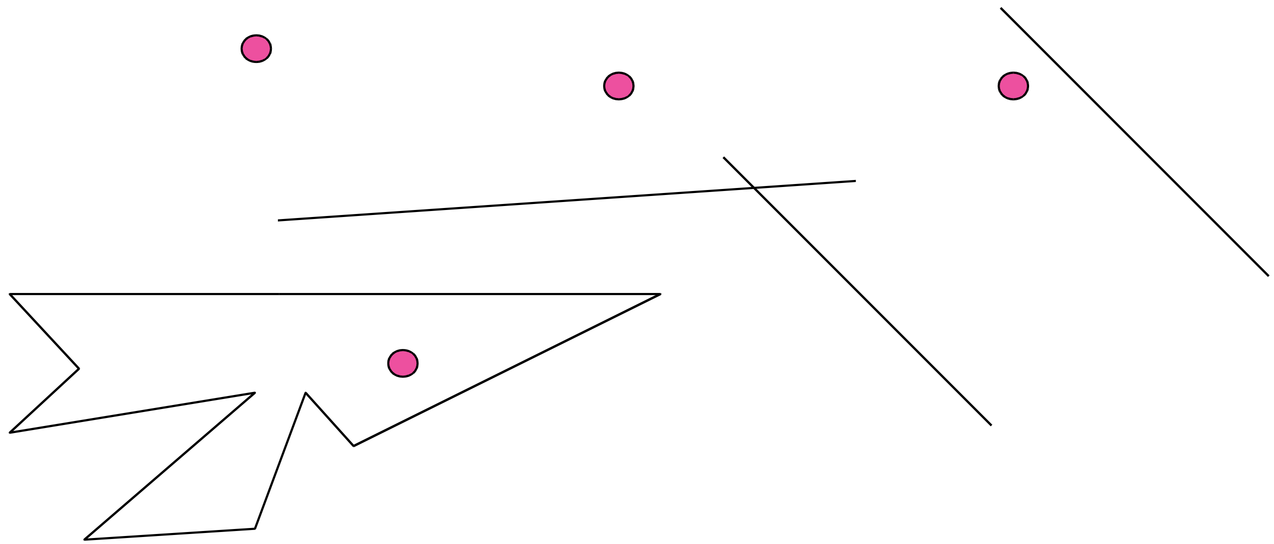

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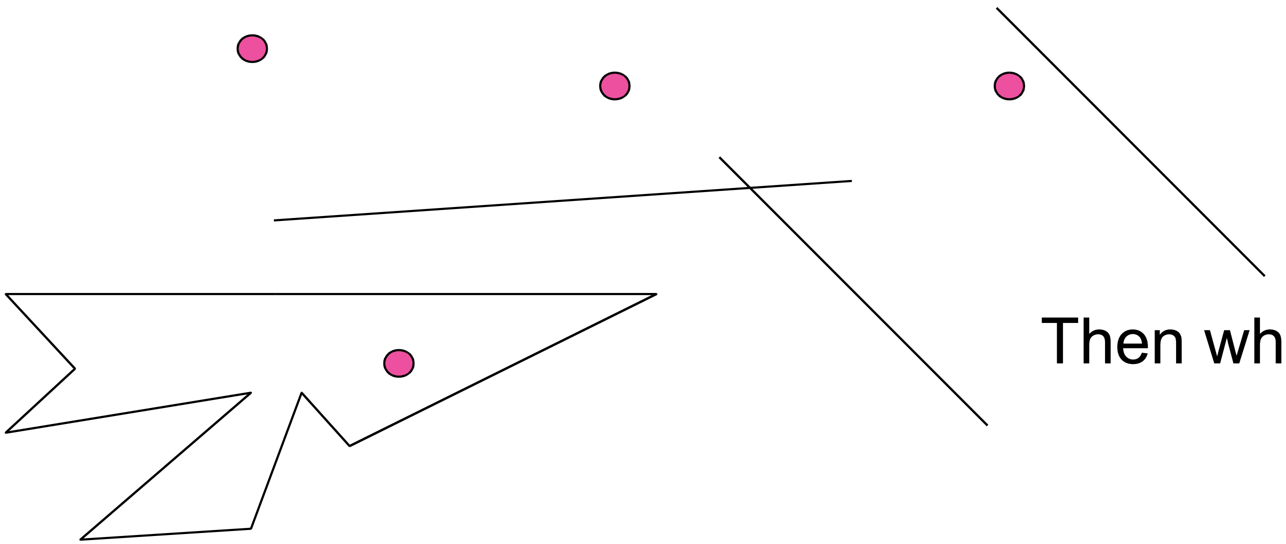

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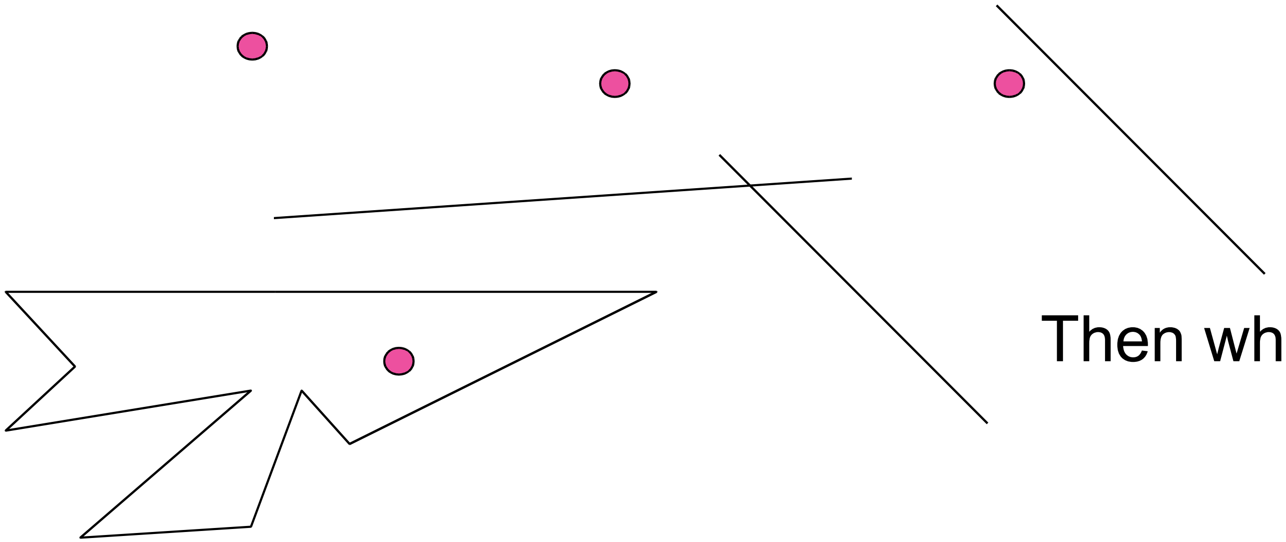

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We want to get answers to the specific questions

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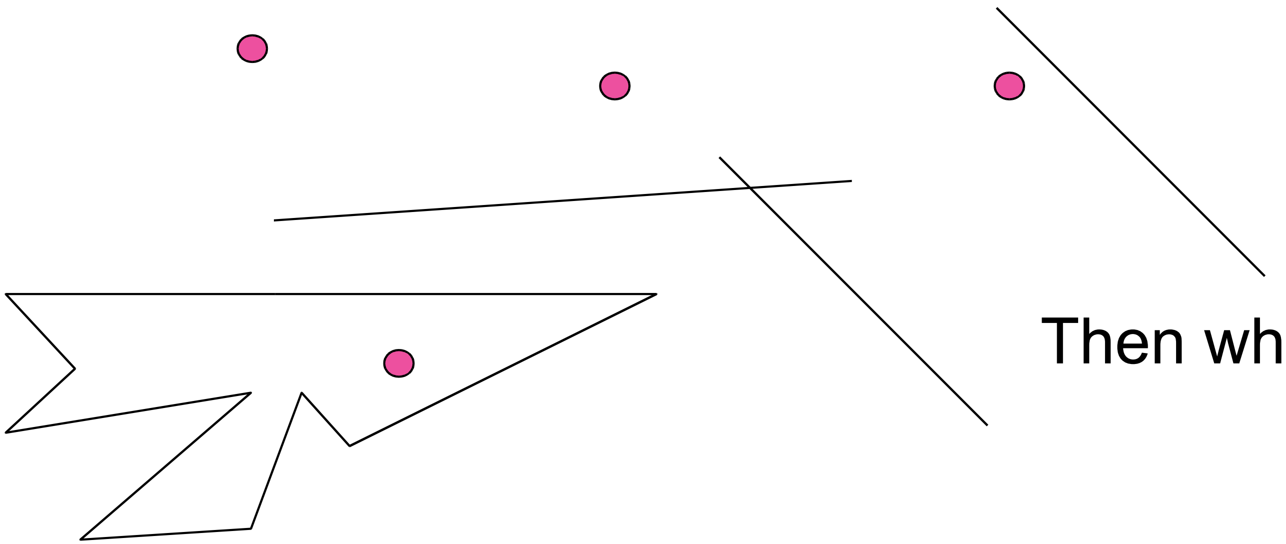

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Closest points to the line segments

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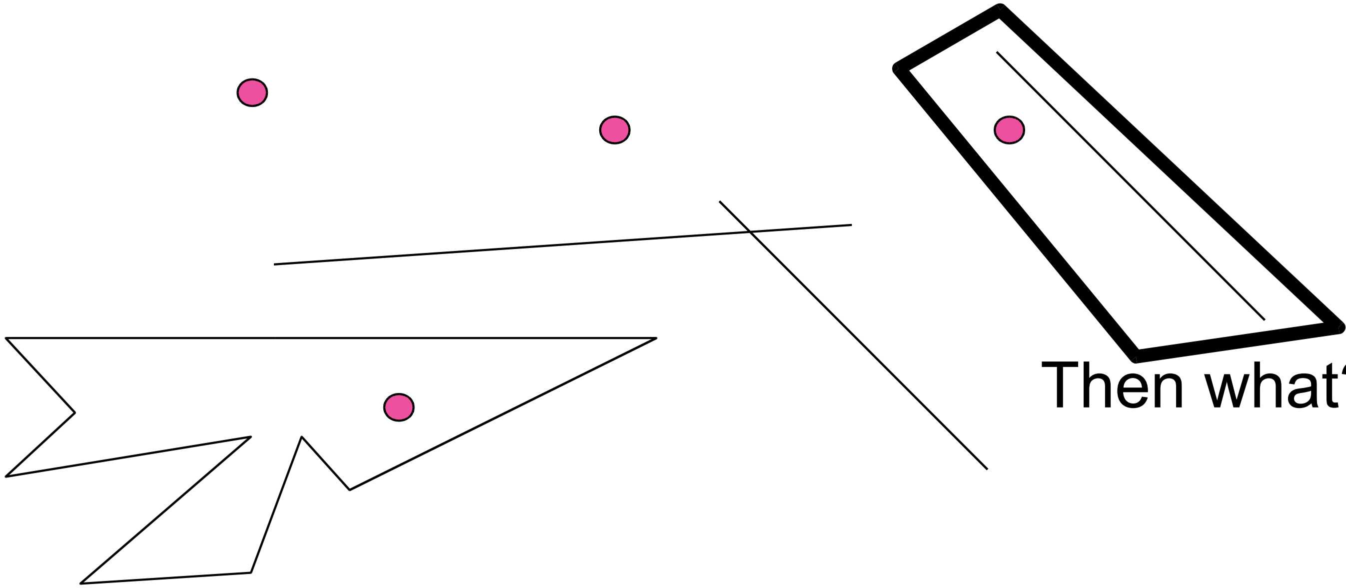

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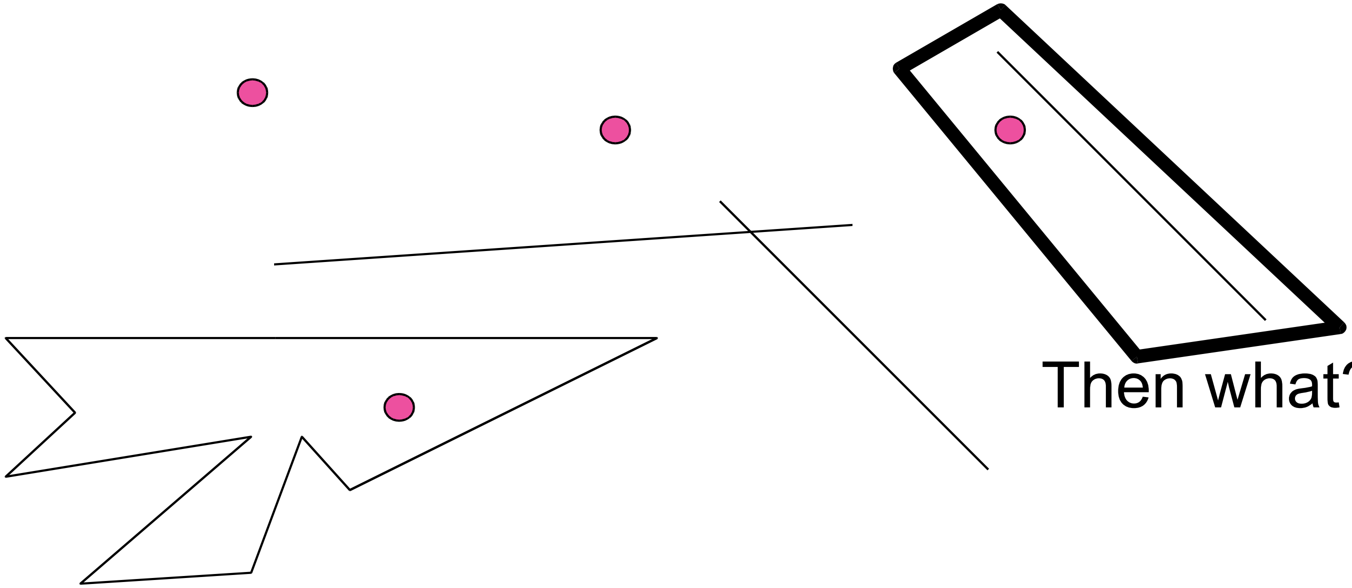

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Point inside the simple polygon

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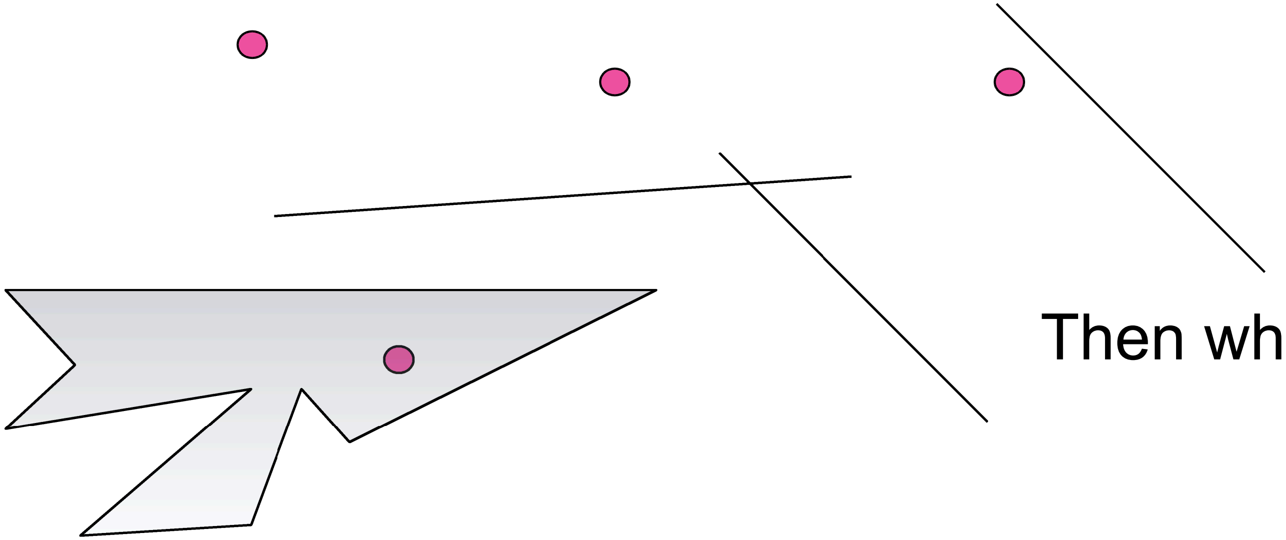

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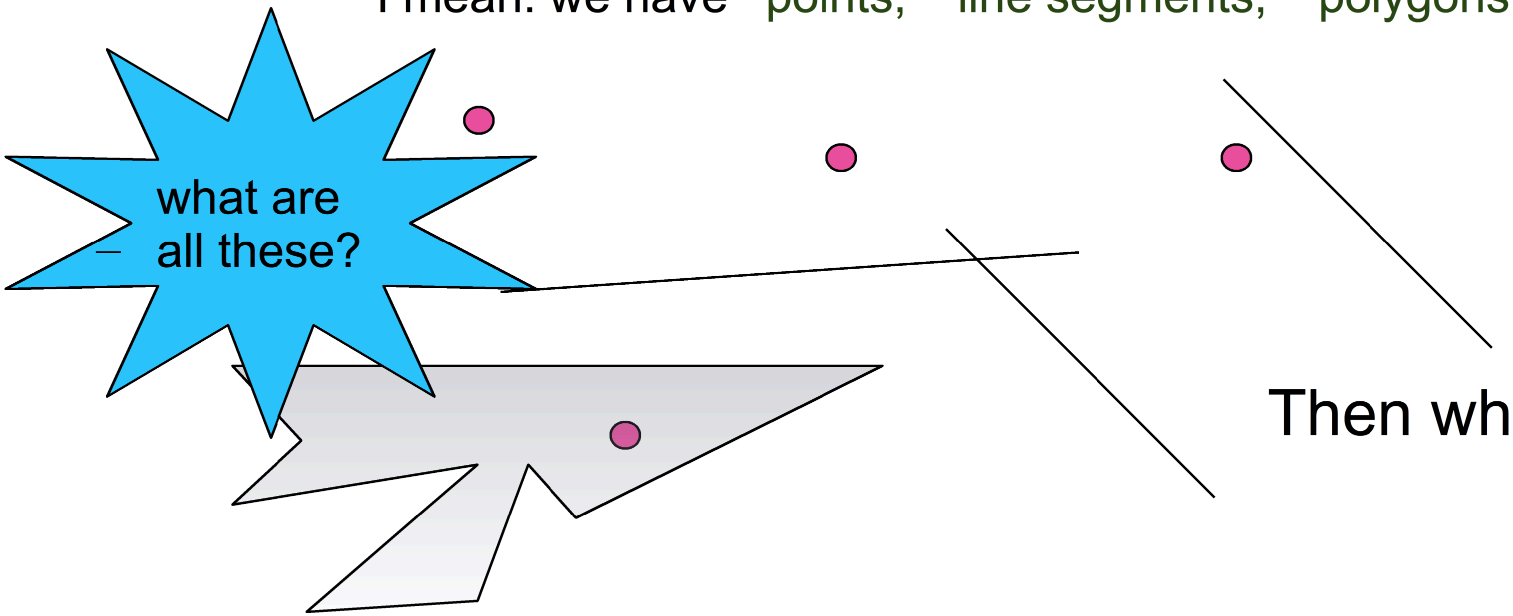

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Closest points to the line segments

Point inside the simple polygon

Can you be a bit Practical??

Planar Point Location

Planar Point Location

Which state has the site/point with

Latitude=   28° 38' N

Longitude= 72° 12' E

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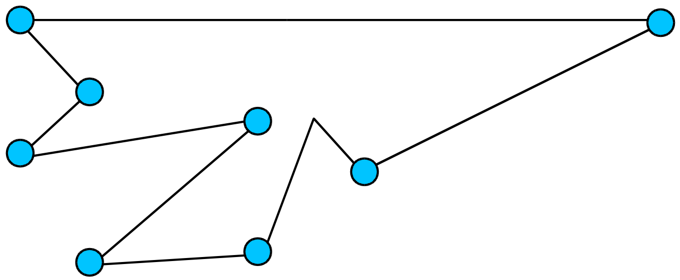
Can we view States as simple polygon?



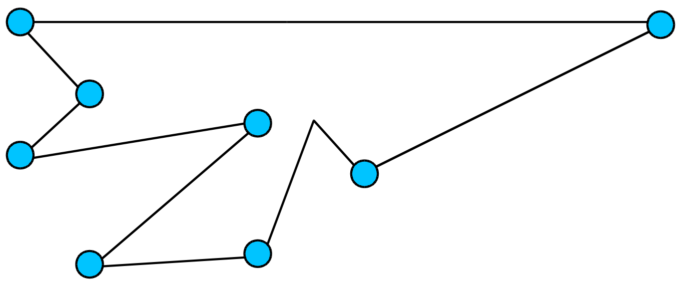
Planar Point Location

Which state has the site/point with

Latitude= 13° 08' 10'' N

Longitude= 80° 27' 40'' E

Can we view States as simple polygon?



simple polygon: Closed region whose boundary is formed by non-intersecting line segments

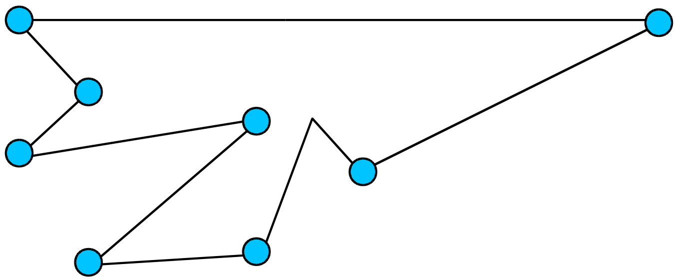
Planar Point Location

Which state has the site/point with

Latitude= 13° 08' 10'' N

Longitude= 80° 27' 40'' E

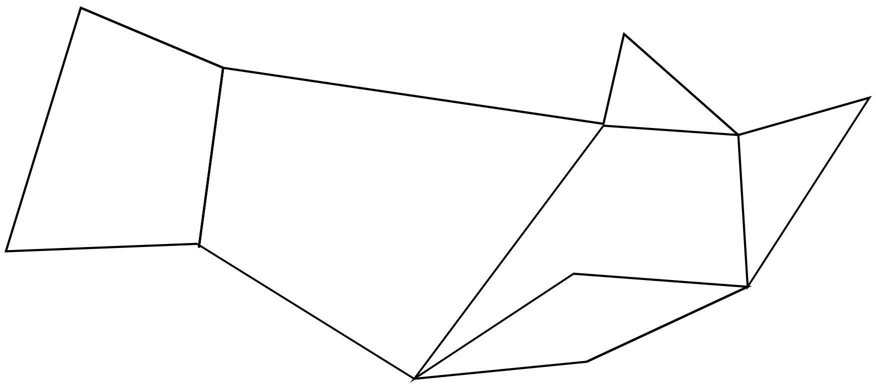
Can we view States as simple polygon? Yes



simple polygon: Closed region whose boundary is formed by non-intersecting line segments

Formally Planar Point Location

Given a planar subdivision S of O(n) vertices/faces/edges



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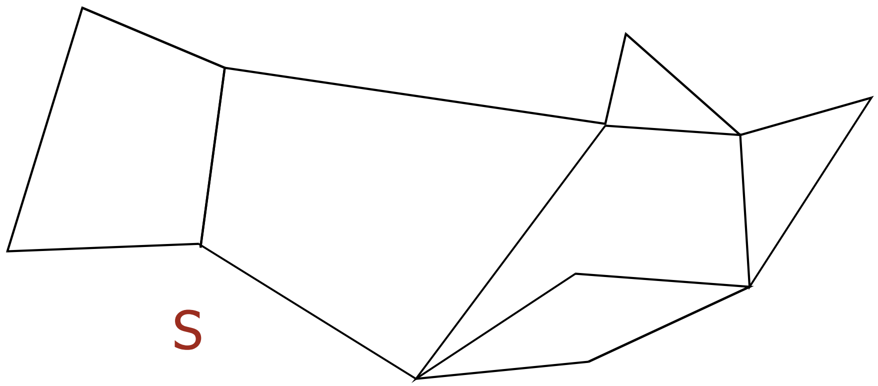

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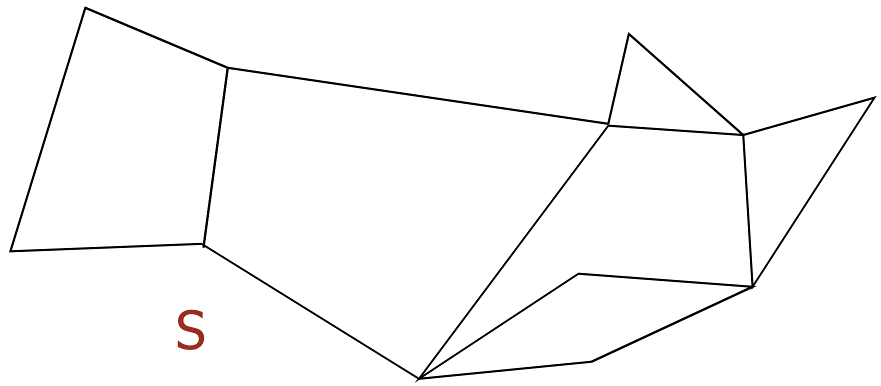

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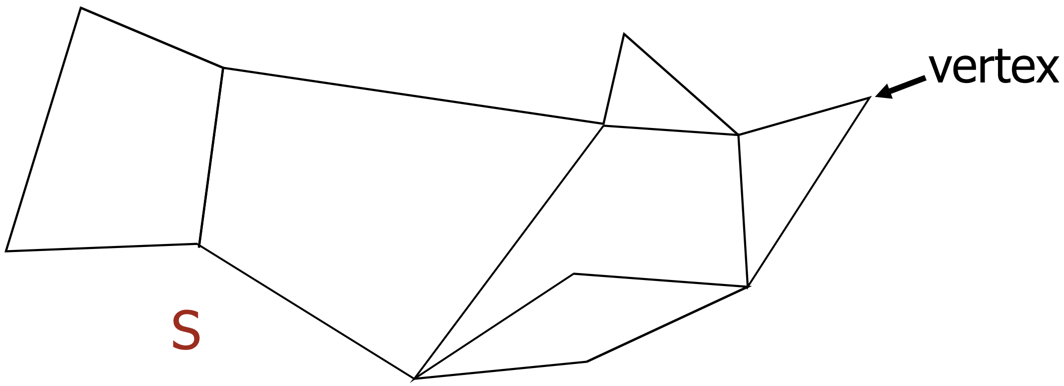

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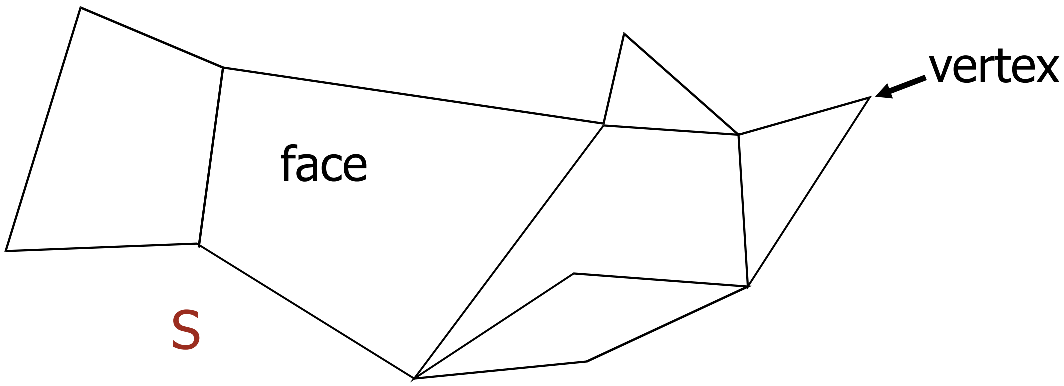

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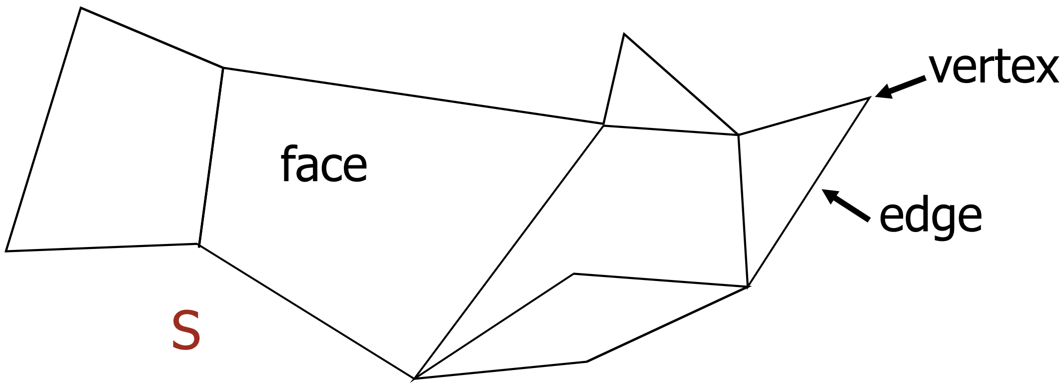

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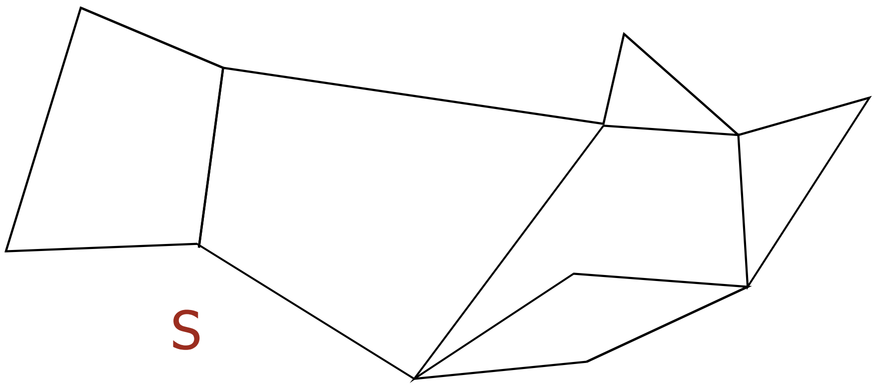
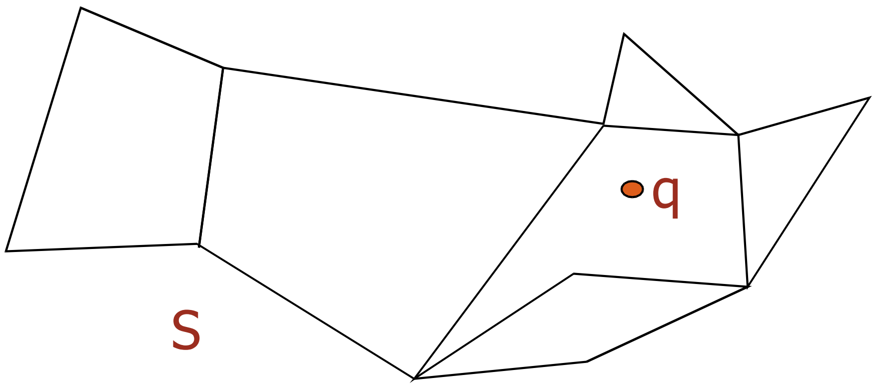


Preprocess S such that:

Formally Planar Point Location

Given a planar subdivision S

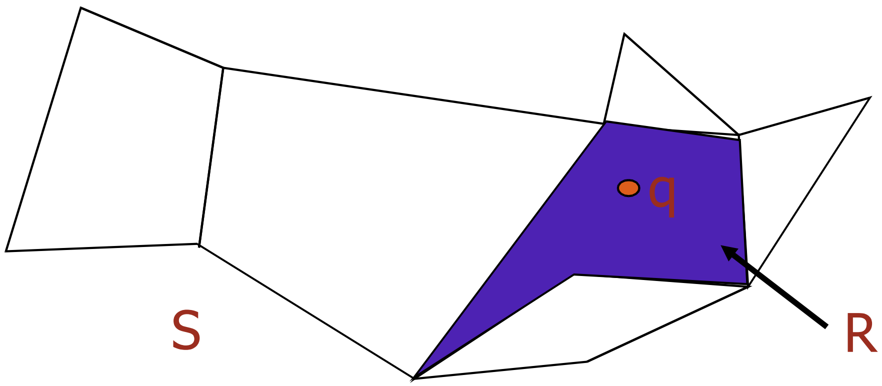


Preprocess S such that:

For any query point q,

Formally Planar Point Location

Given a planar subdivision S

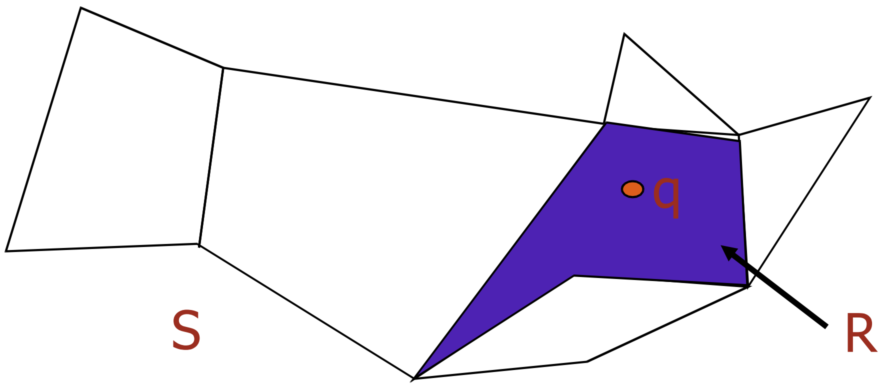


Preprocess S such that:

For any query point q,

The region/face R containing q can be reported efficiently.

Questions?



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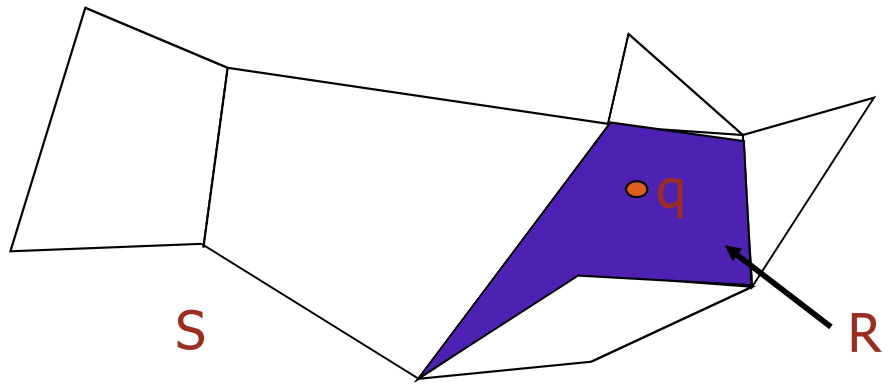
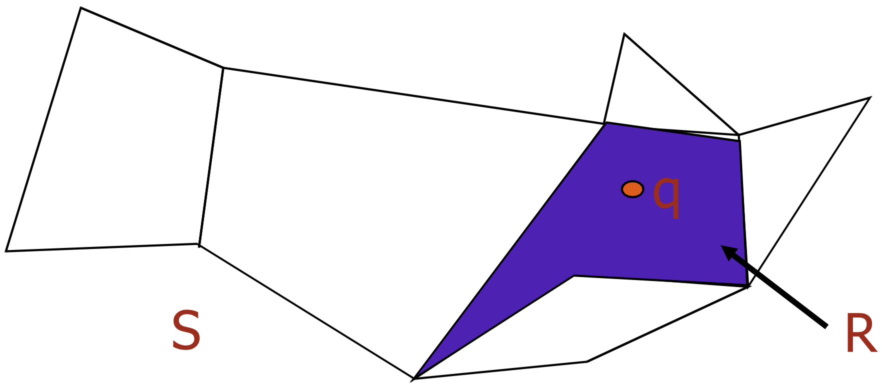


1. How much is the preprocessing Time?

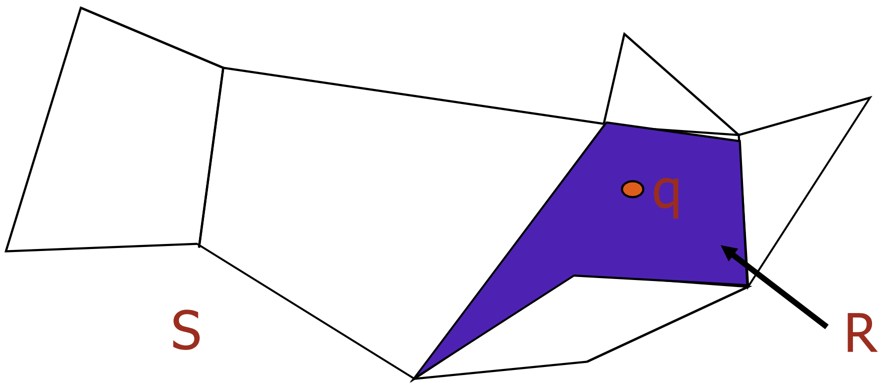
Questions?



1. How much is the preprocessing Time?

2. How much space is required?

Questions?



1. How much is the preprocessing Time?
2. How much space is required?
3. How much is the query time?

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For convex polygon

P $\rightarrow$ The Geometric Objects is a Convex Polygon with n edges

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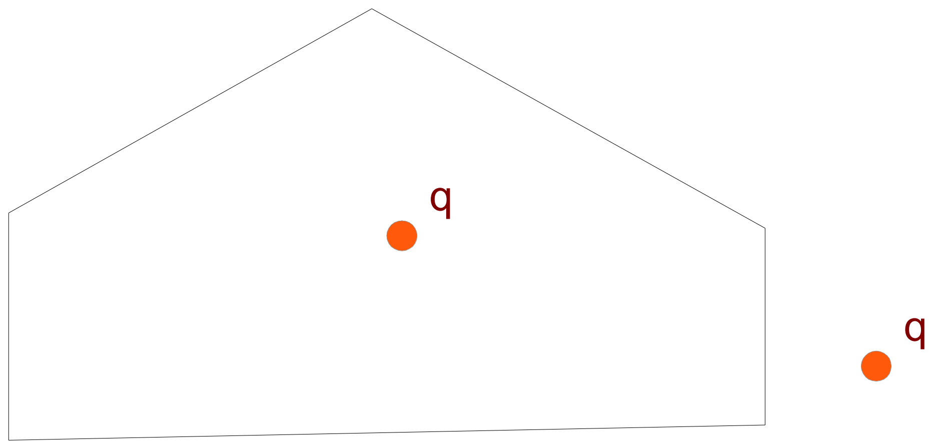

For convex polygon

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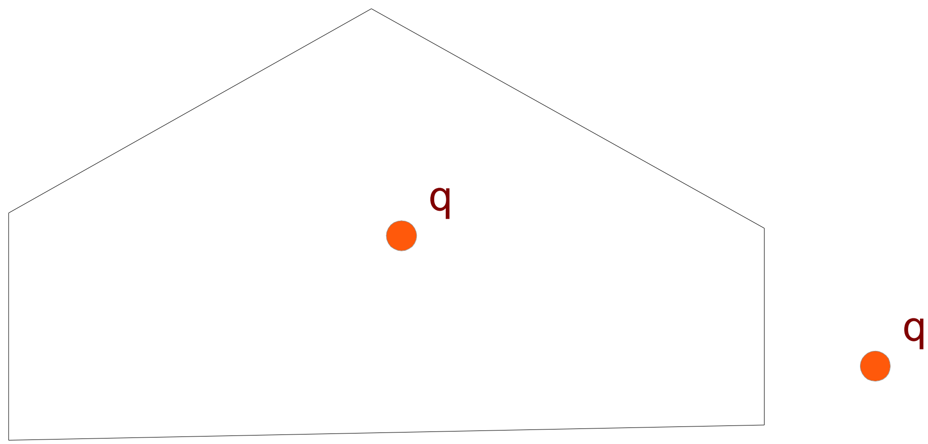
Whether q is inside or outside?



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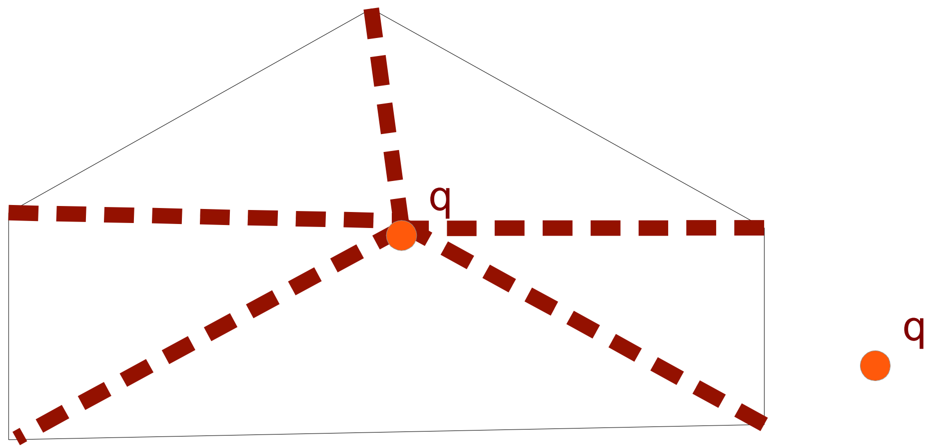
Whether q is inside or outside?

Clue!! from area



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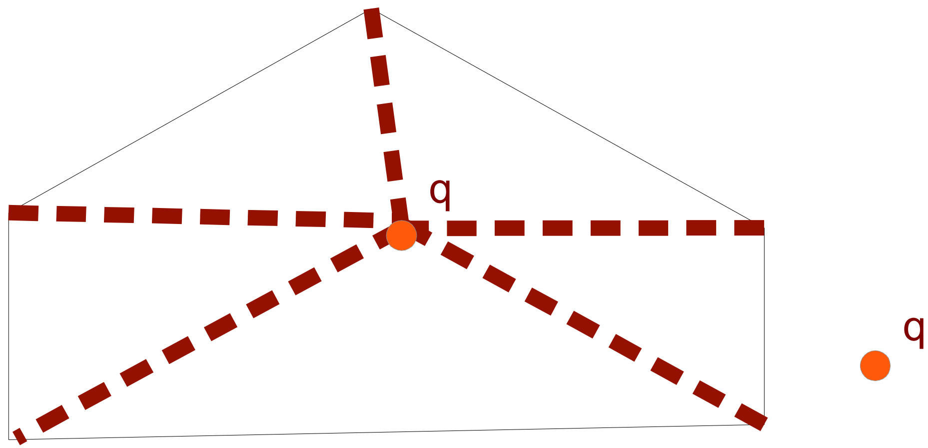

Complexity:

For convex polygon

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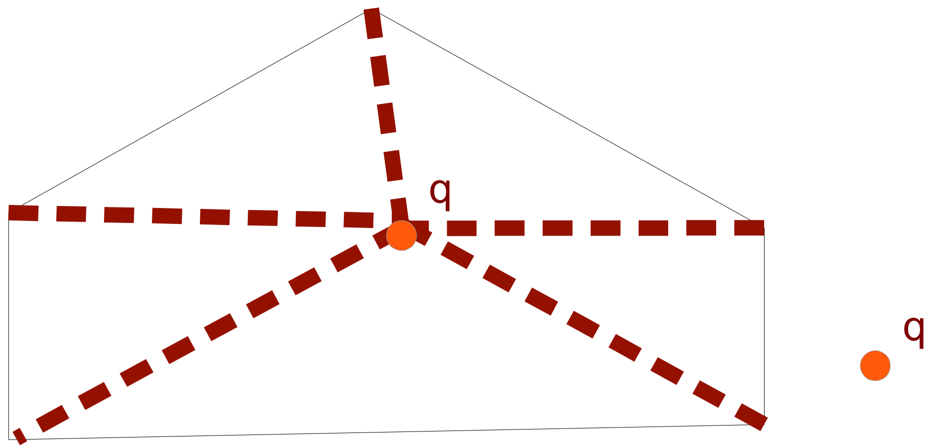

Complexity: O(n)

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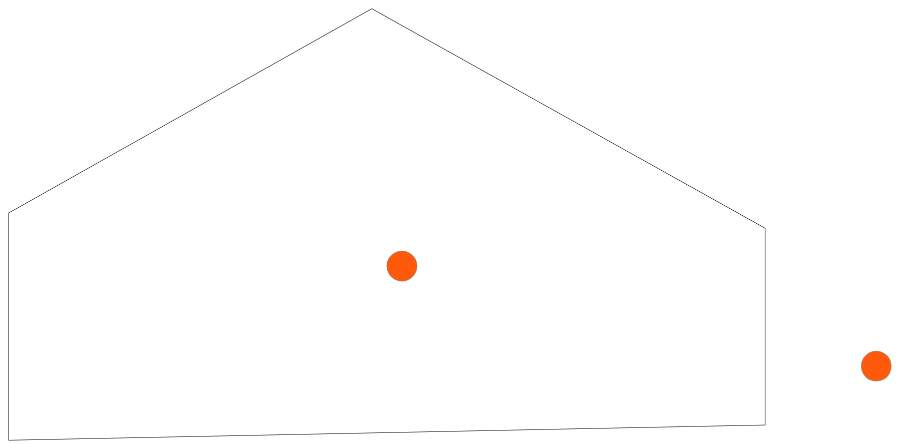
Now preprocess P to answer the question quickly



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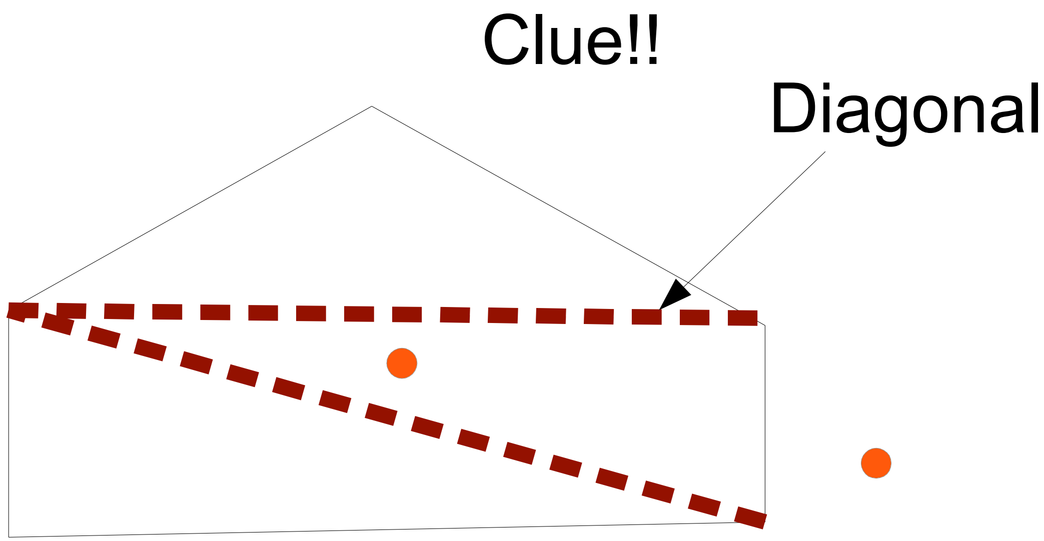

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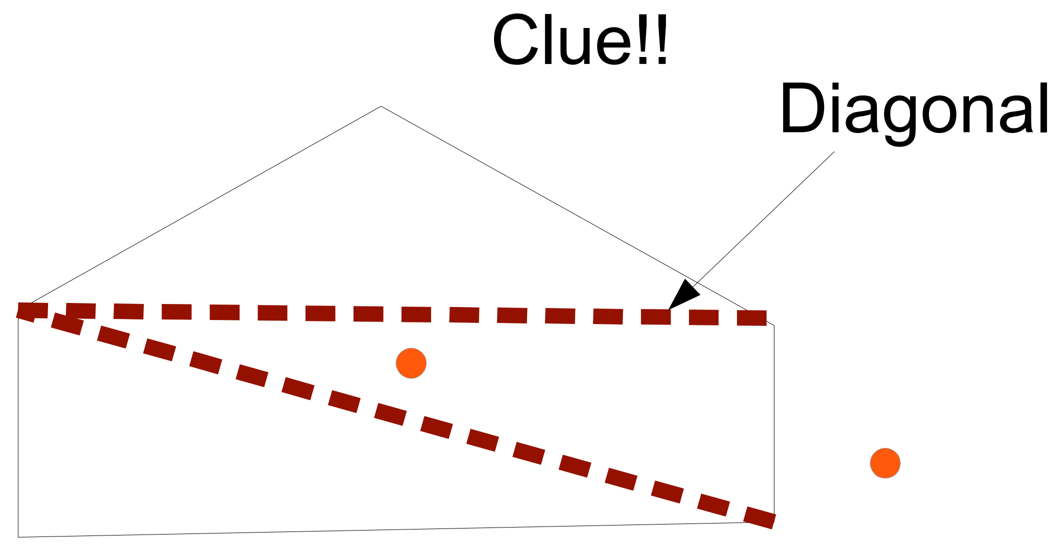

Triangulated Convex polygon

For convex polygon

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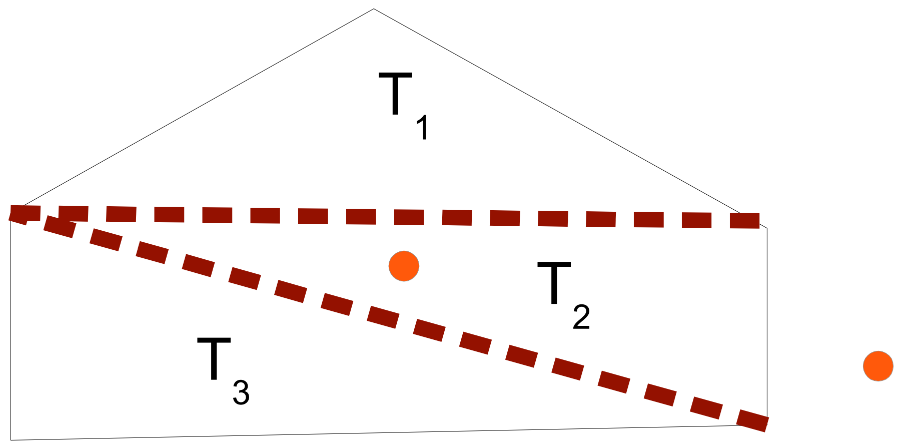
Clue!!



For convex polygon

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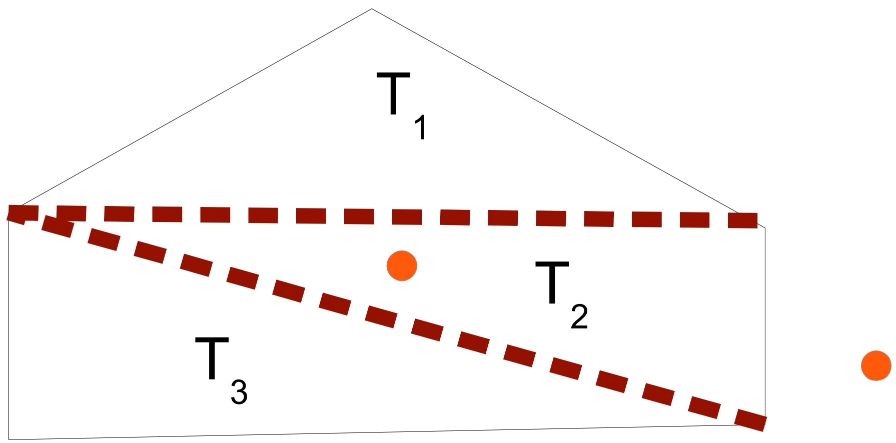

May we have an angular order of the triangles please!!

For convex polygon

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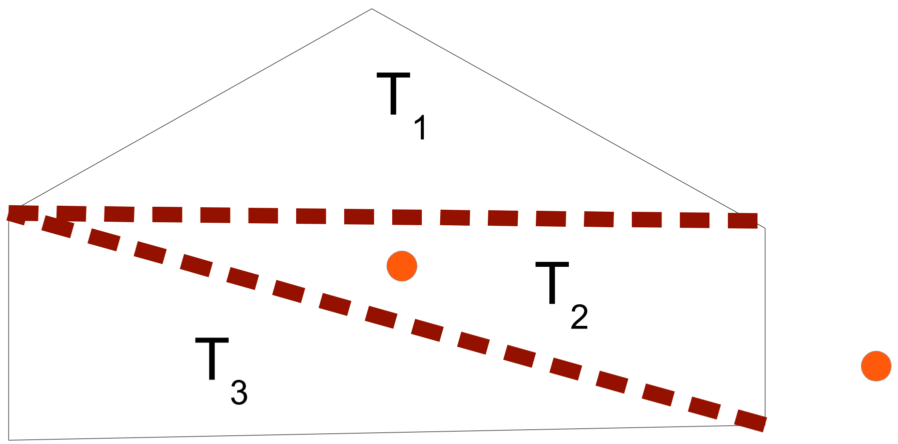
Clue!!



May we have an angular order of the triangles please!!

And then!!

Results

P → The Geometric Objects is a Convex Polygon with n edges

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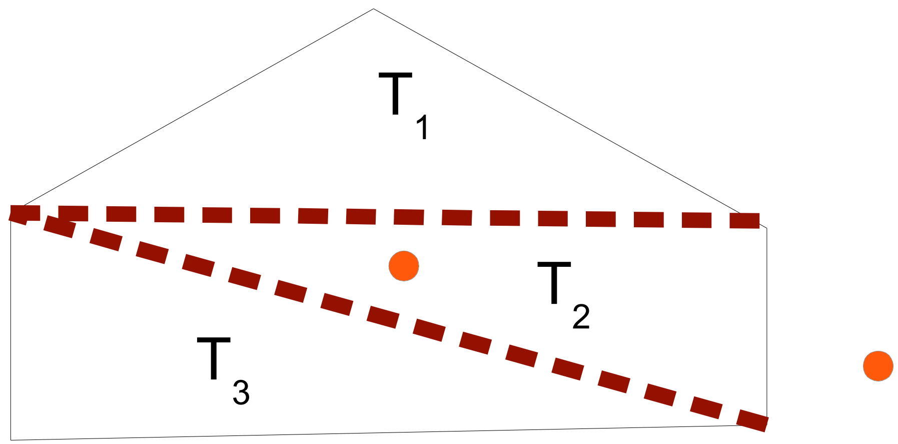

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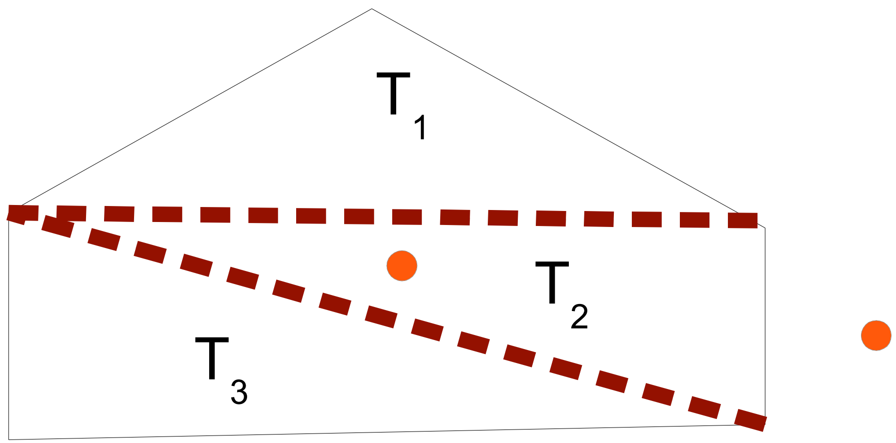

Preprocessing Time: O(n)

Preprocessing space requirement: O(n)

Query Time: O(log n)

For simple polygon

P $\rightarrow$ The Geometric Objects is a simple Polygon with n edges

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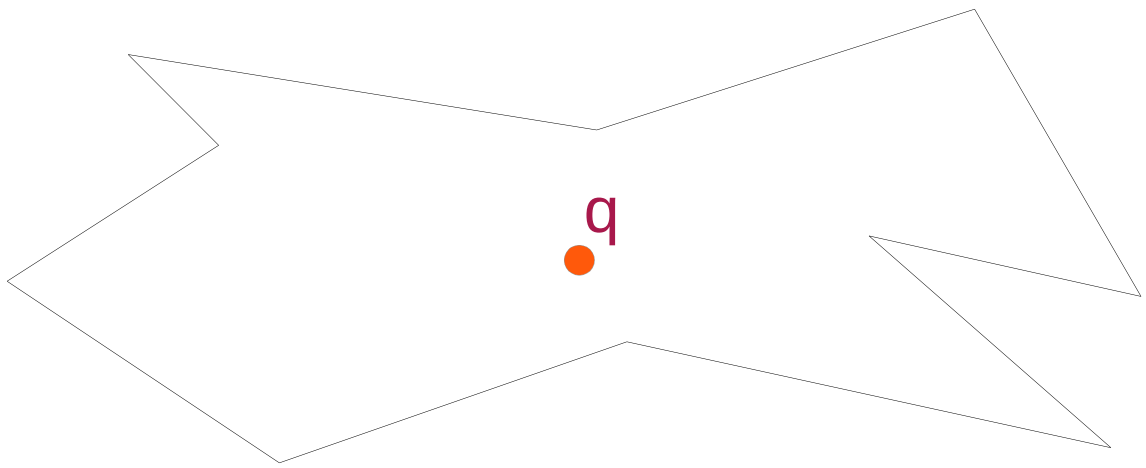
Whether q is inside or outside?



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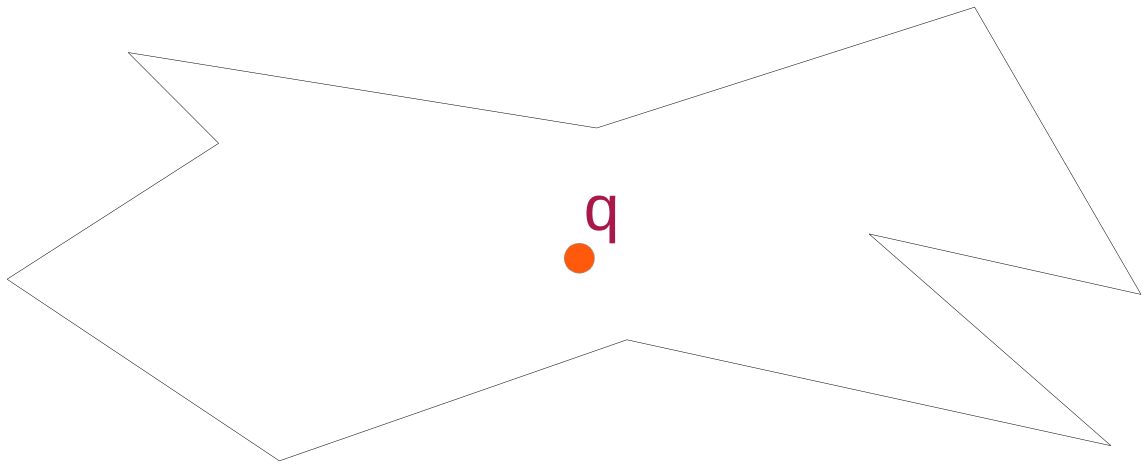
What about Brute Force!!



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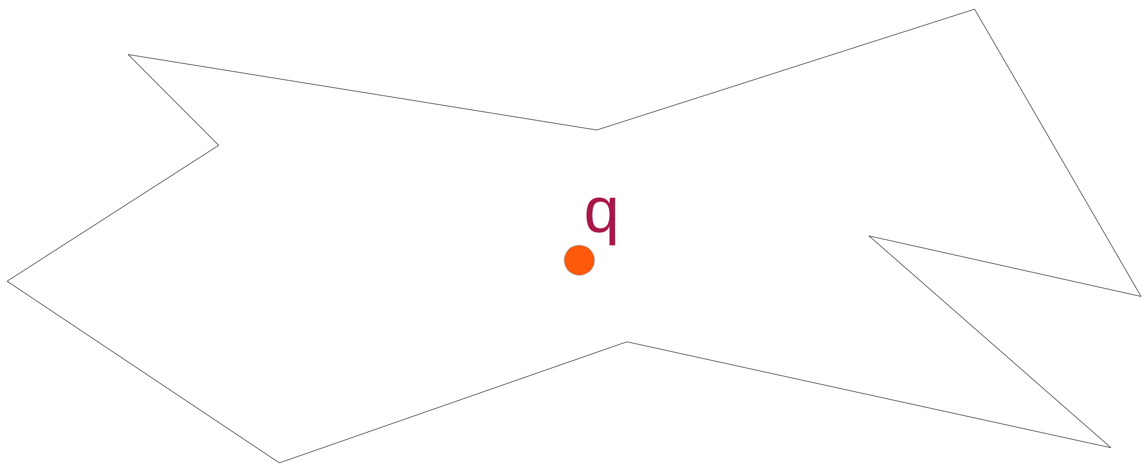

Can we use the convex polygon method/ area method?

For simple polygon

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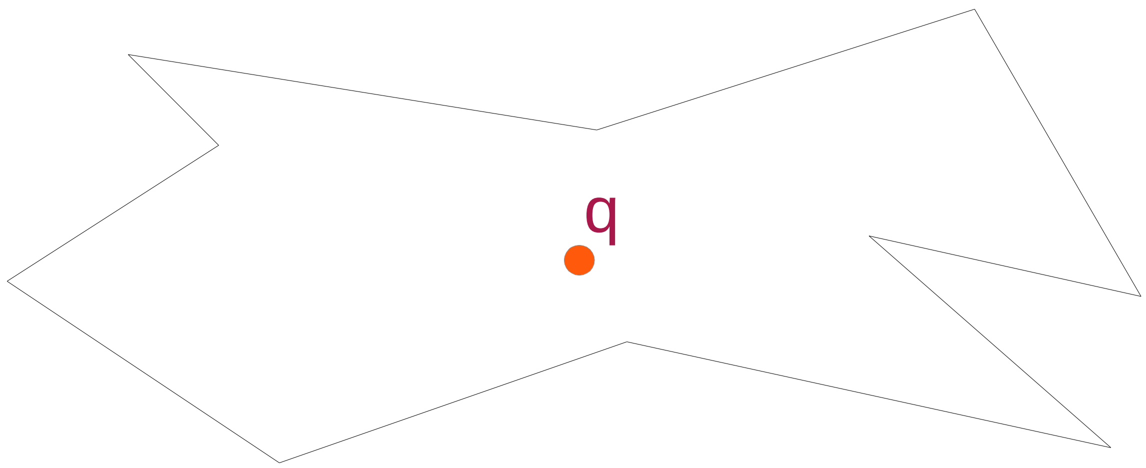

Can we use the convex polygon method/ area method?

Not easily!!

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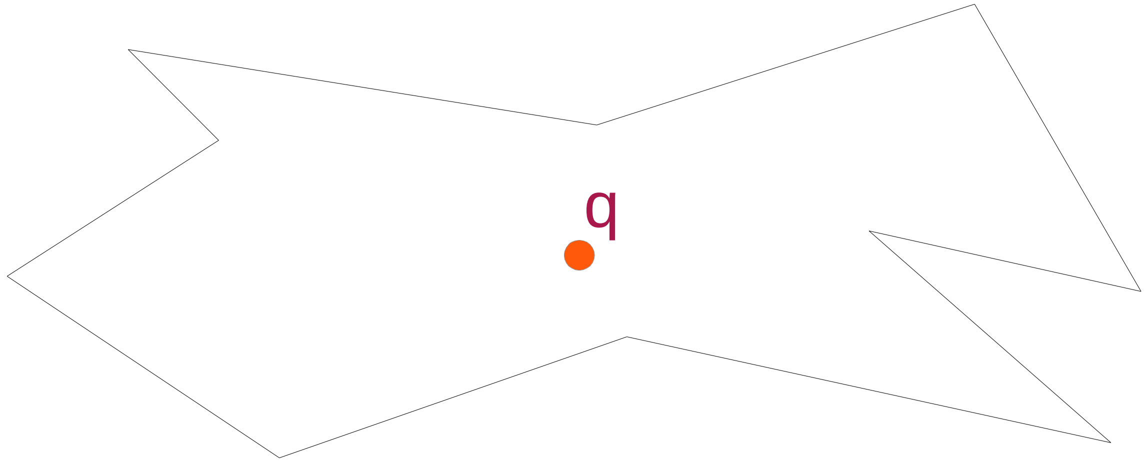

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Then how do we solve??

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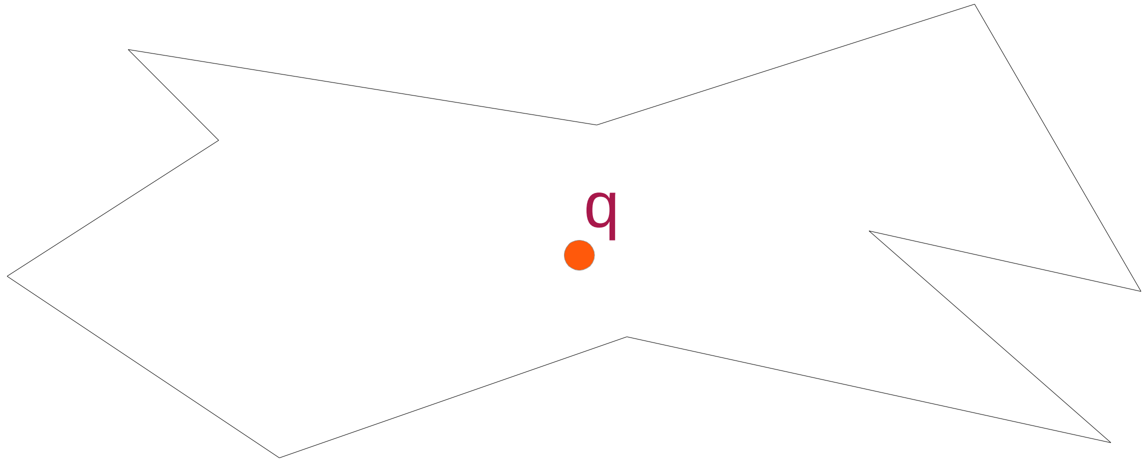

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Shoot a horizontal ray R from q towards positive x-direction

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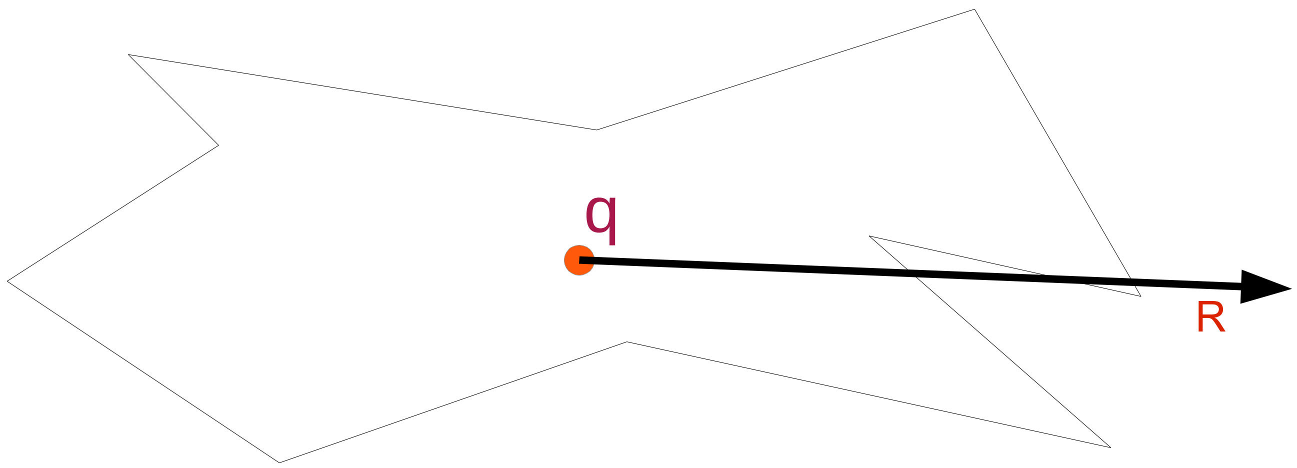

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Odd number of intersection between R and P implies ==>>

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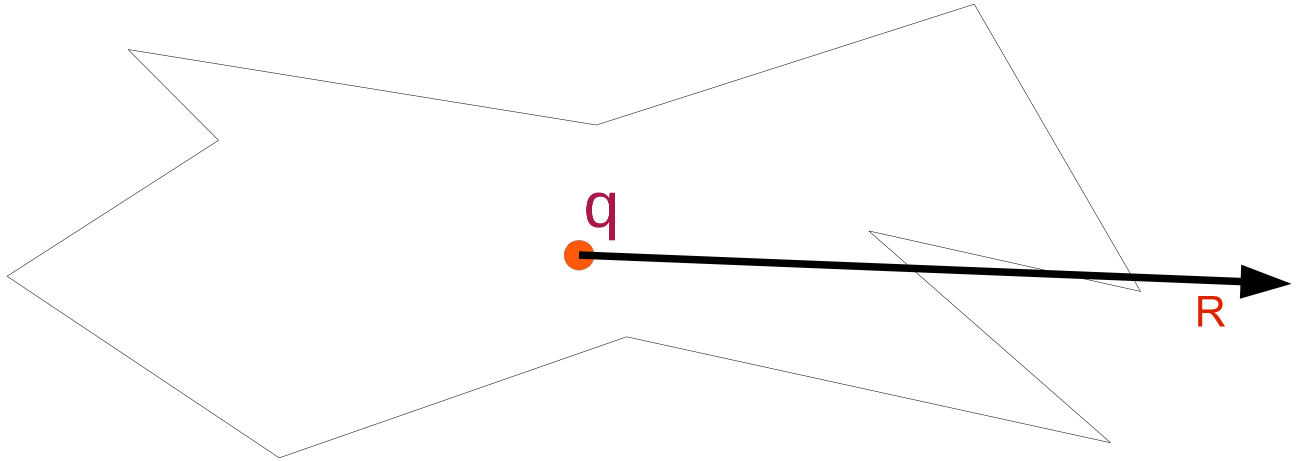

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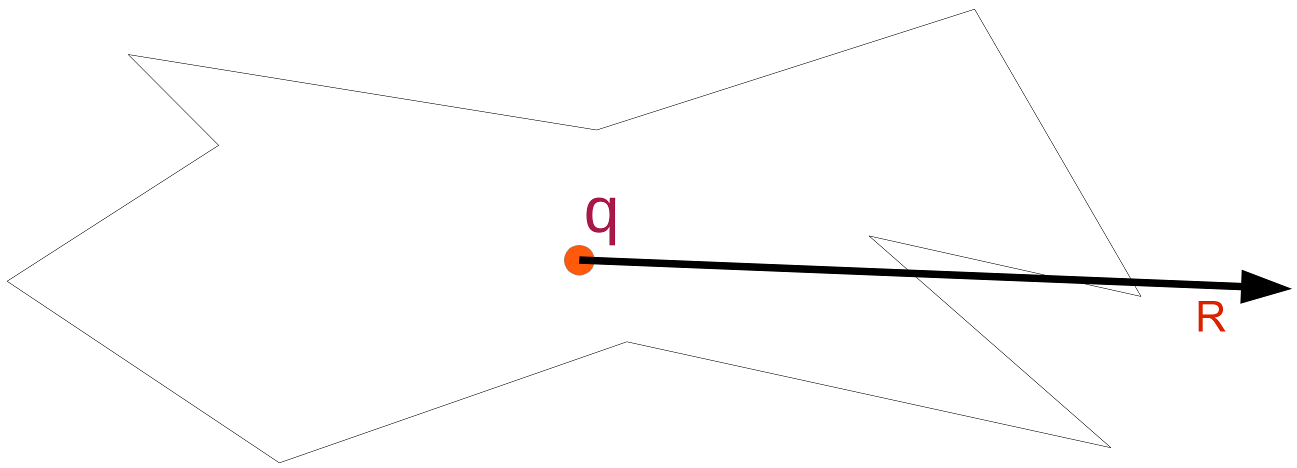

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Complexity:

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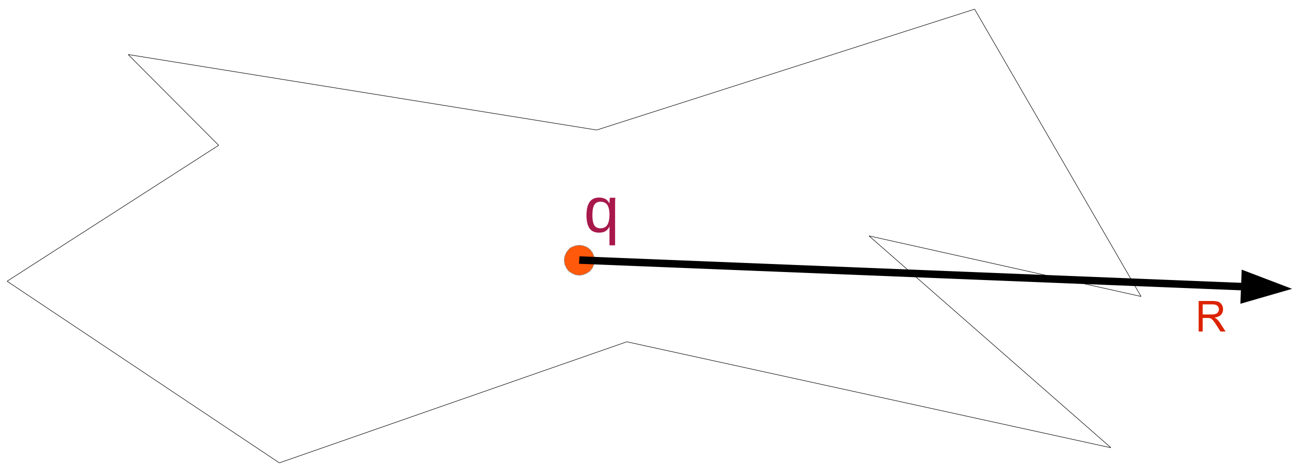

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Complexity: O(n)

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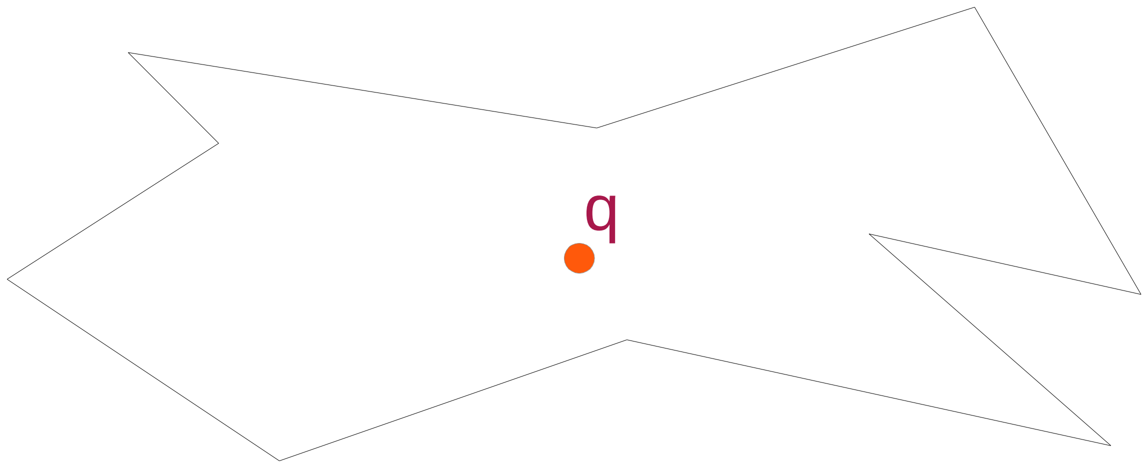

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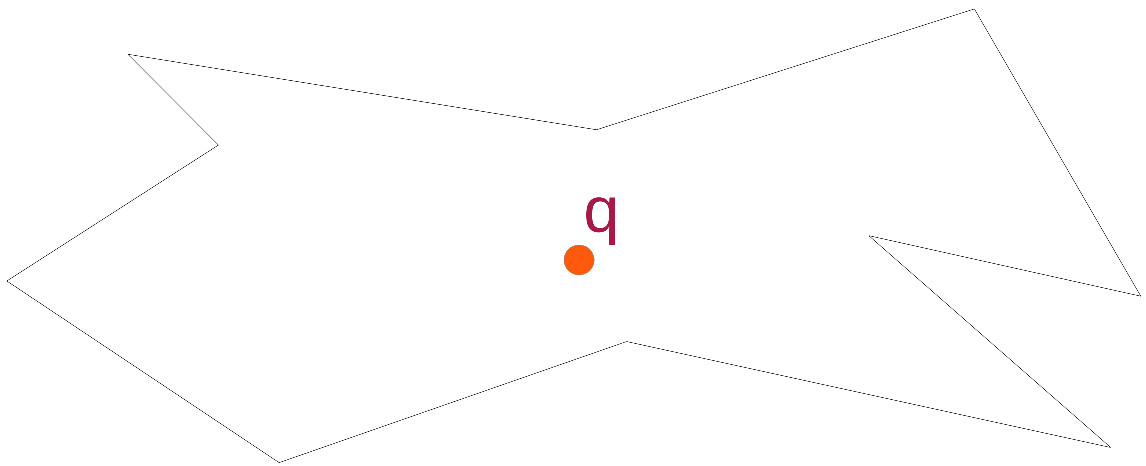

Can we use the convex polygon triangulation method?

For simple polygon

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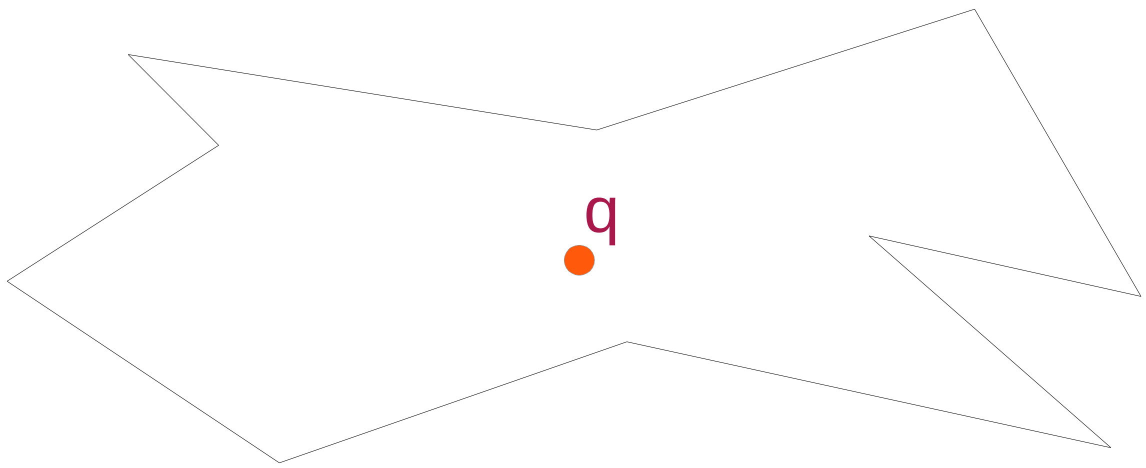

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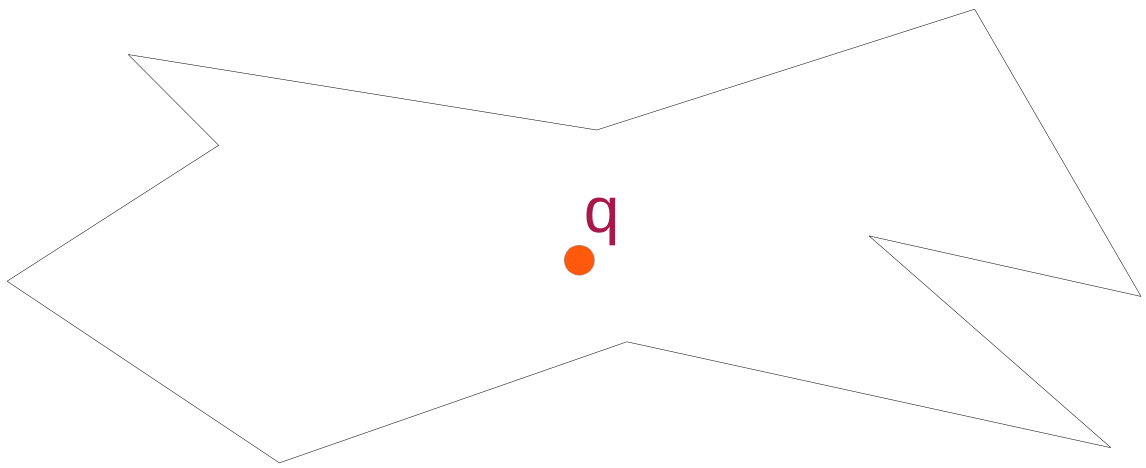

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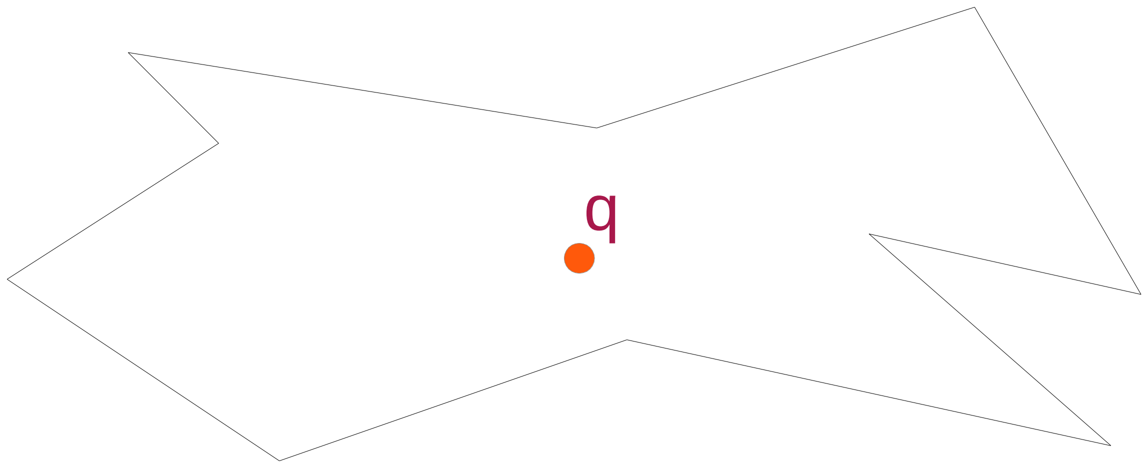

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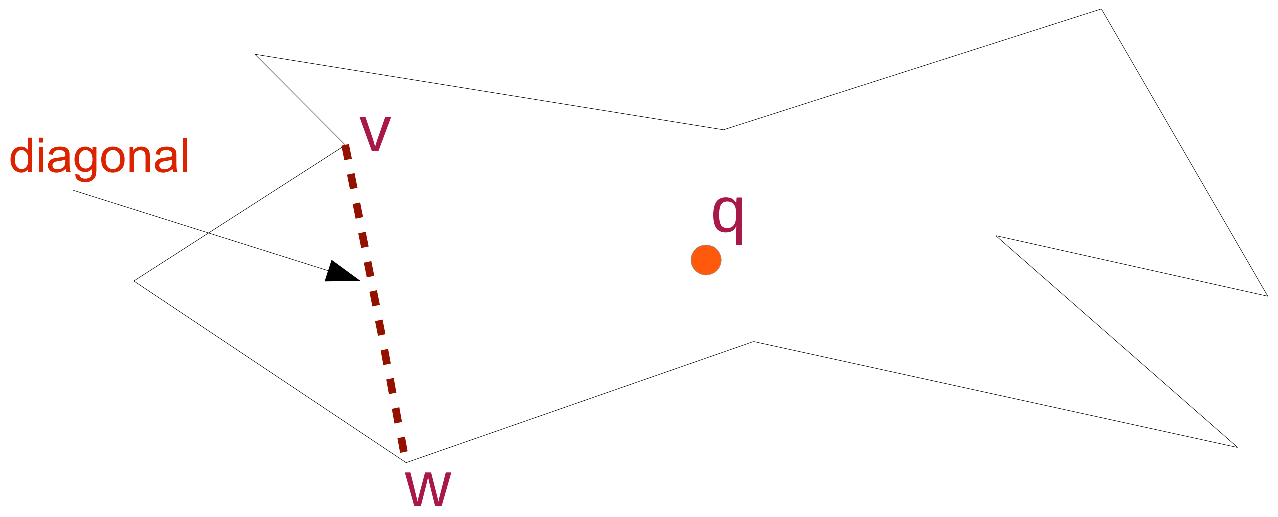

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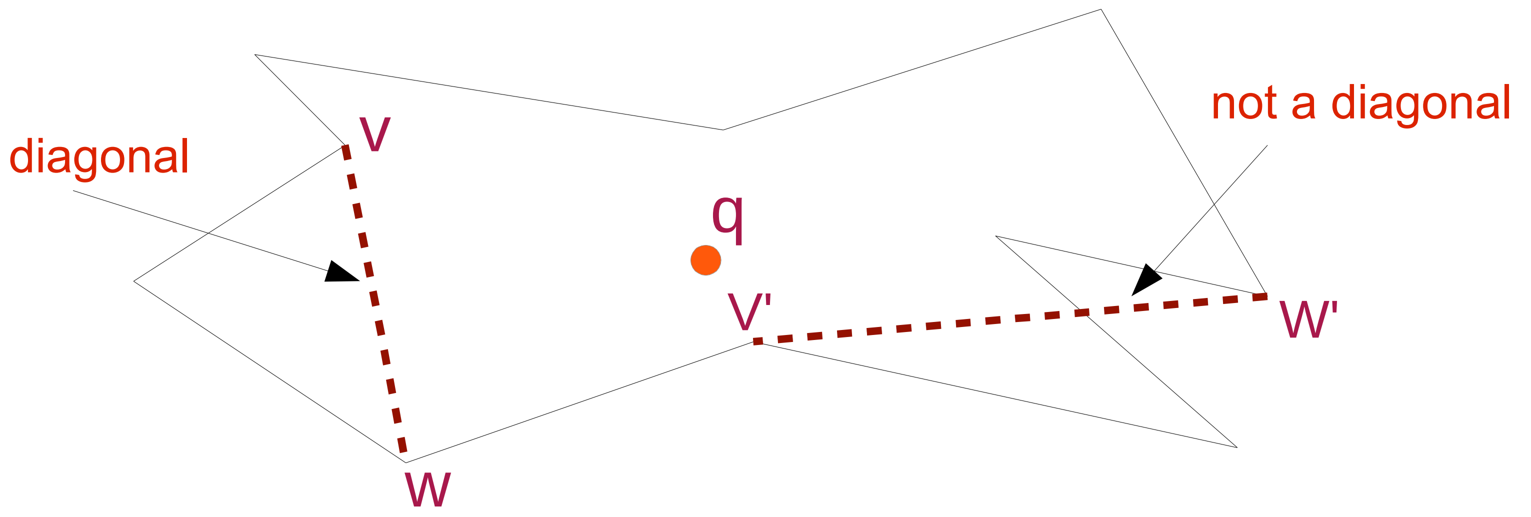

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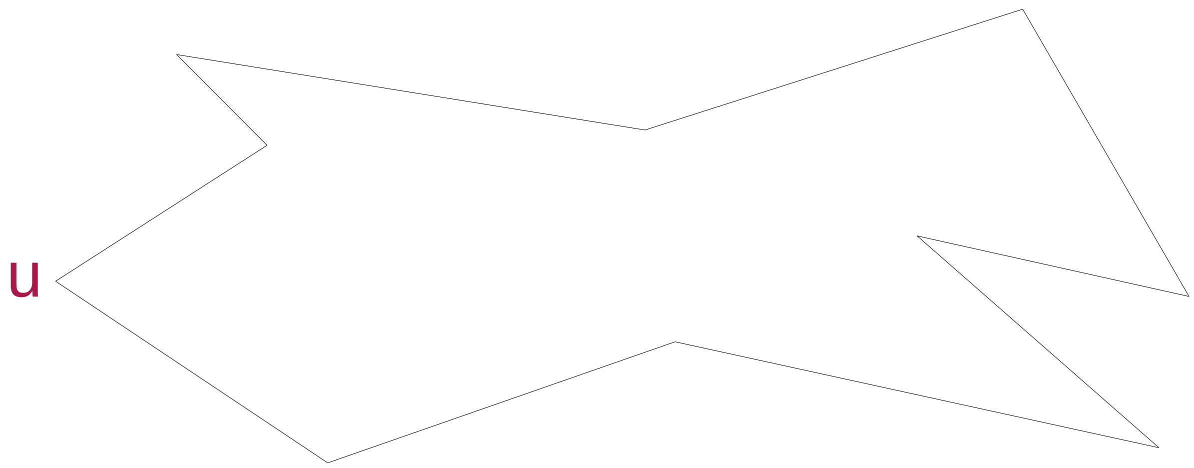

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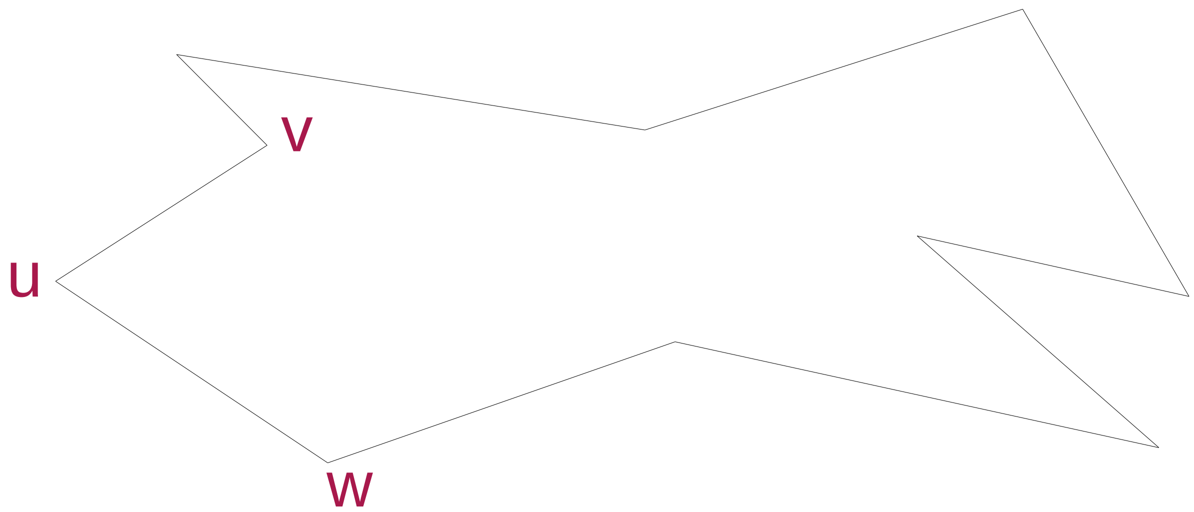

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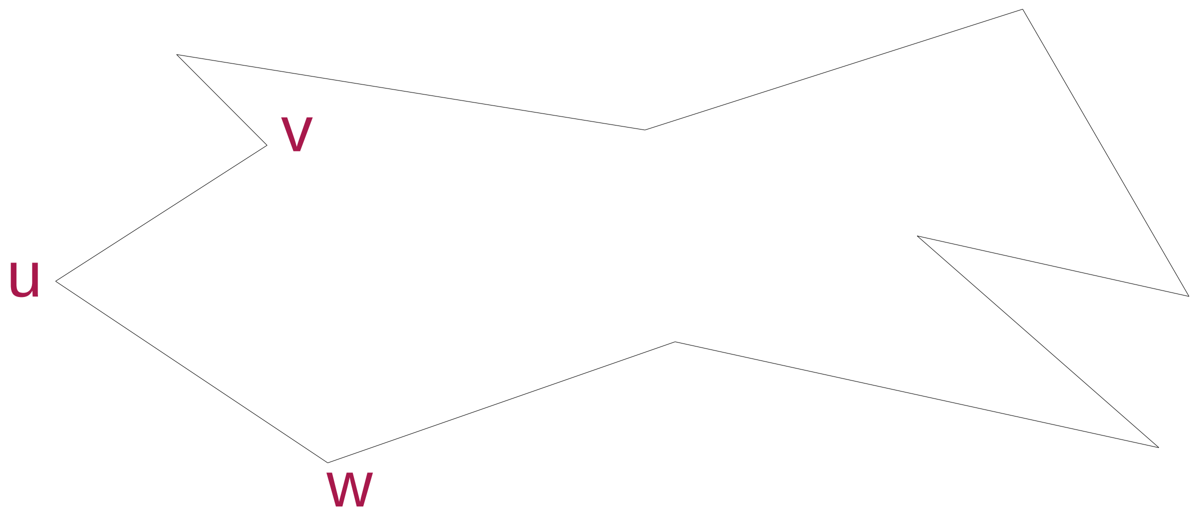

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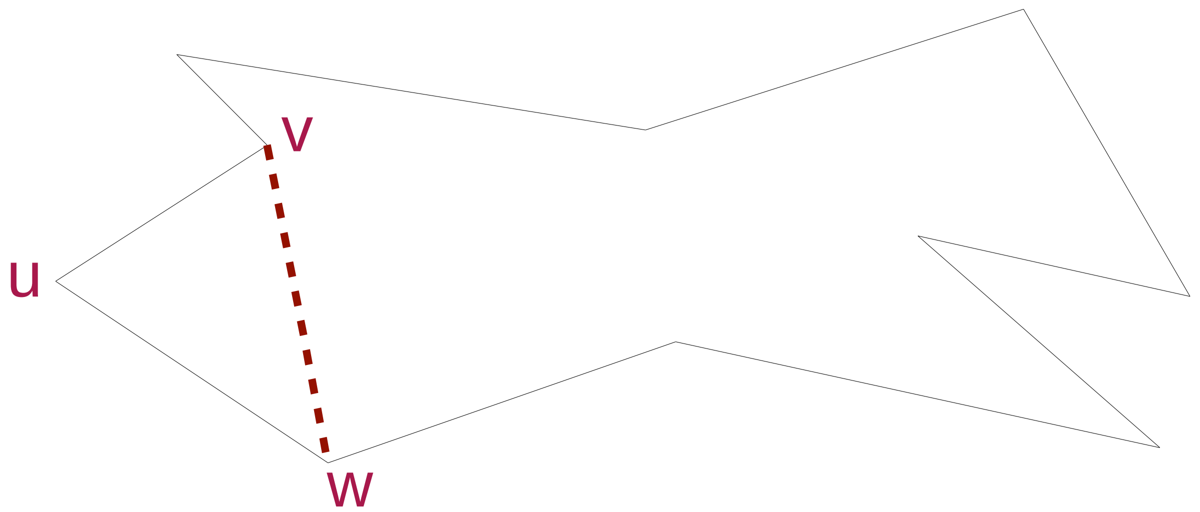

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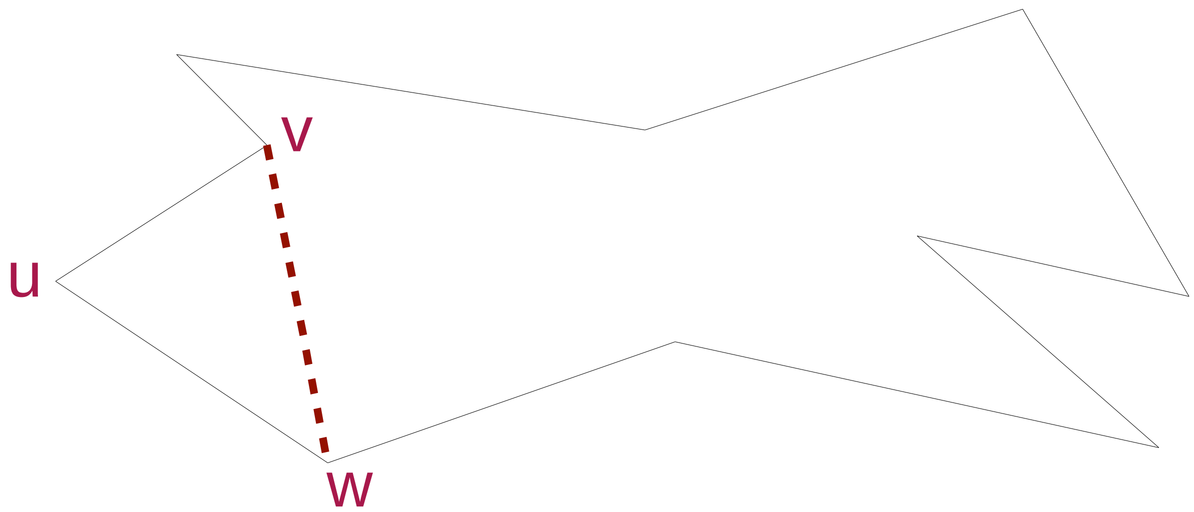

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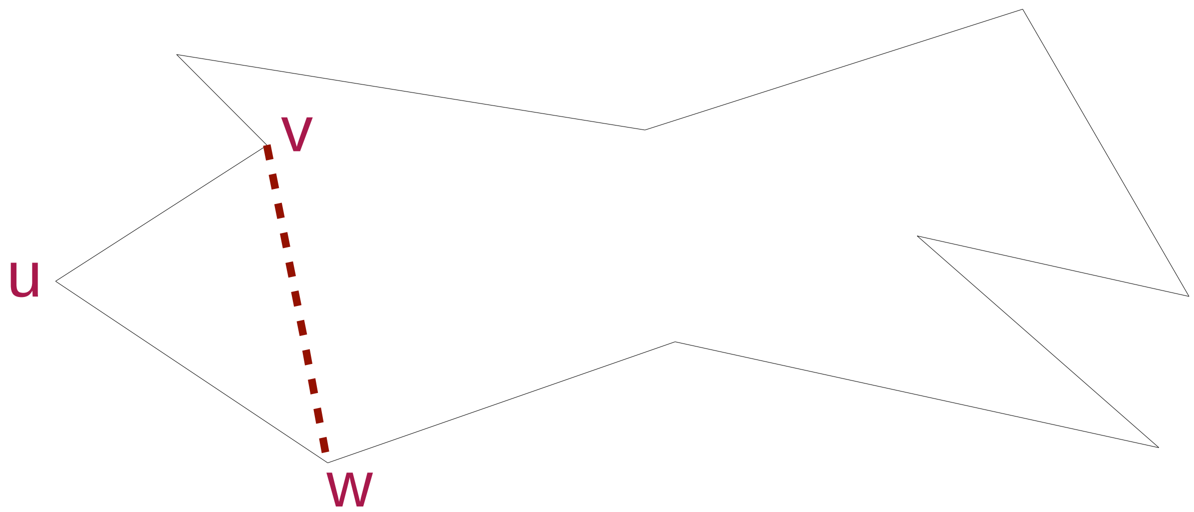

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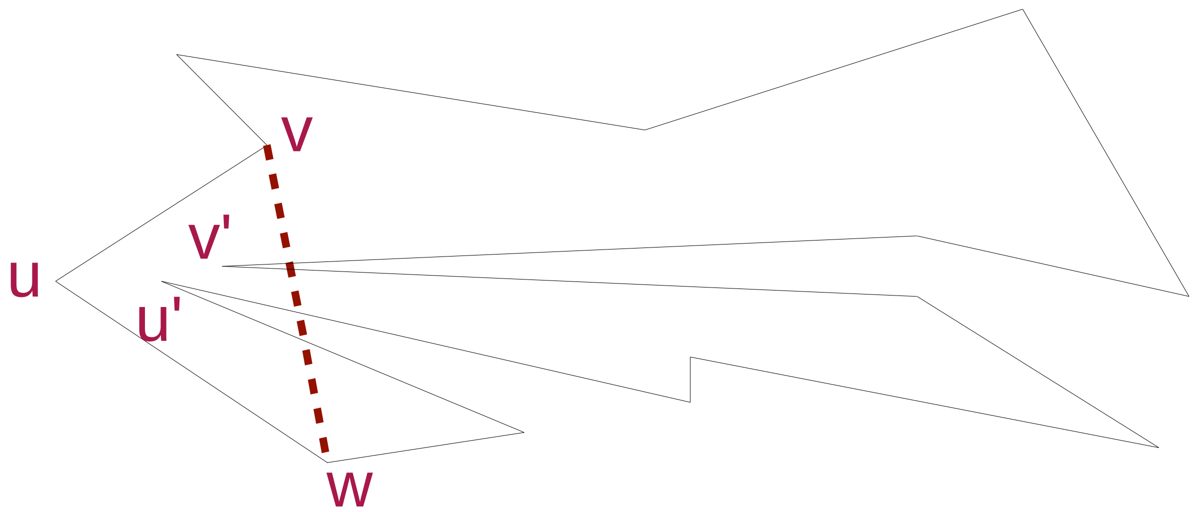

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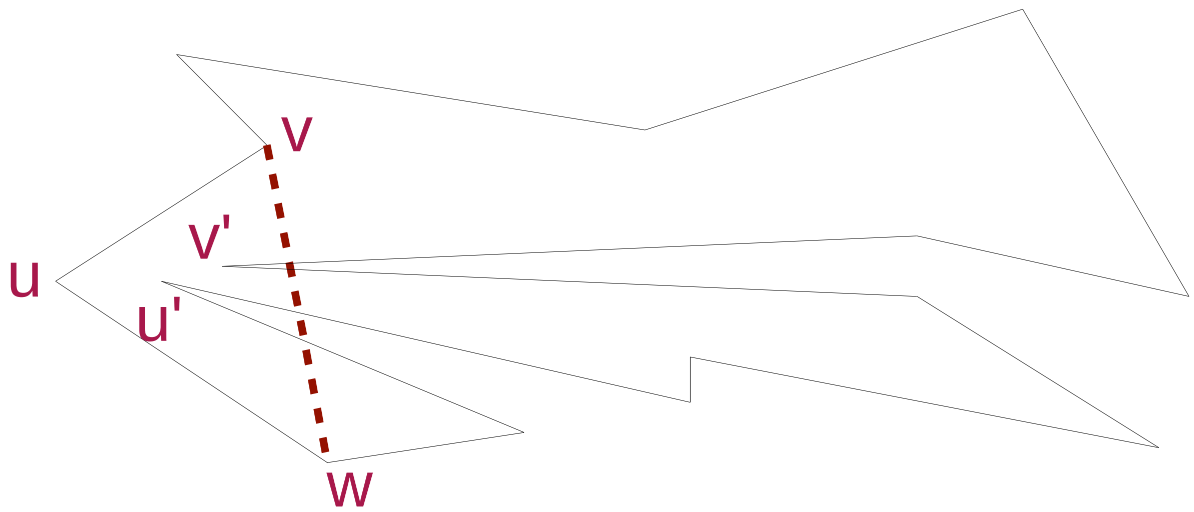

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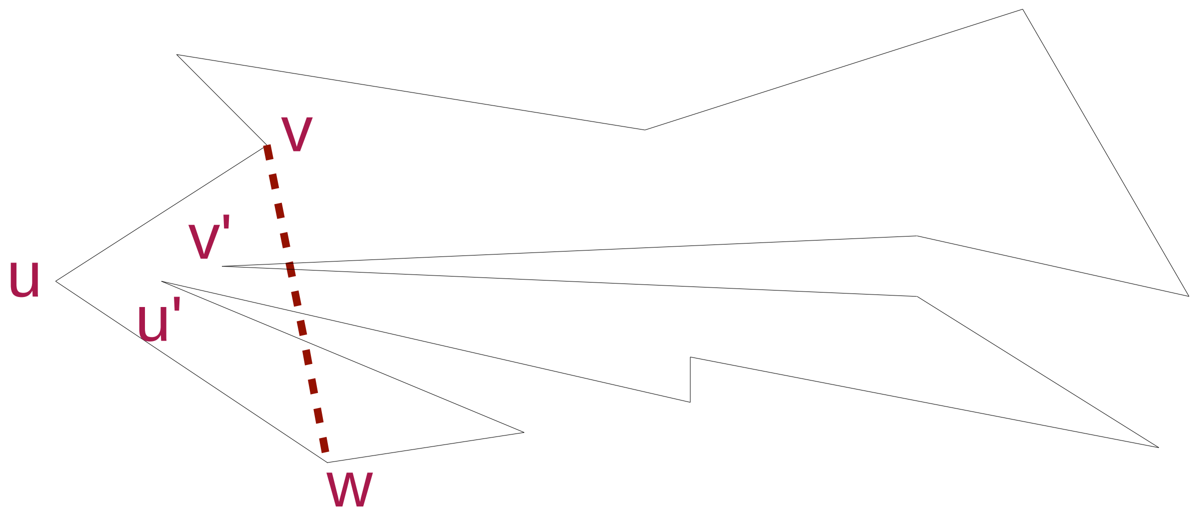

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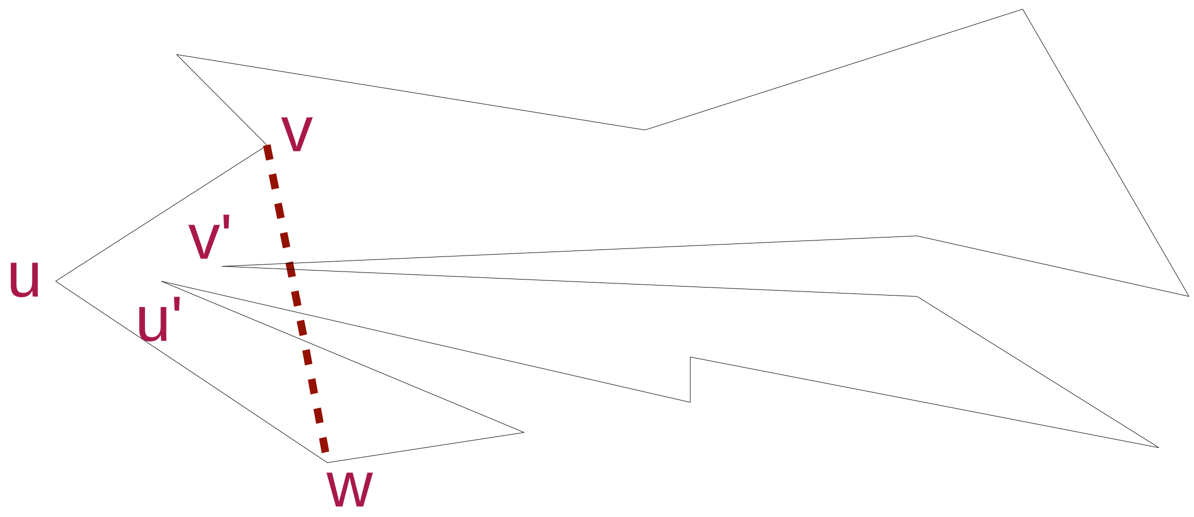
Can we find a vertex u' inside triangle uvw such that uu' is a diagonal?



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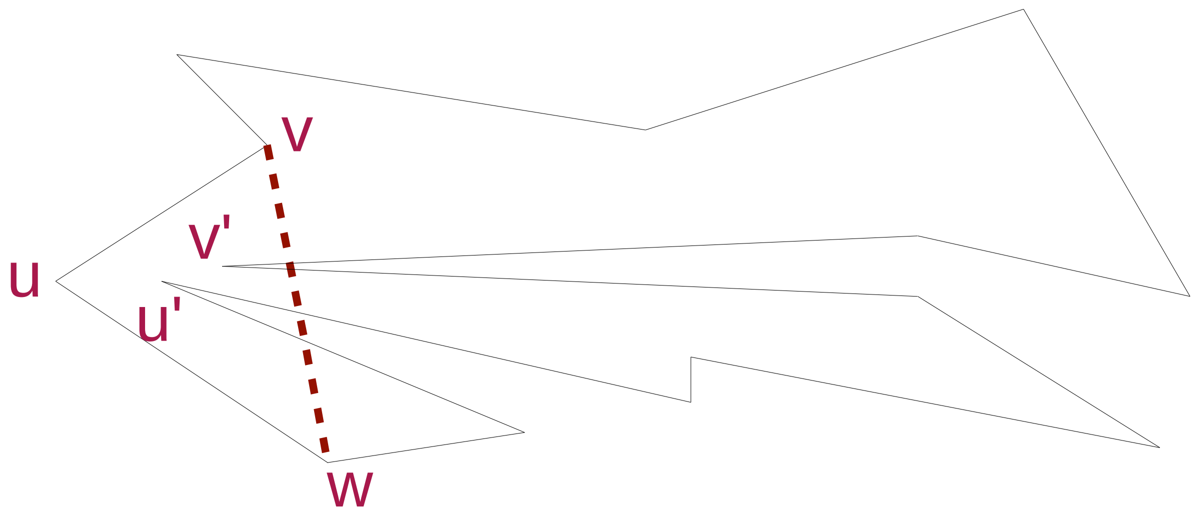

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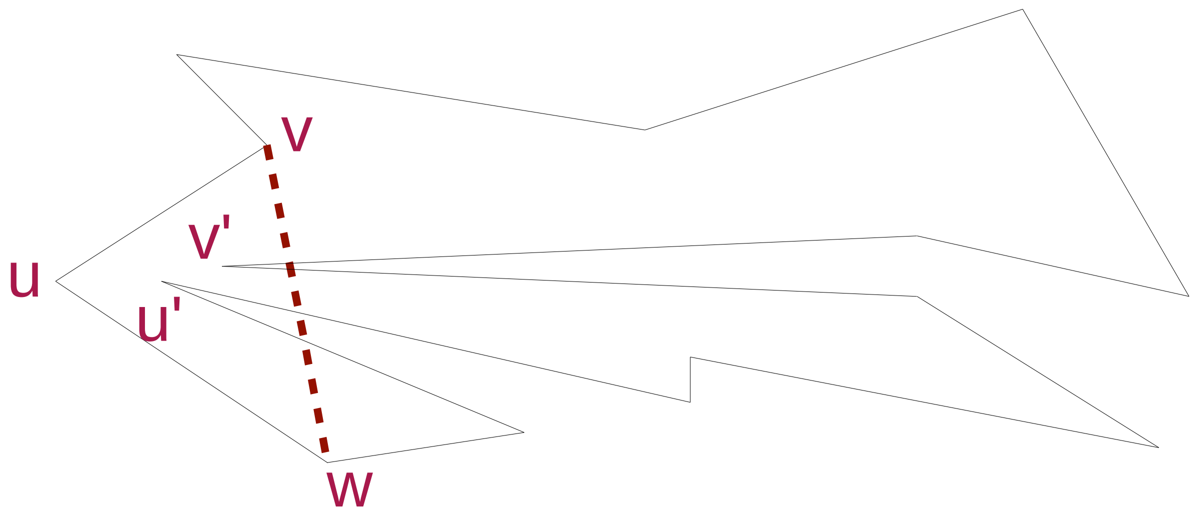

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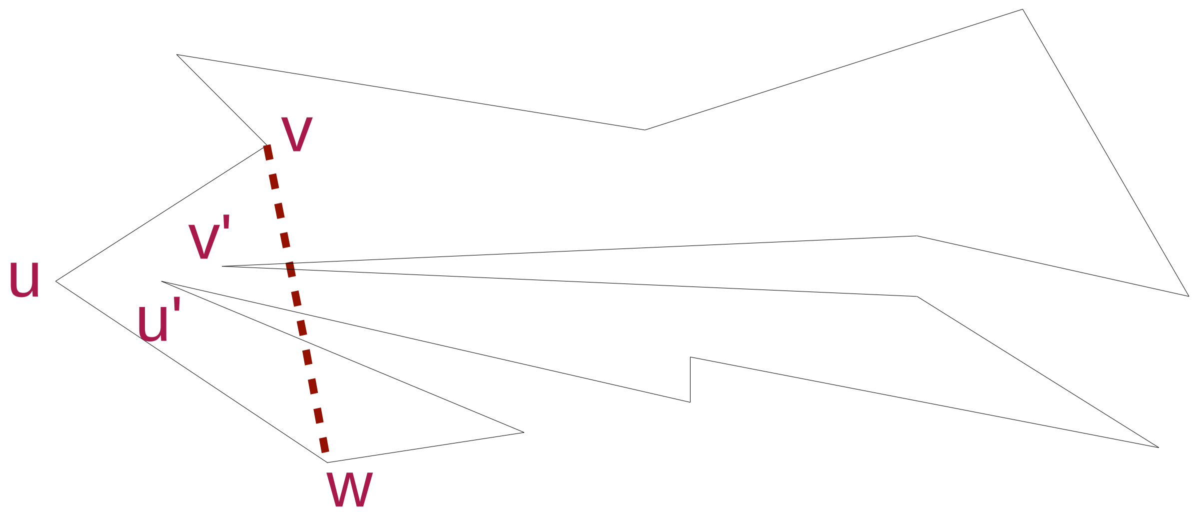

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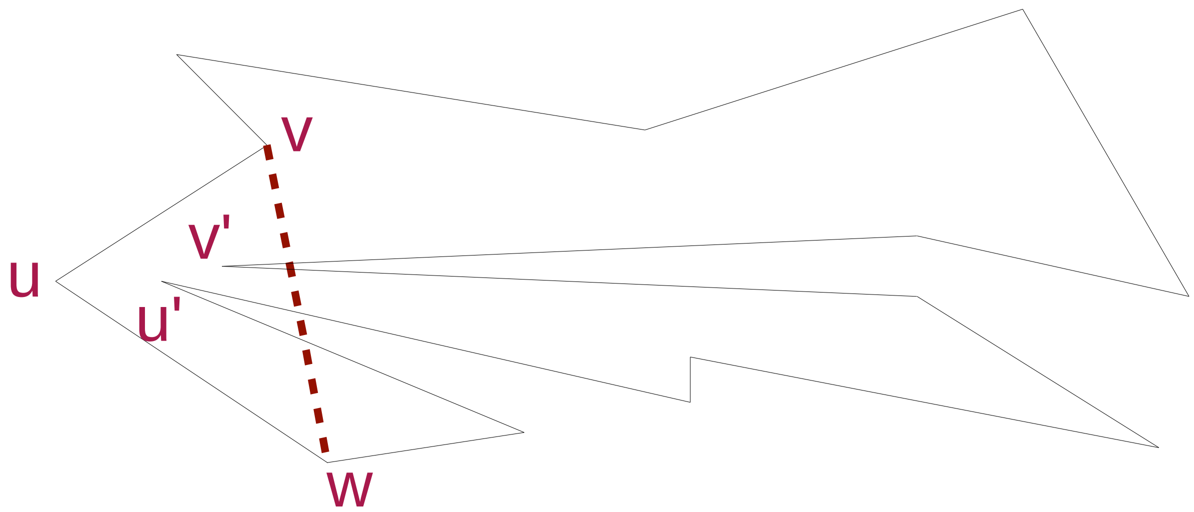

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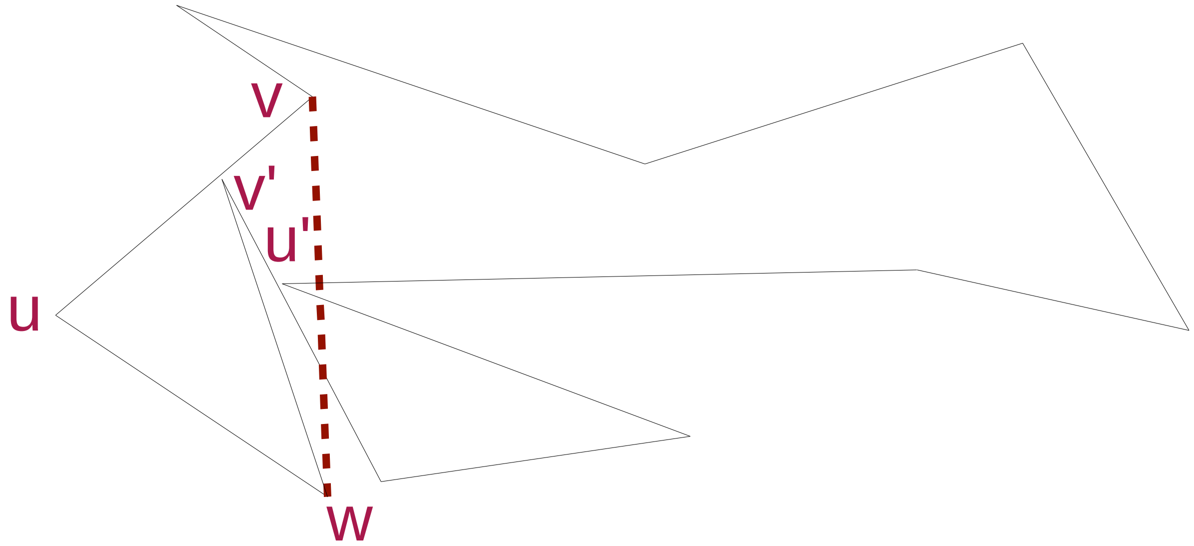

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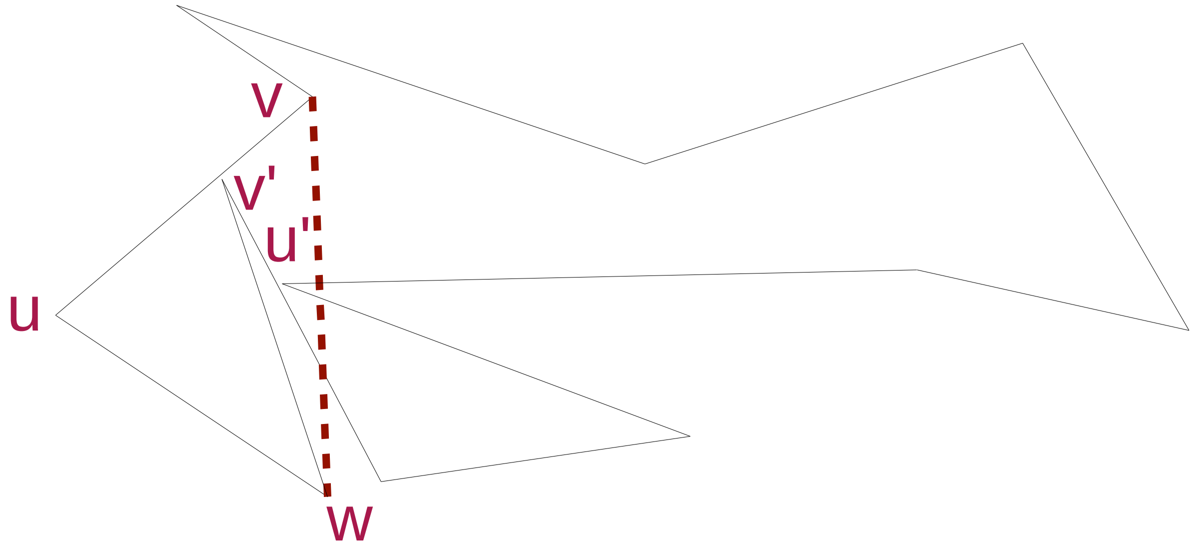

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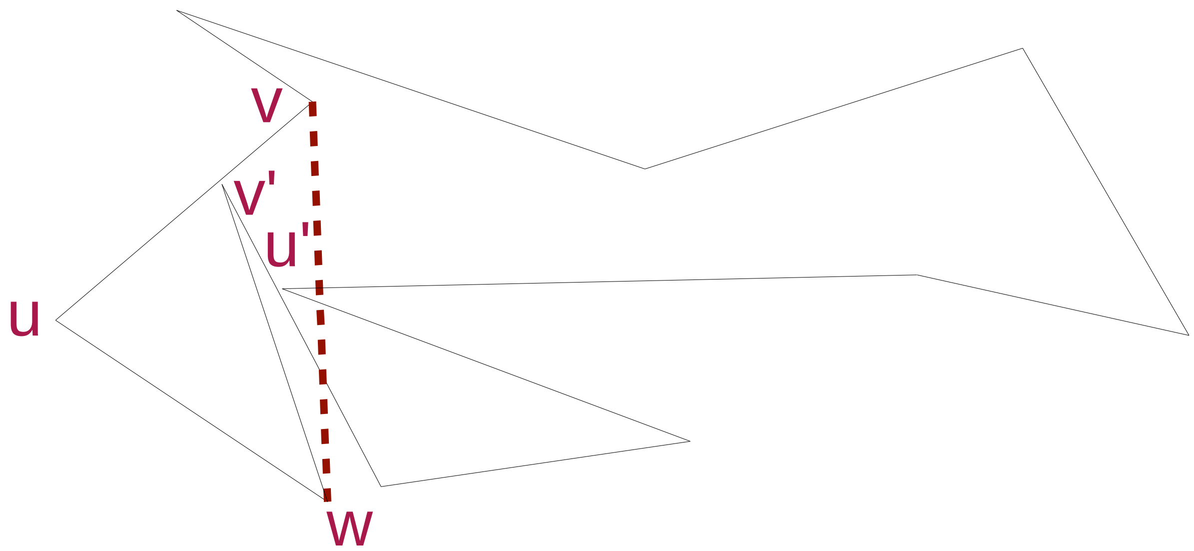

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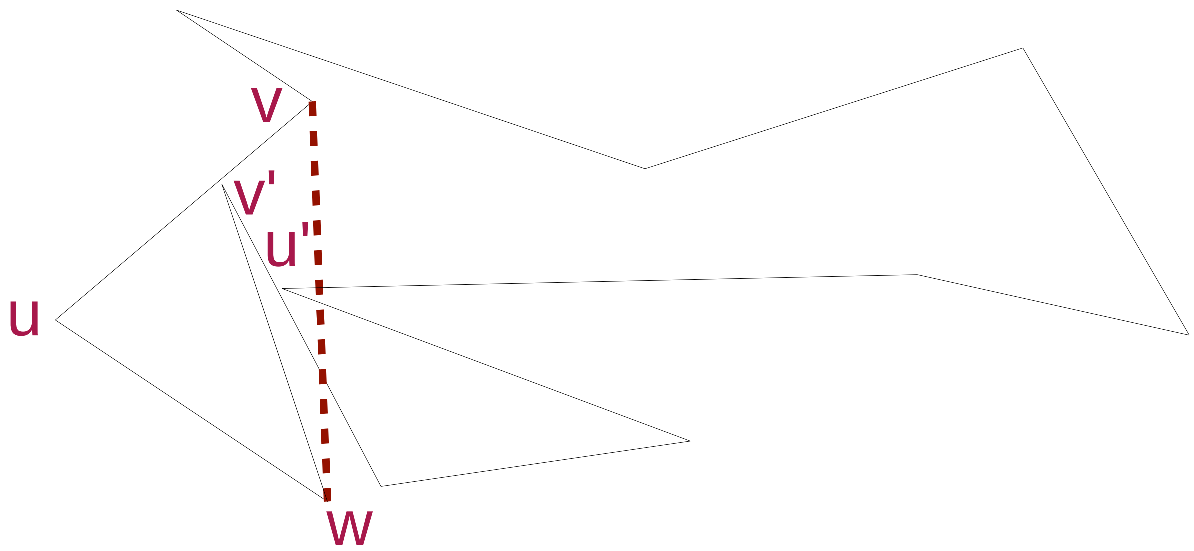

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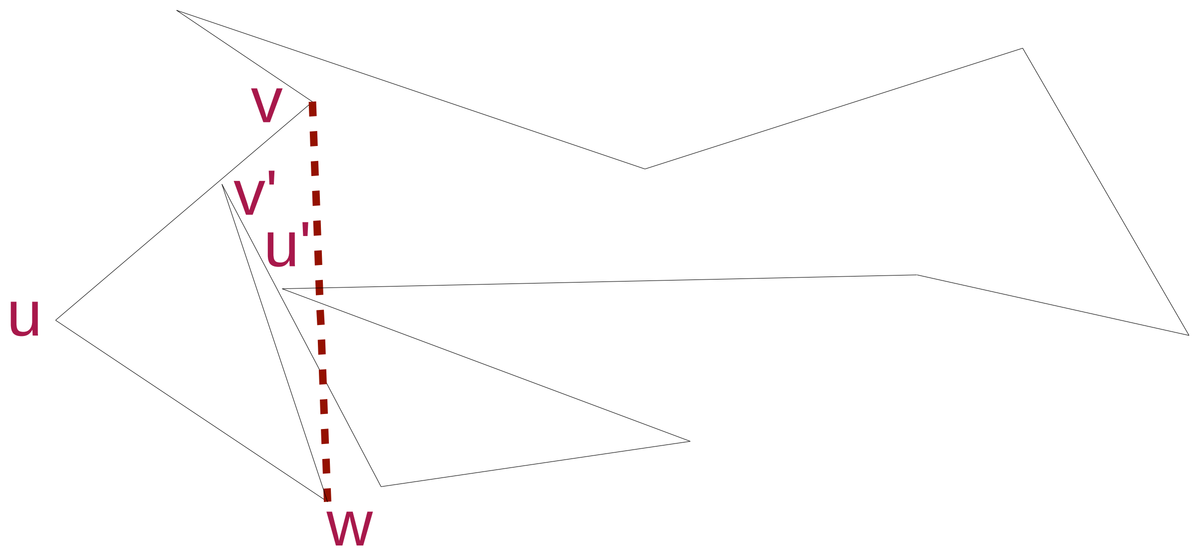

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Complexity: O(n)

Triangulation in quadratic time

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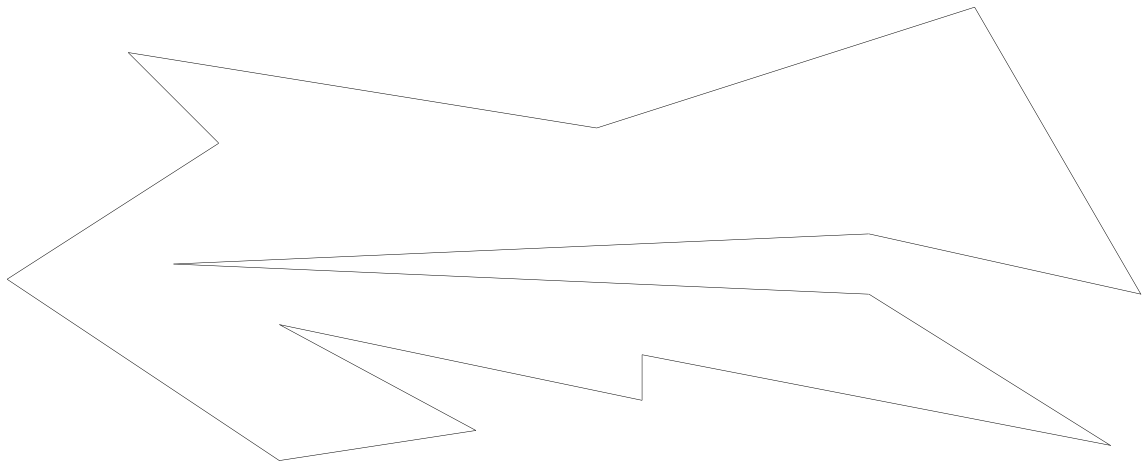

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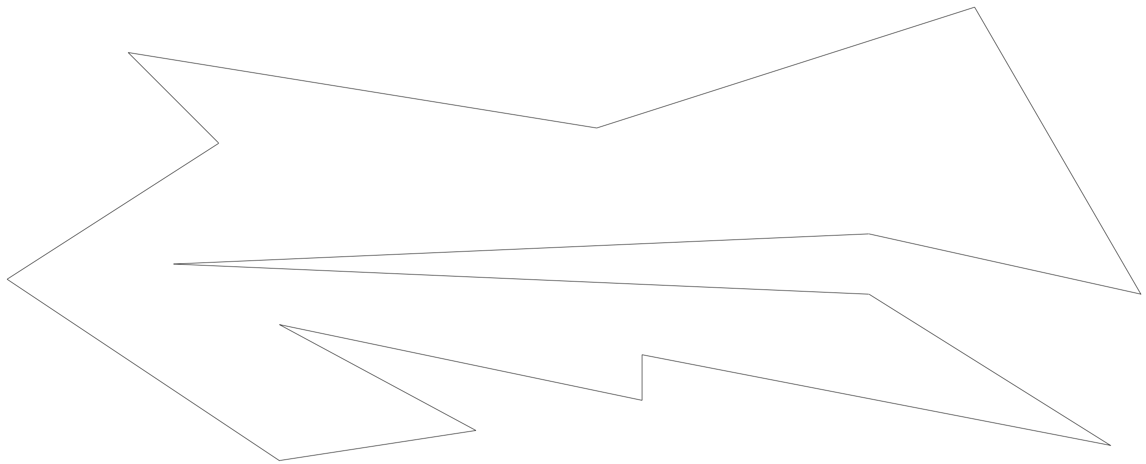

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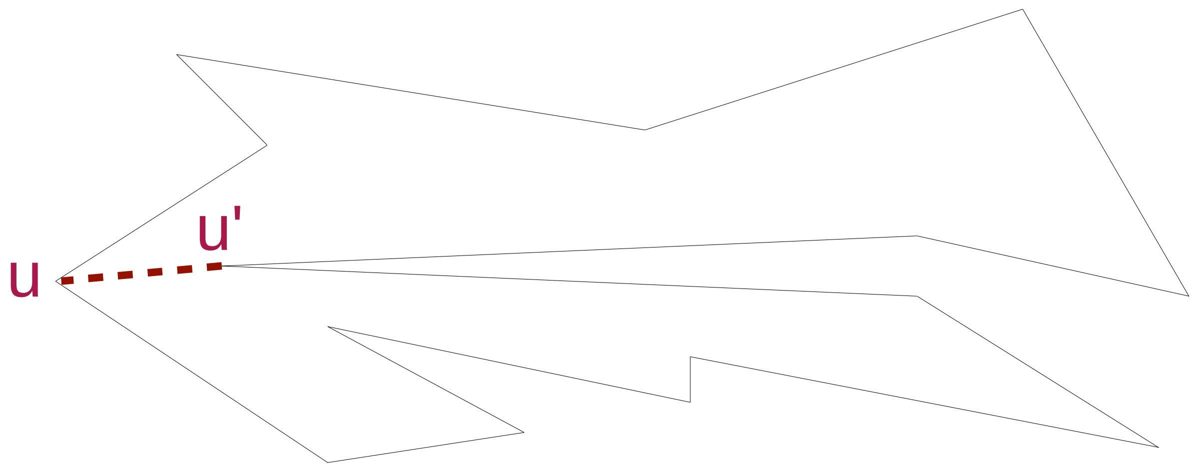

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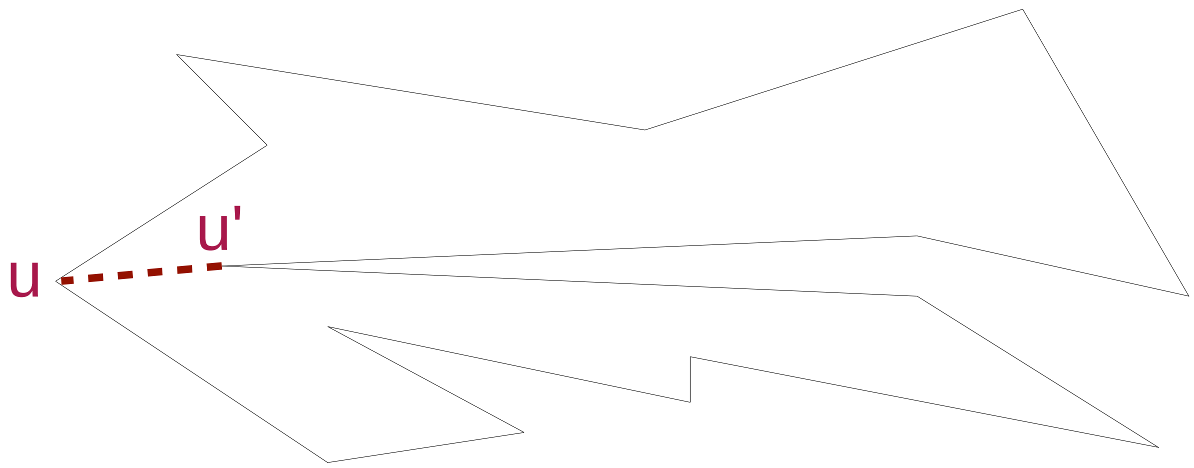

Diagonal uu' partitions P in two smaller size polygon, say P_1 and P_2

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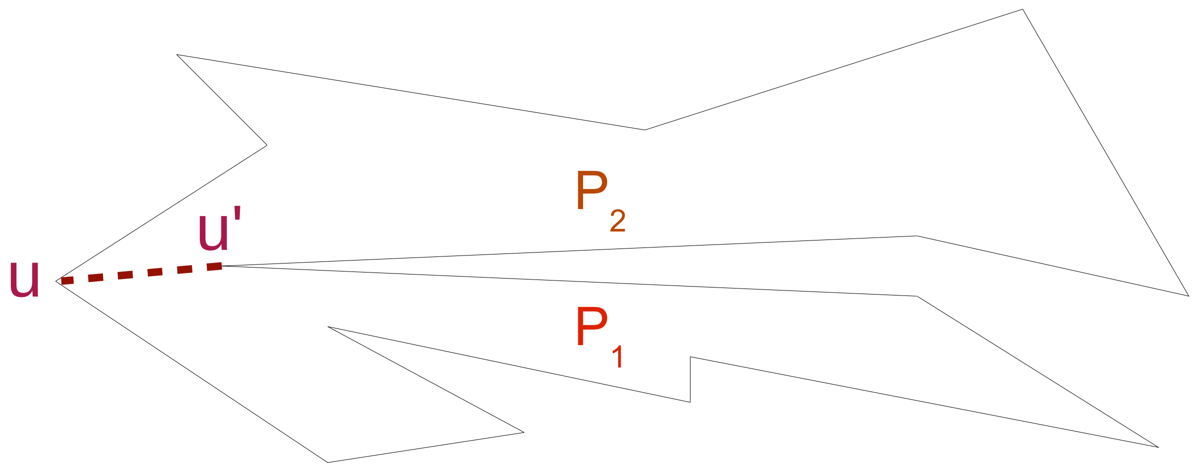

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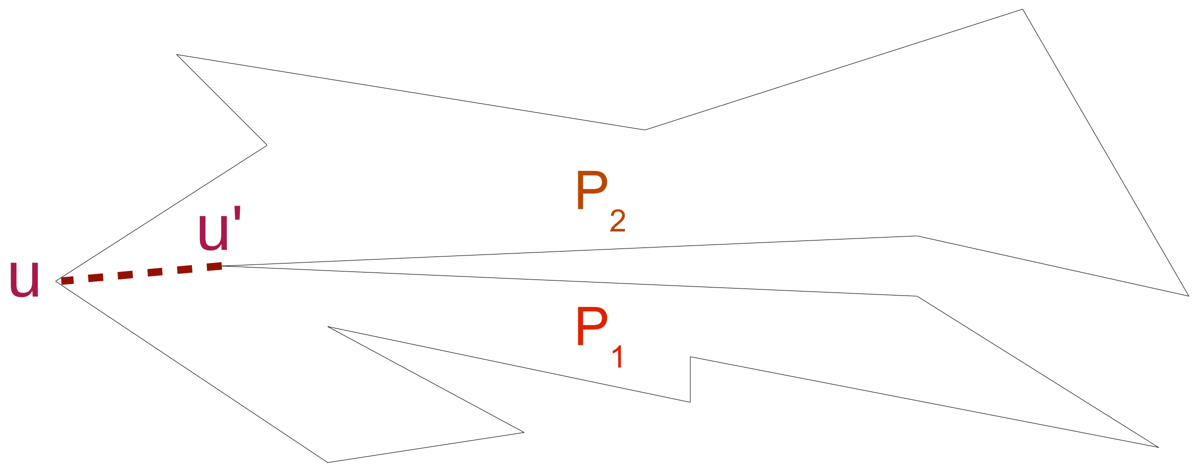

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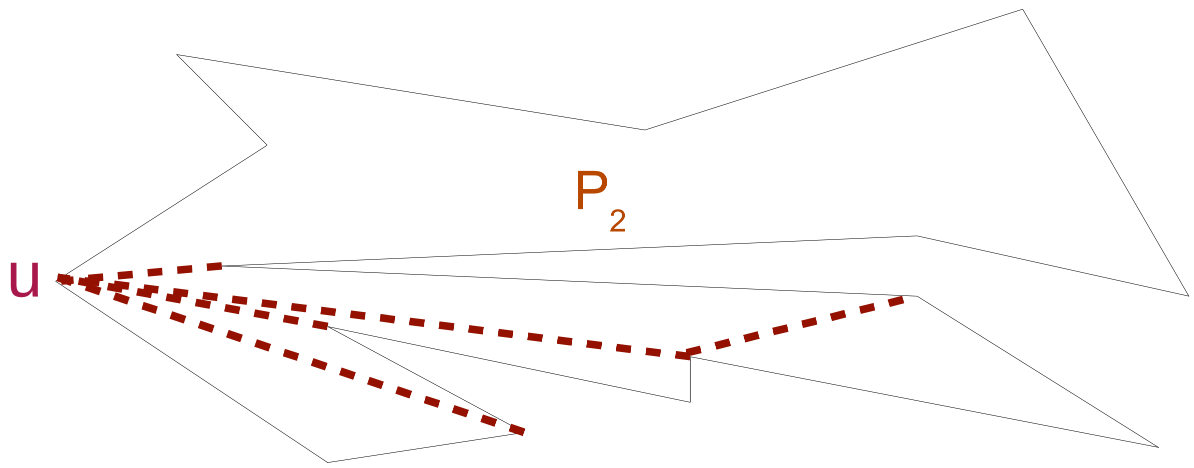

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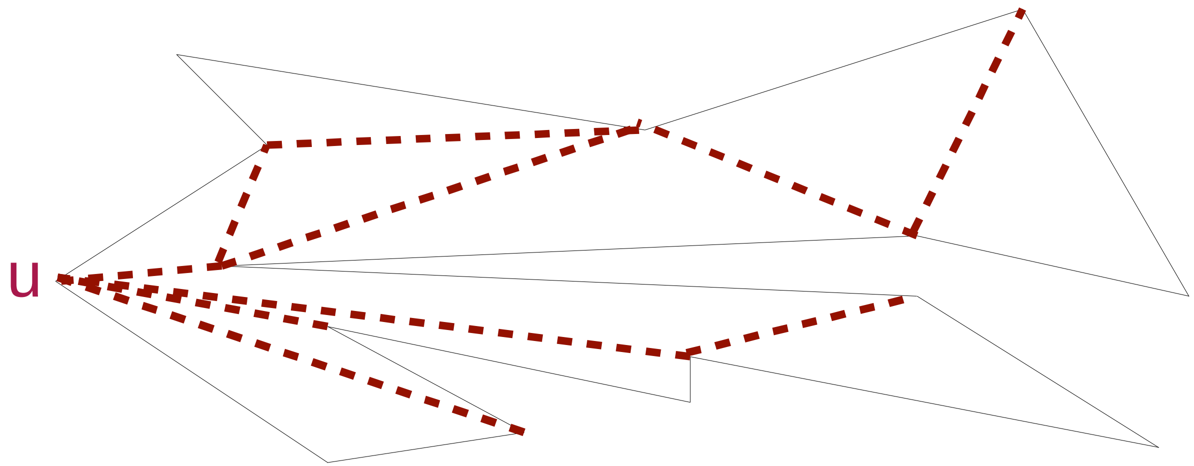

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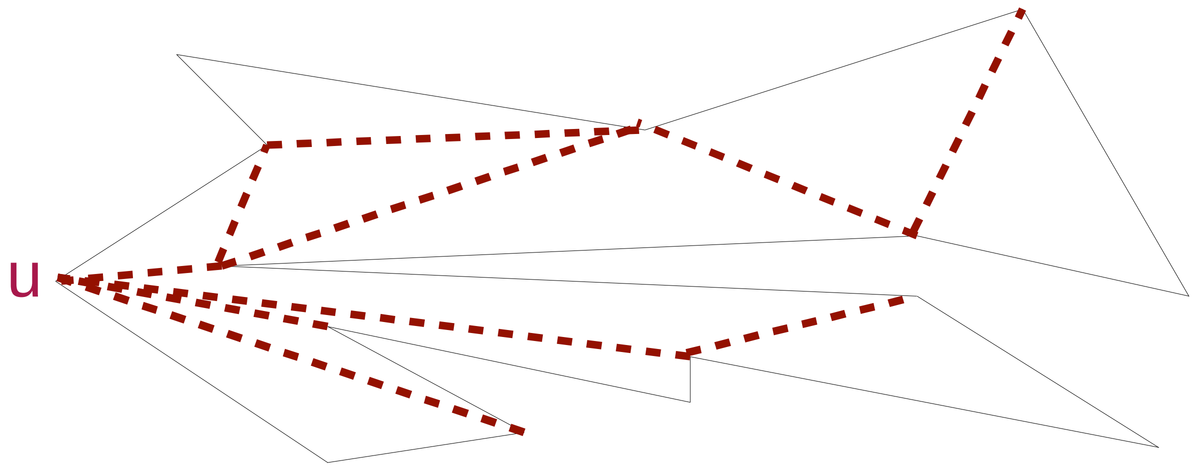

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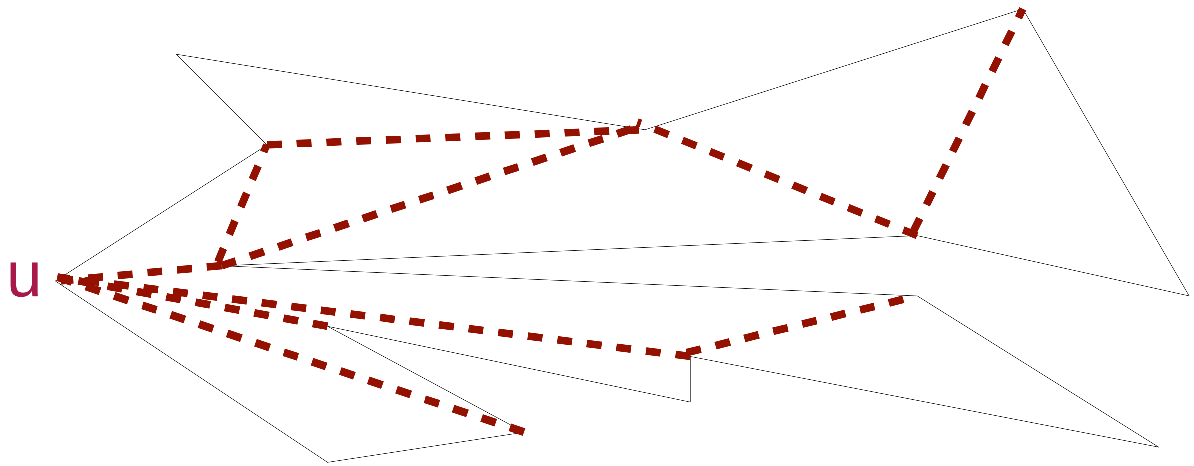

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Complexity: $O(n^2)$

Some results on triangulation

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Chazelle proposed a linear time algorithm for triangulation

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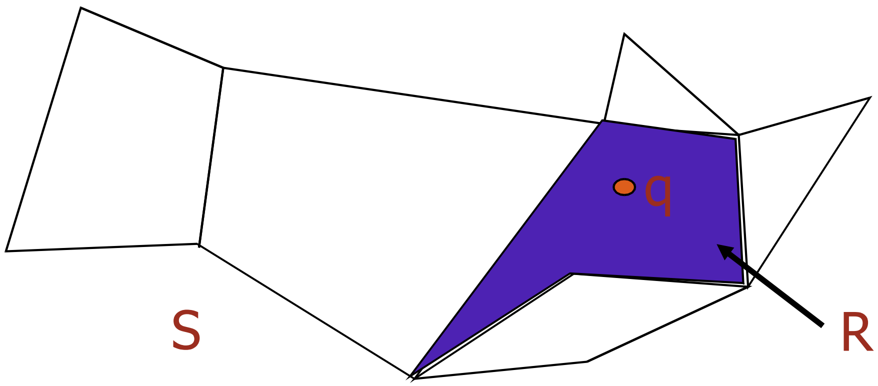
But we want planar point location not triangulation !!

Planar Point Location

Given a planar subdivision S



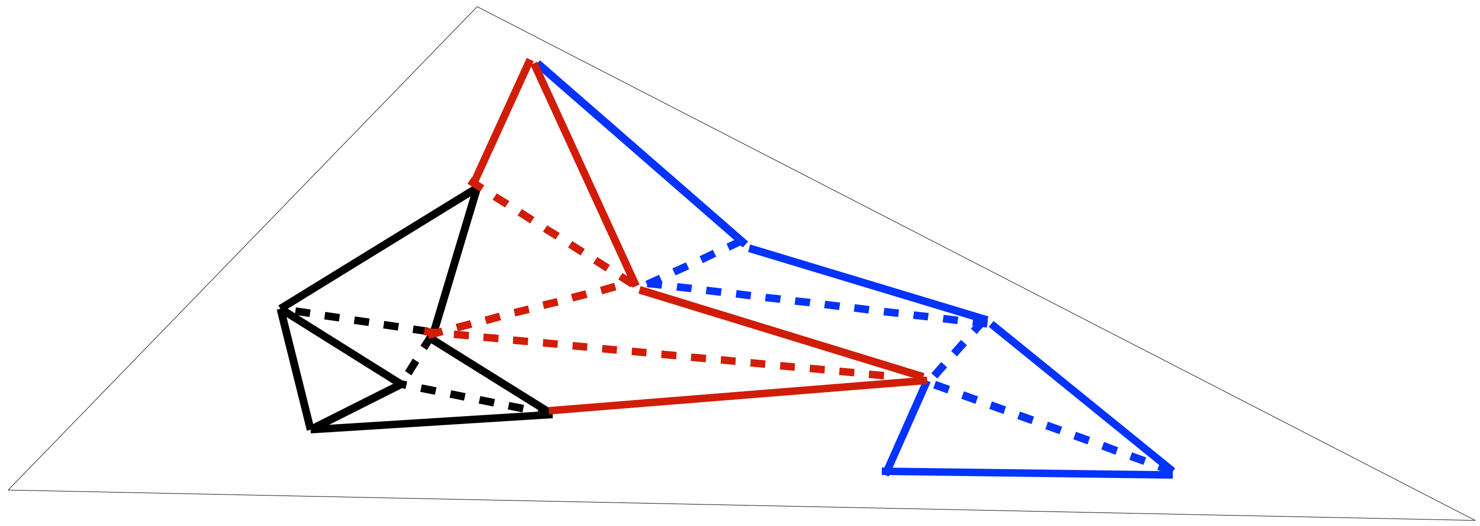
Preprocess S such that:

For any query point q,

The region/face R containing q can be reported efficiently.

Outline of Kirkpatrick Planar Point Location

Given a triangulated planar subdivision S inside a bounded triangle



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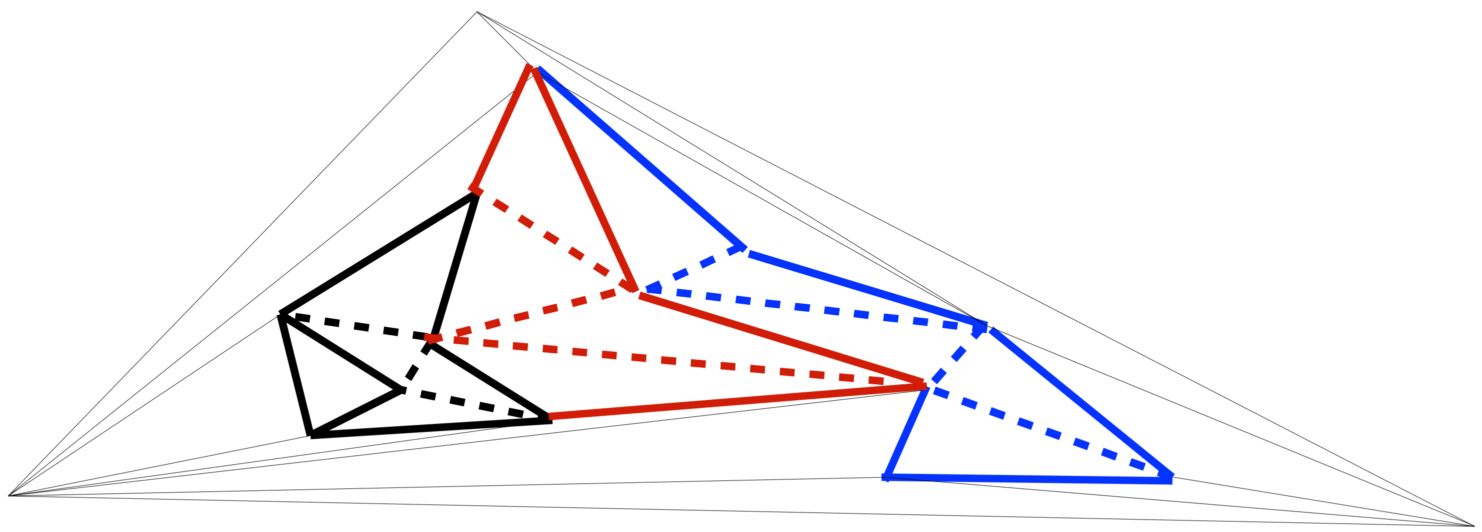

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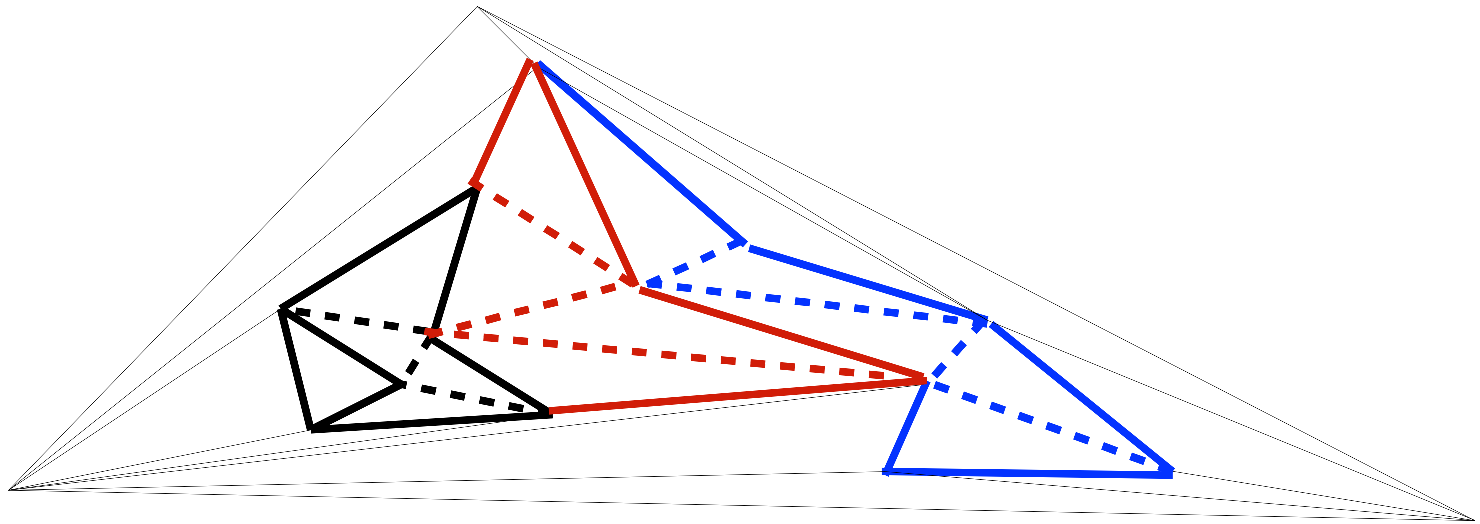
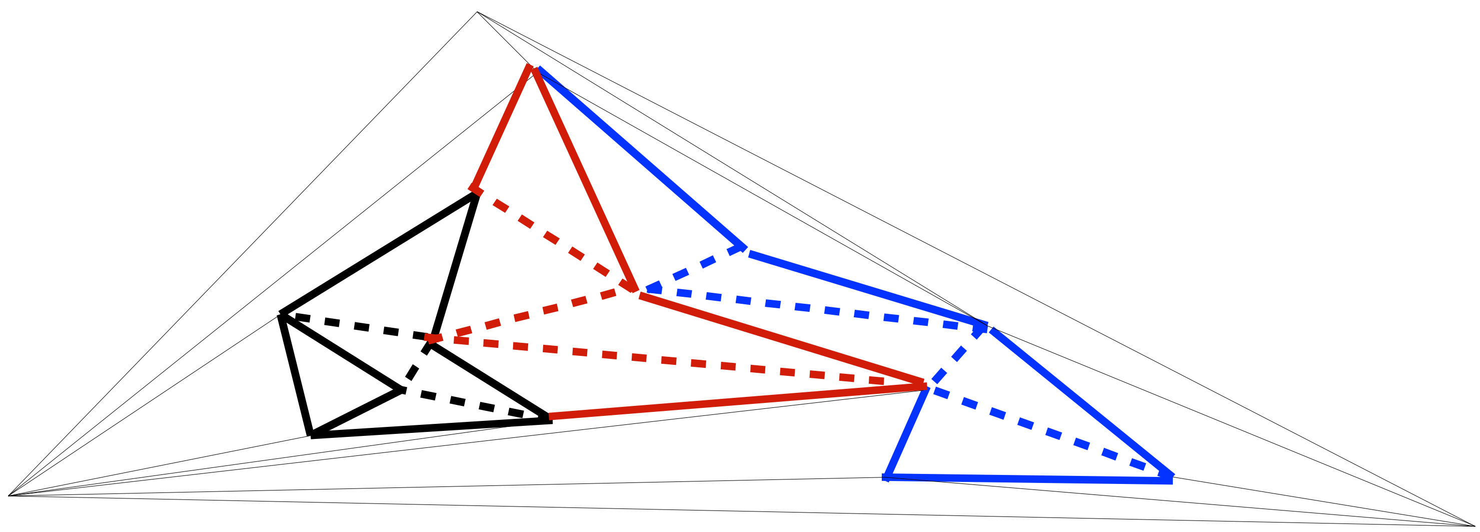


Fact : S has n/2 vertices with degree at most 9 [because S is planar]

Outline of Kirkpatrick Planar Point Location

Given a triangulated planar subdivision S inside a bounded triangle



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Corollary : S has n/18 mutually non-adjacent vertices H with degree at most 9

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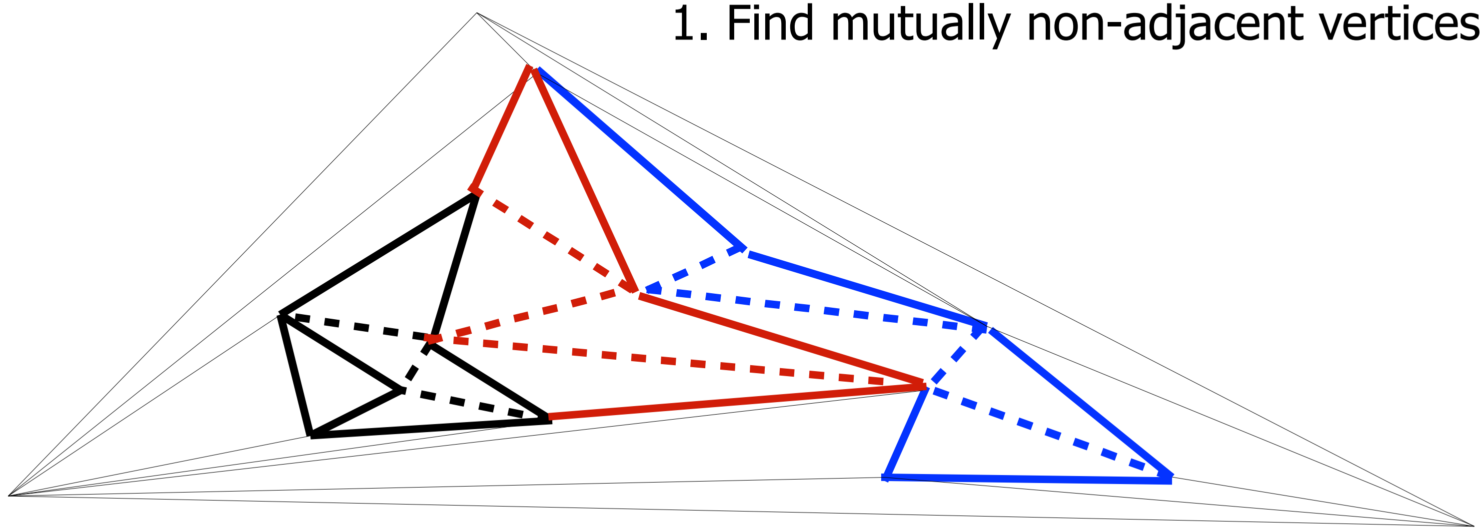
1. Find mutually non-adjacent vertices H



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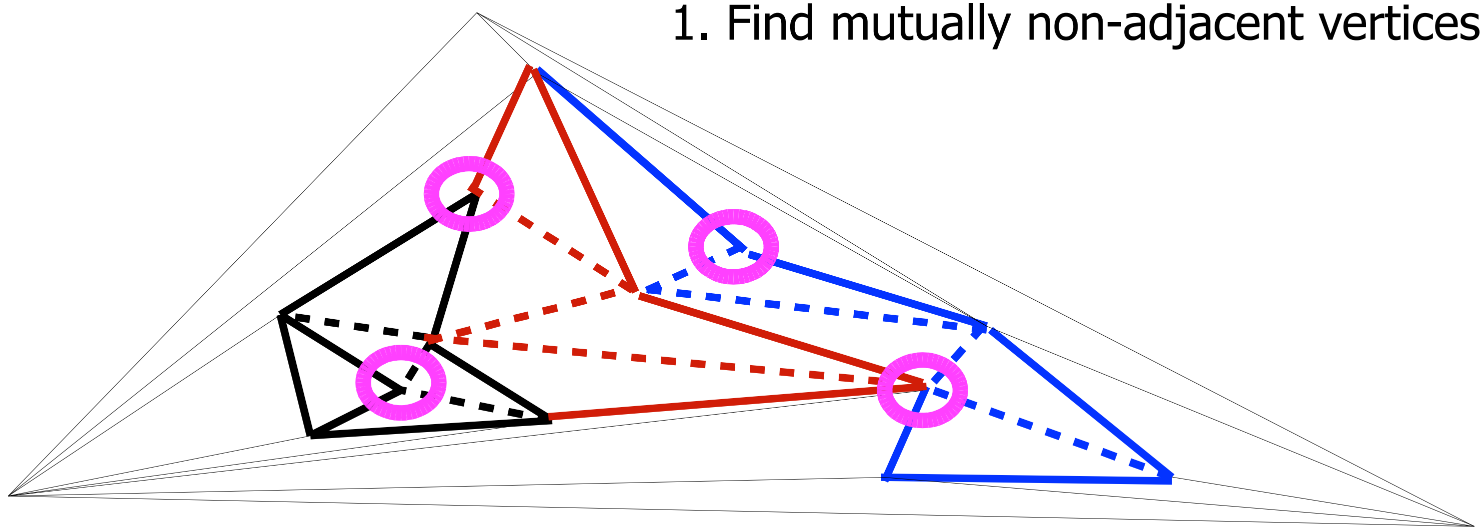
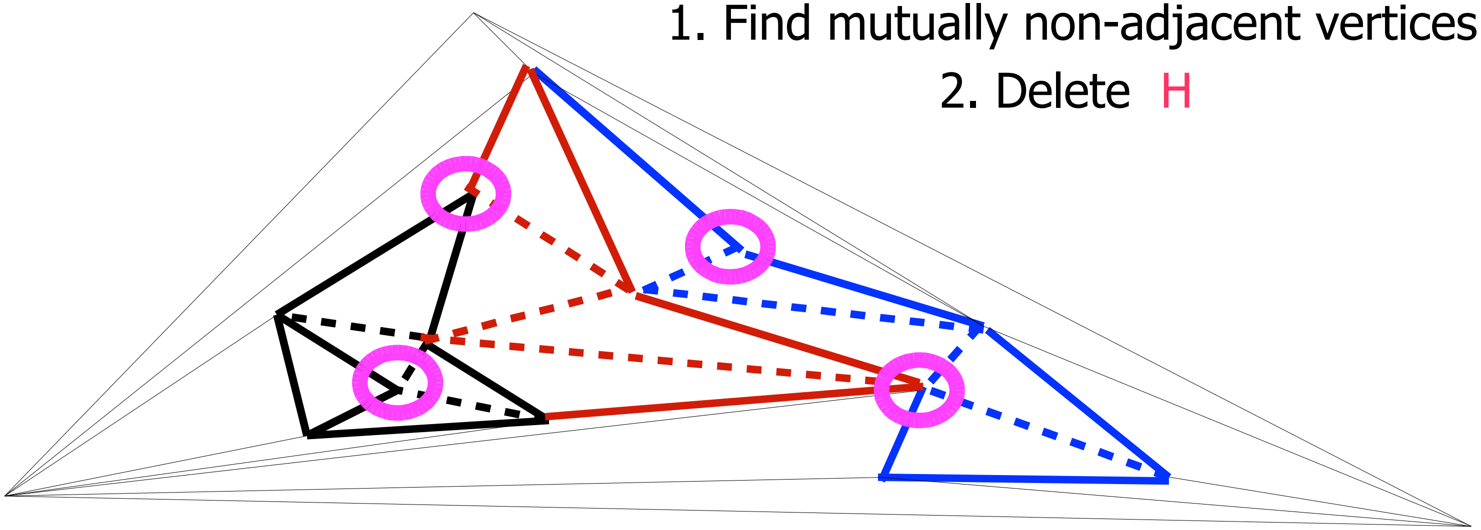


Corollary : S has n/18 mutually non-adjacent vertices H with degree at most 9

Outline of Kirkpatrick Planar Point Location

Given a triangulated planar subdivision S inside a bounded triangle

1. Find mutually non-adjacent vertices H
2. Delete H



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Given a triangulated planar subdivision S inside a bounded triangle

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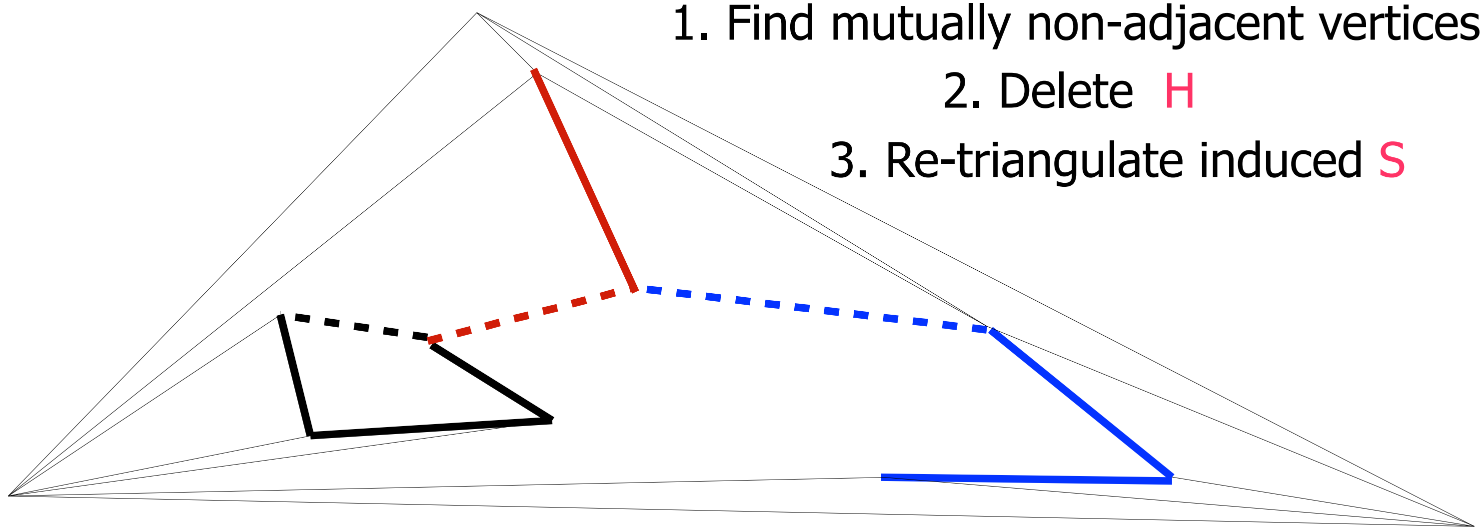
3. Re-triangulate induced S



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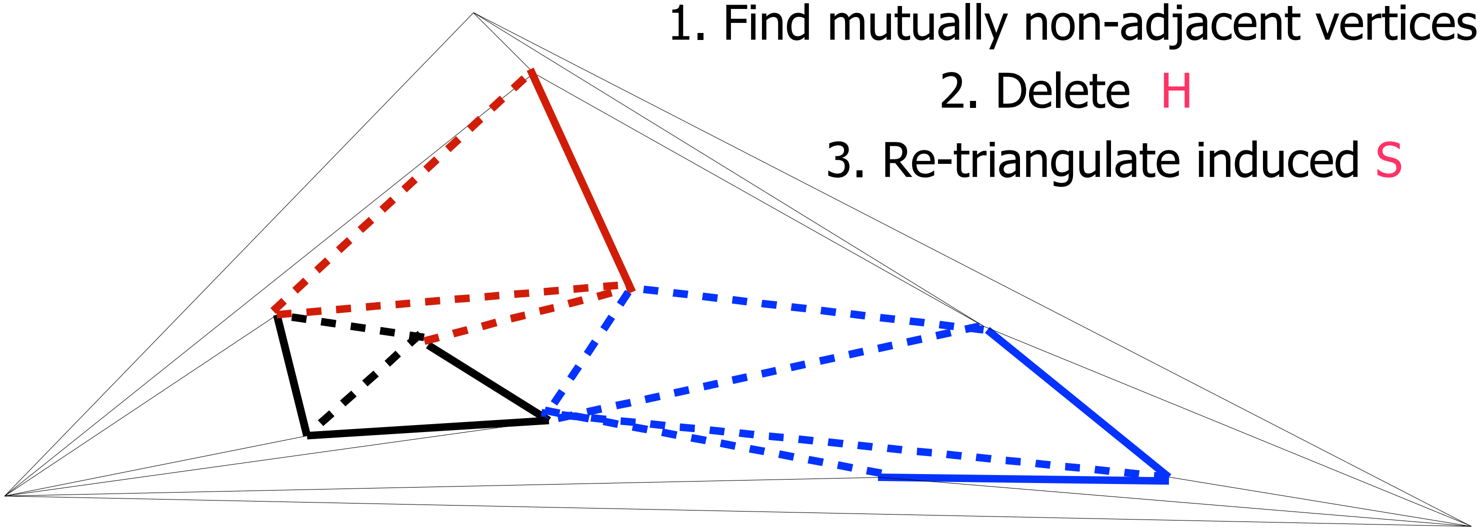

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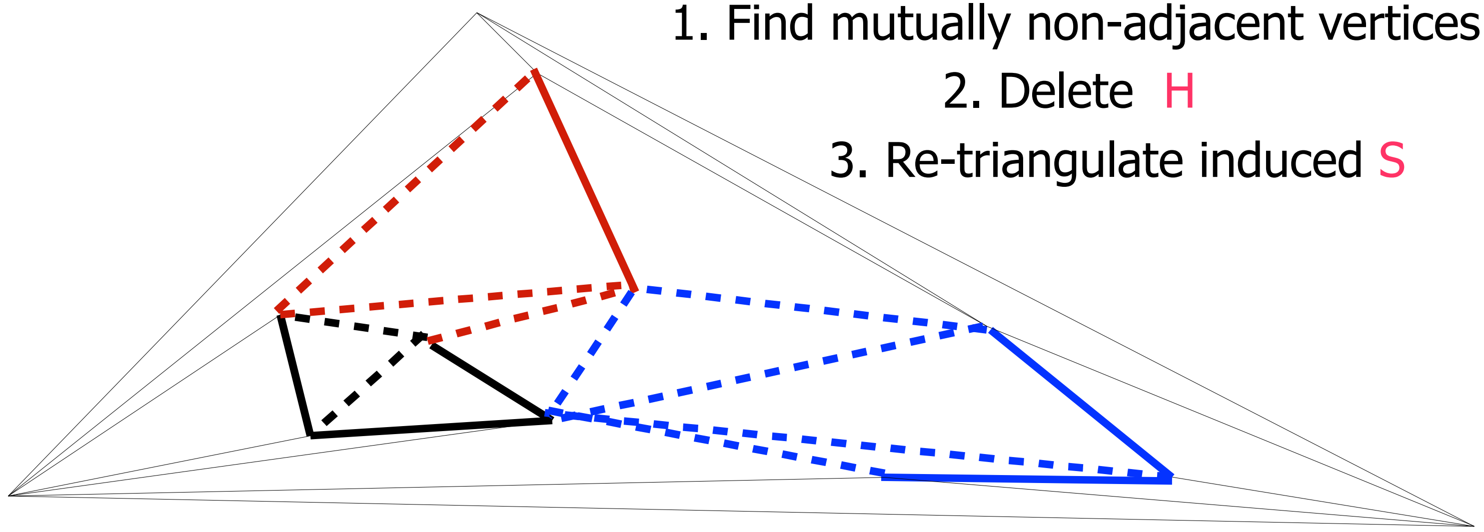

Recursively do the same until only the three outside vertices are left

Corollary : S has n/18 mutually non-adjacent vertices H with degree at most 9

Outline of Kirkpatrick Planar Point Location

Given a triangulated planar subdivision S inside a bounded triangle

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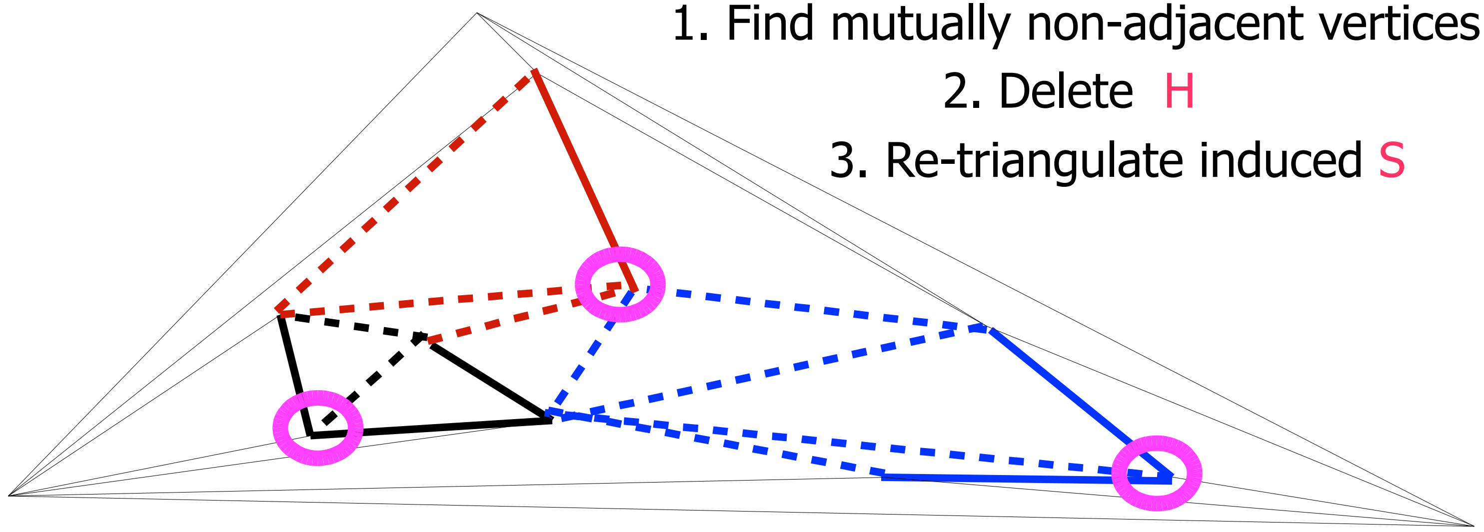

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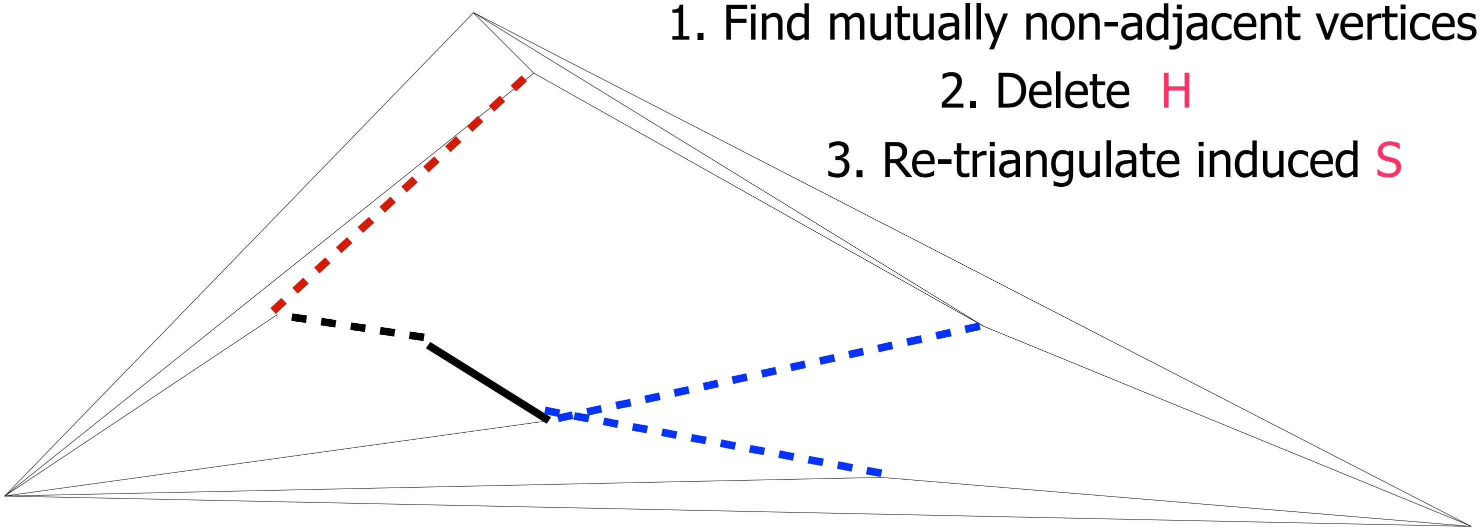

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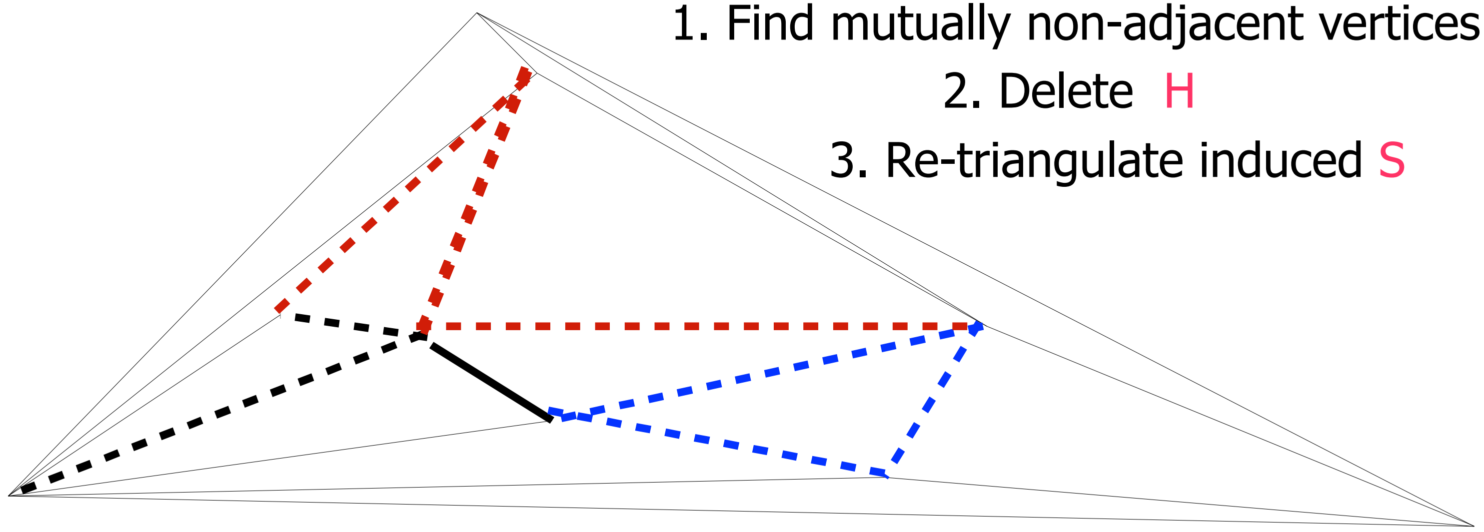

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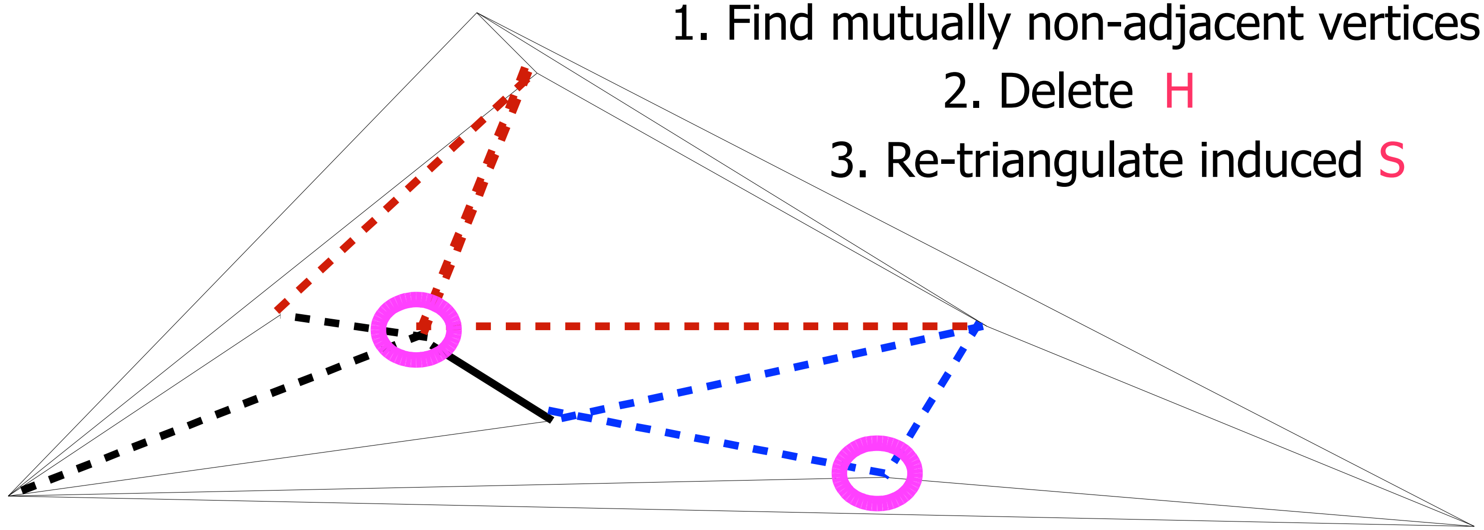

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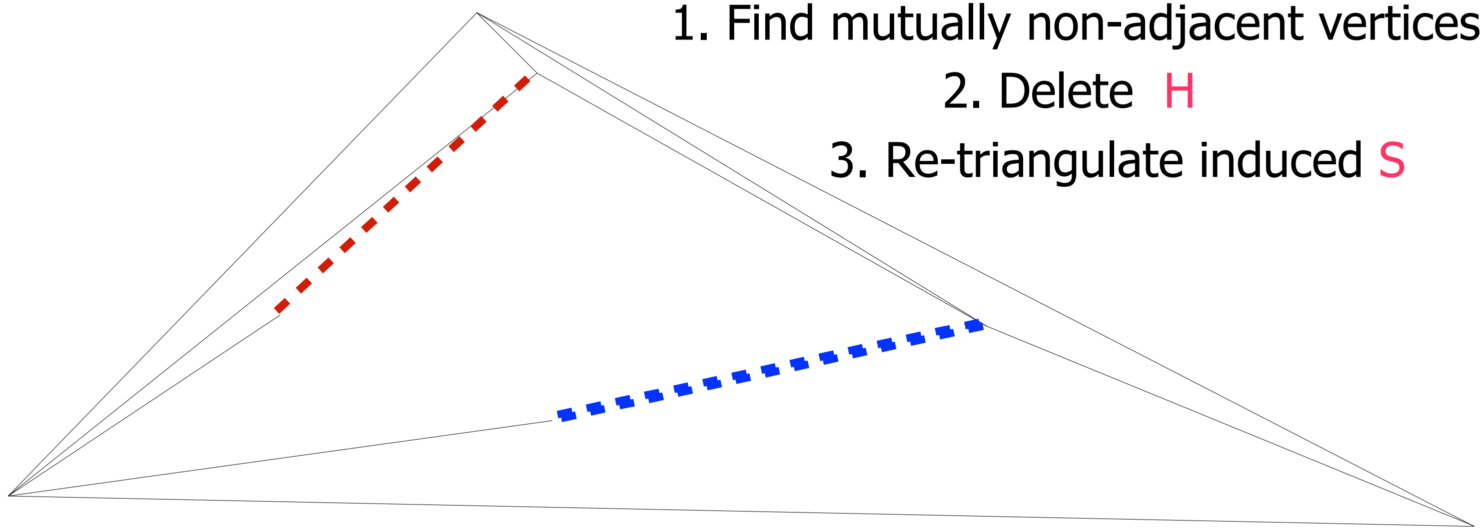

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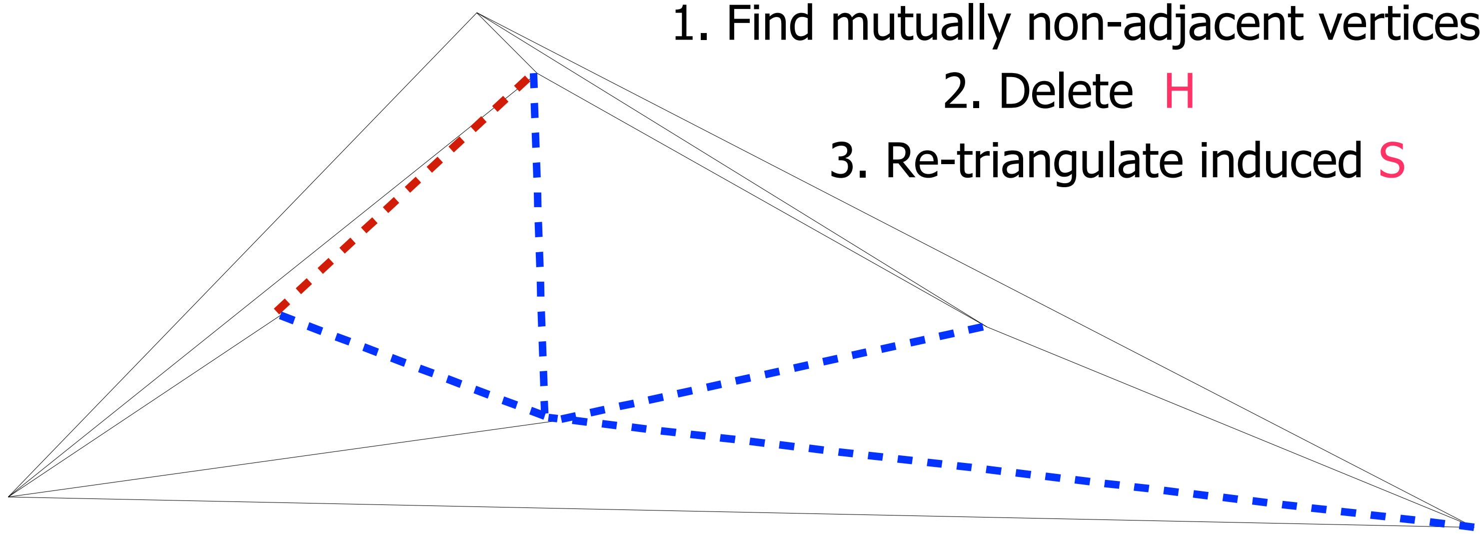

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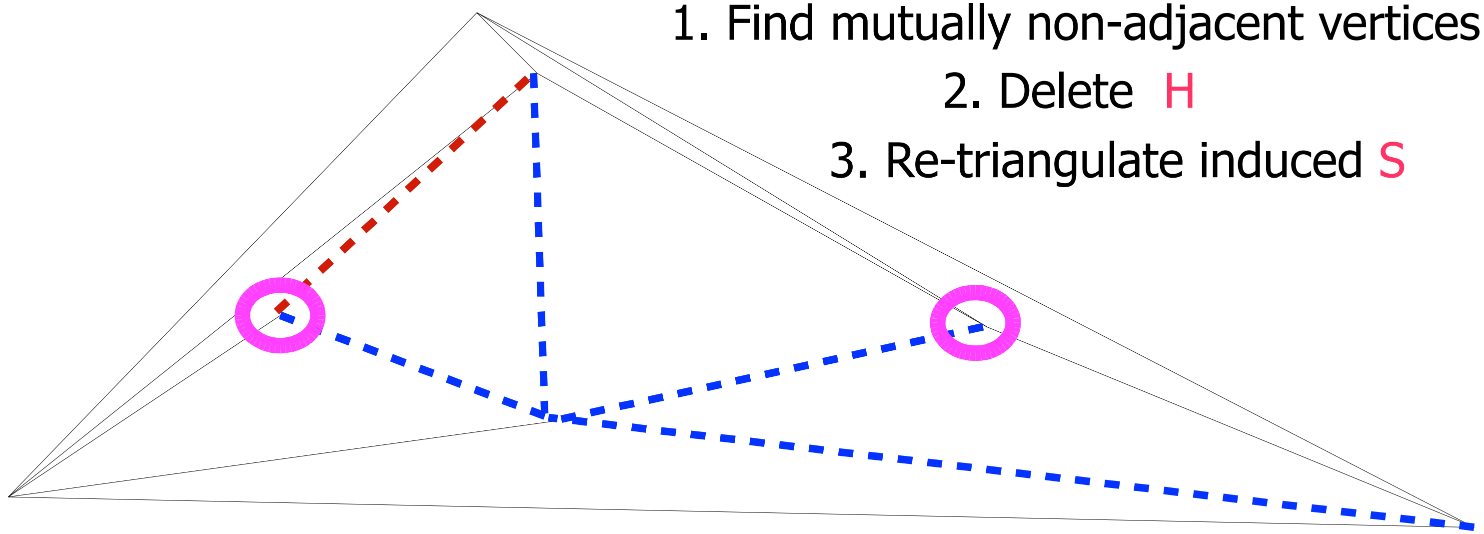

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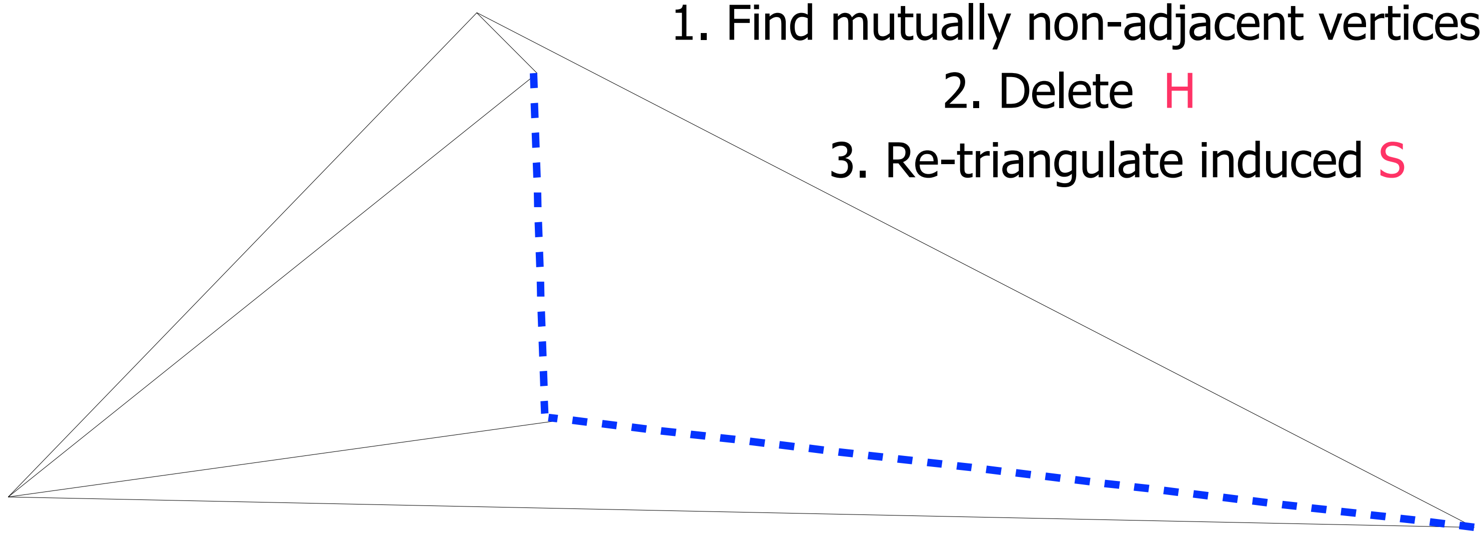

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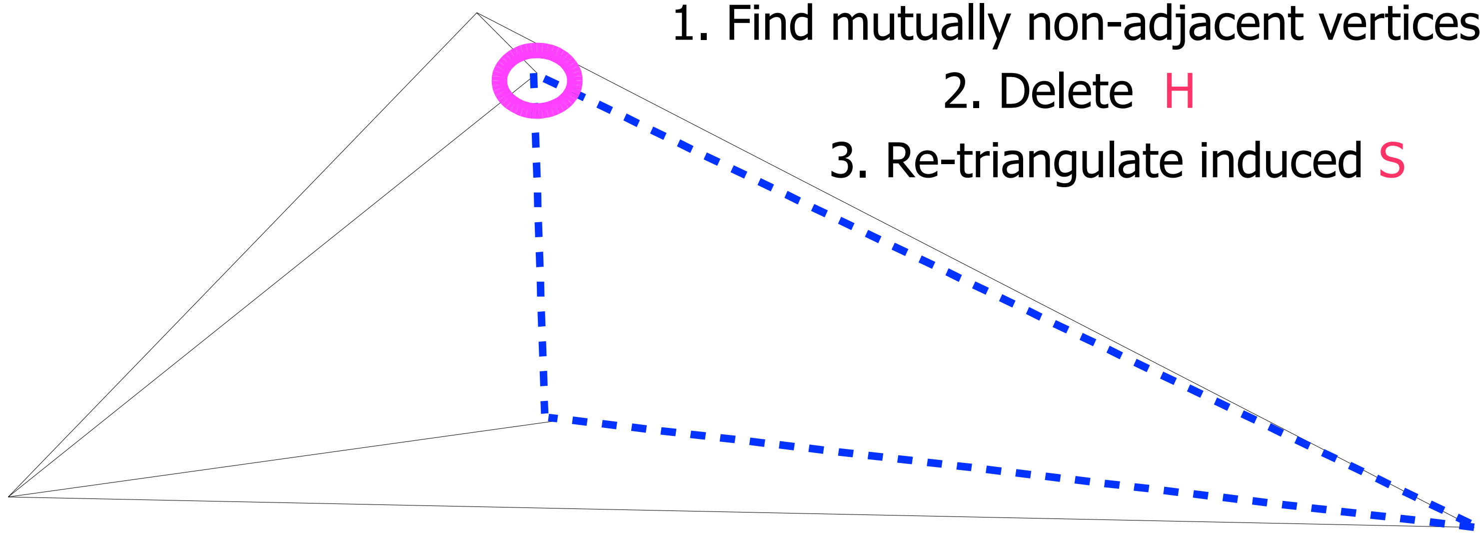

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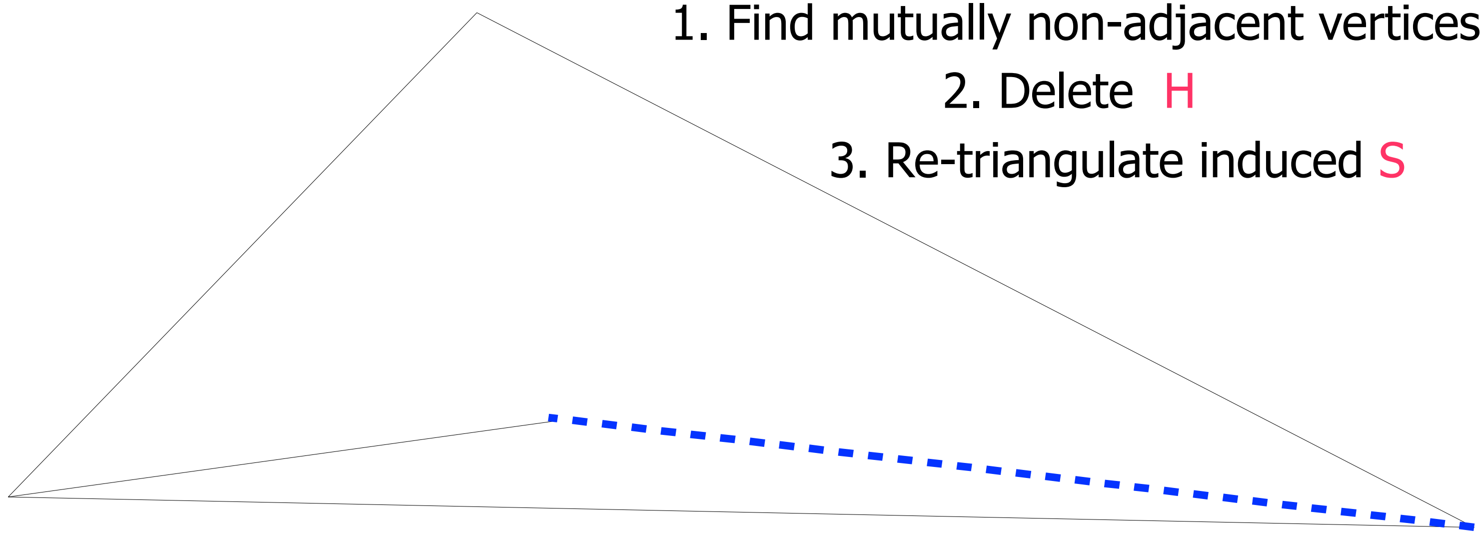

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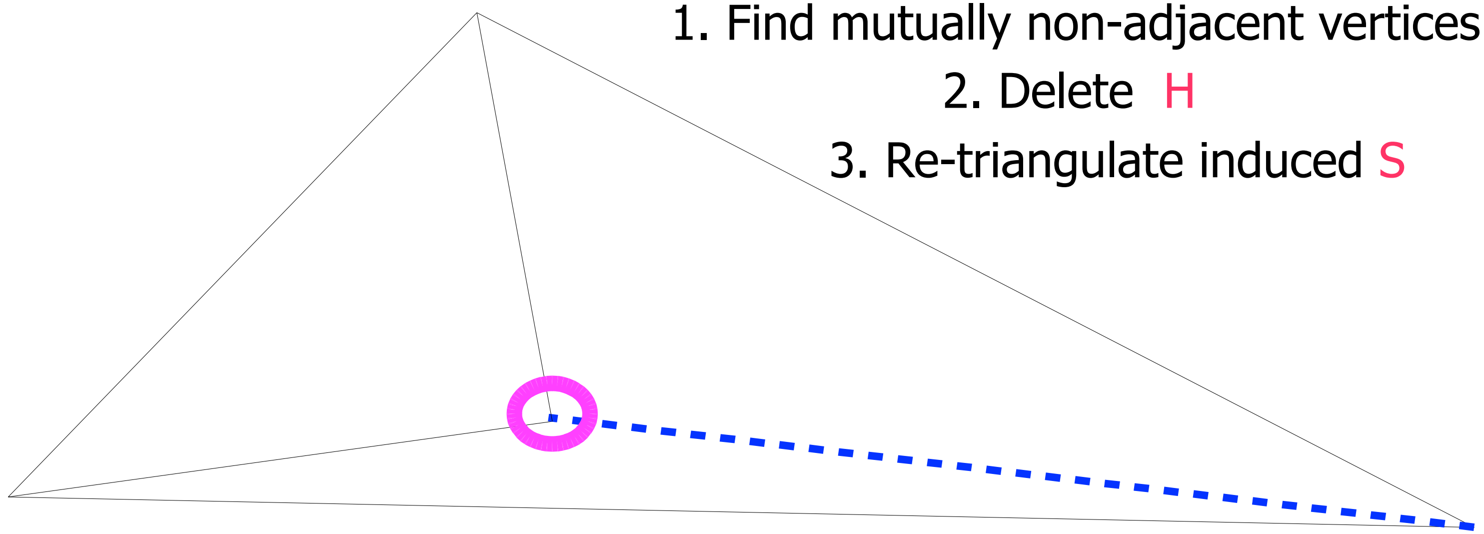

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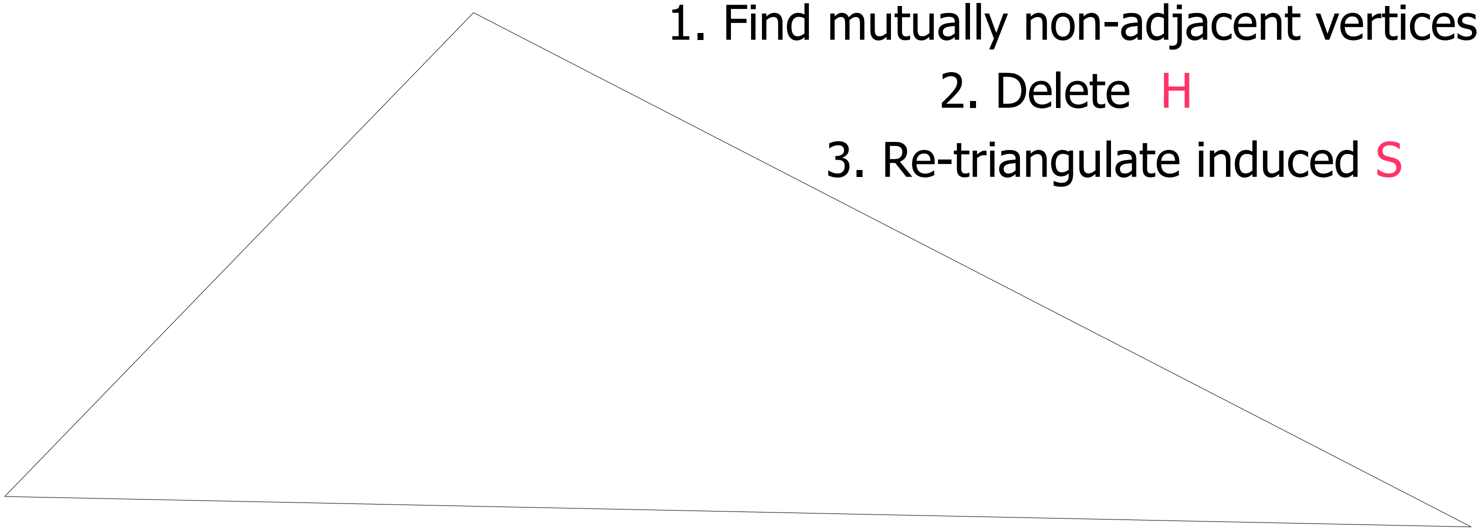

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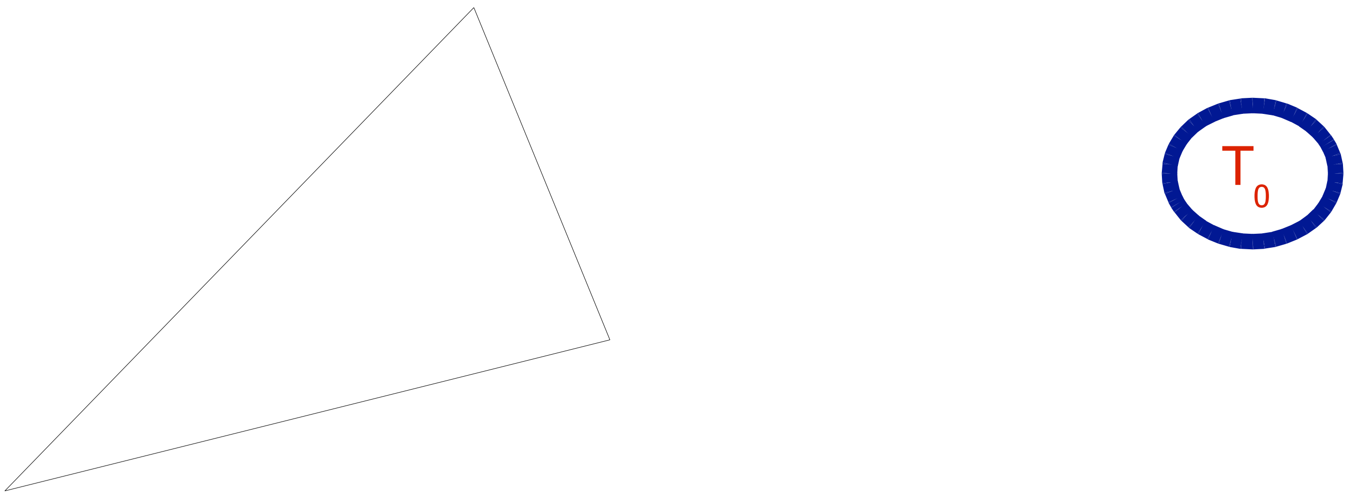

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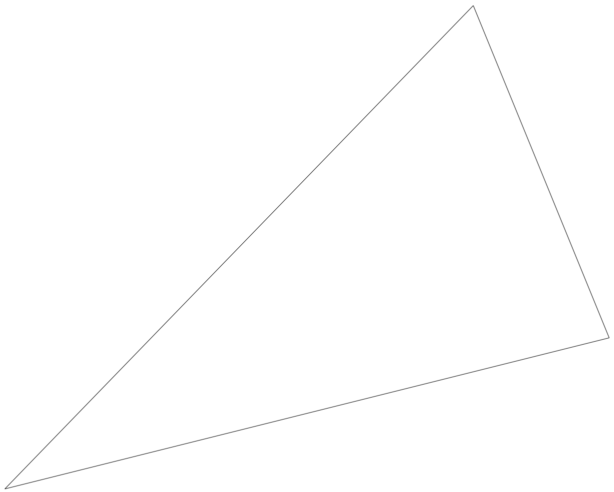

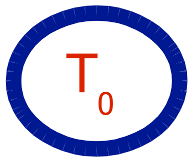

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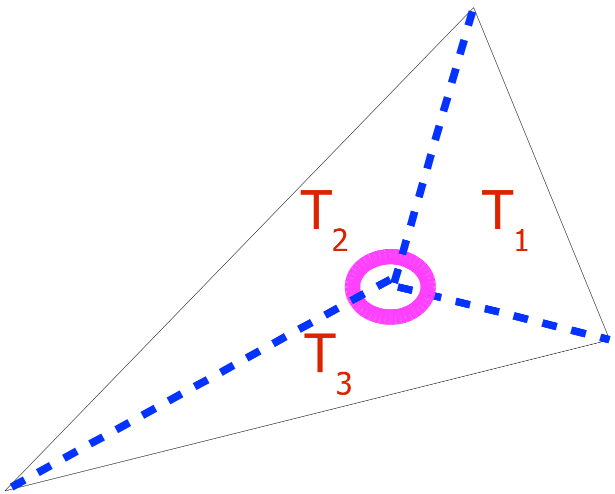

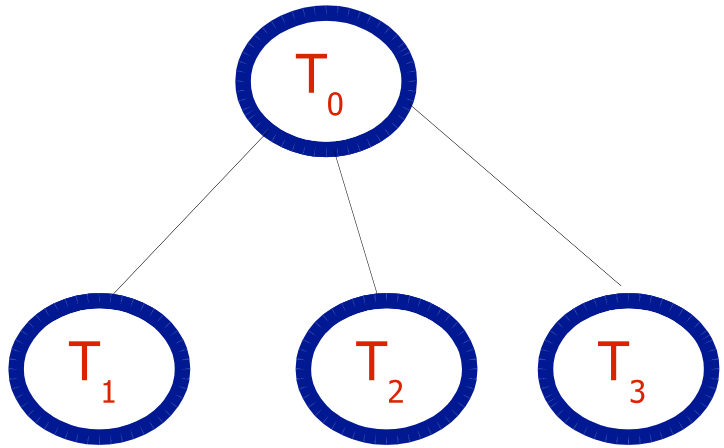

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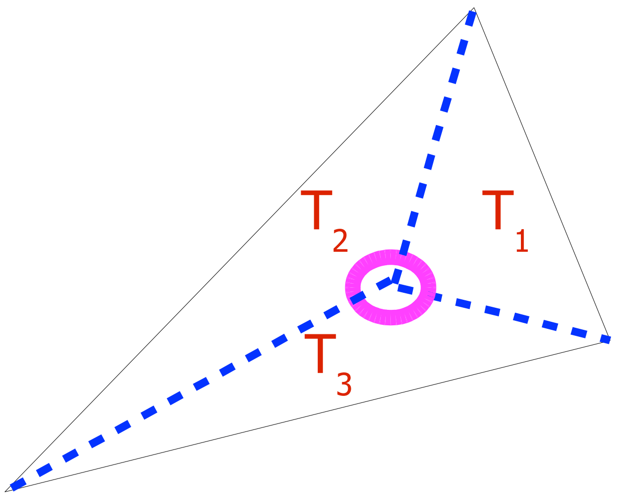

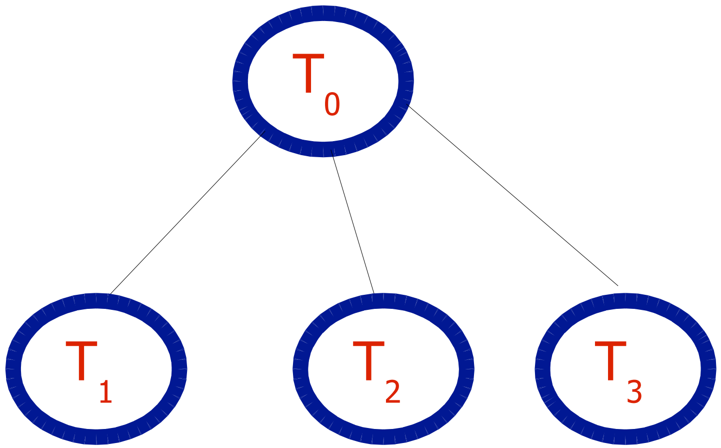

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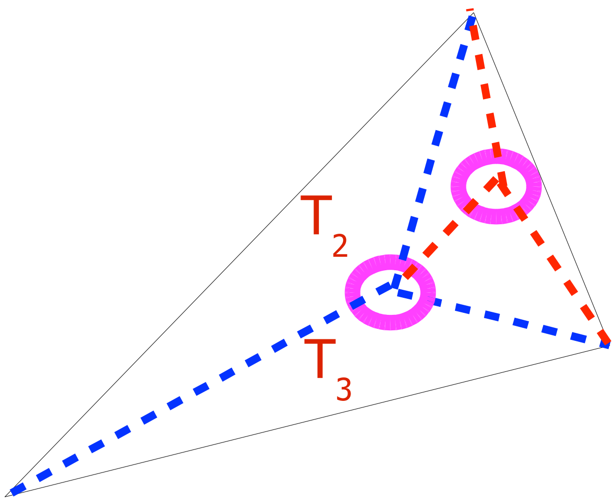

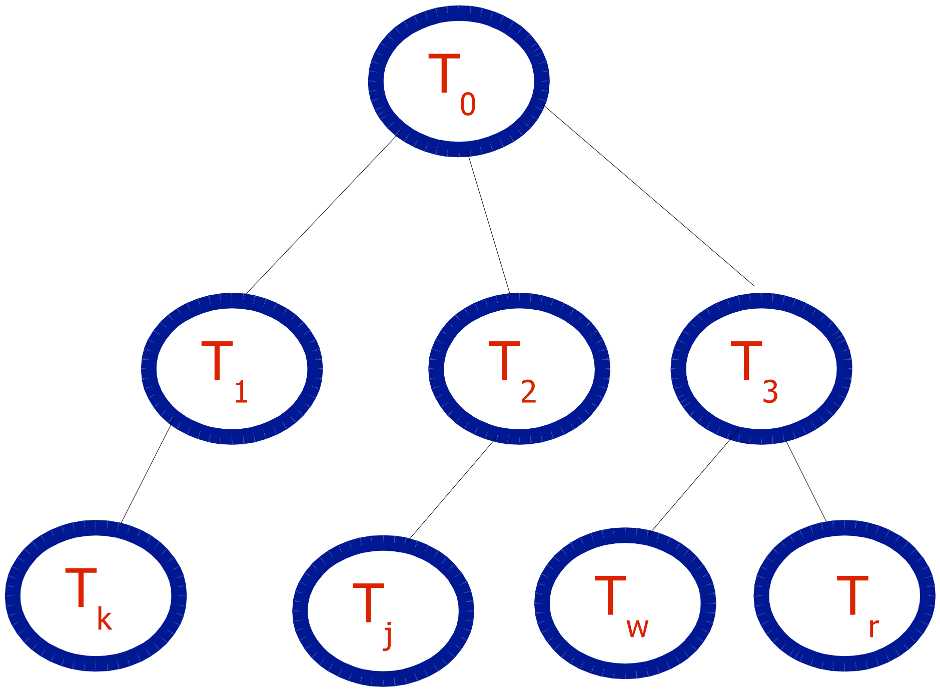

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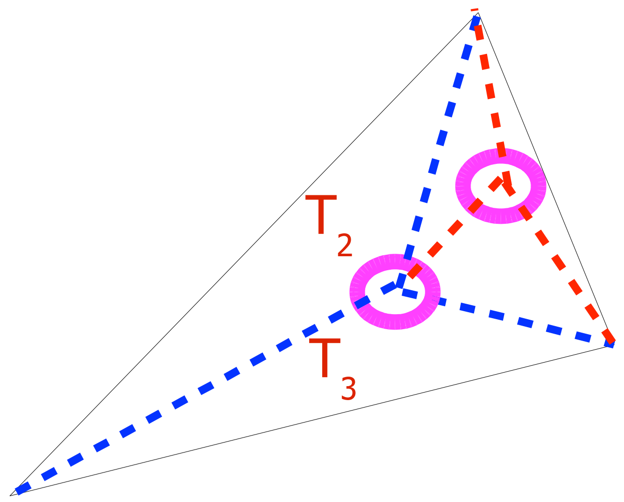

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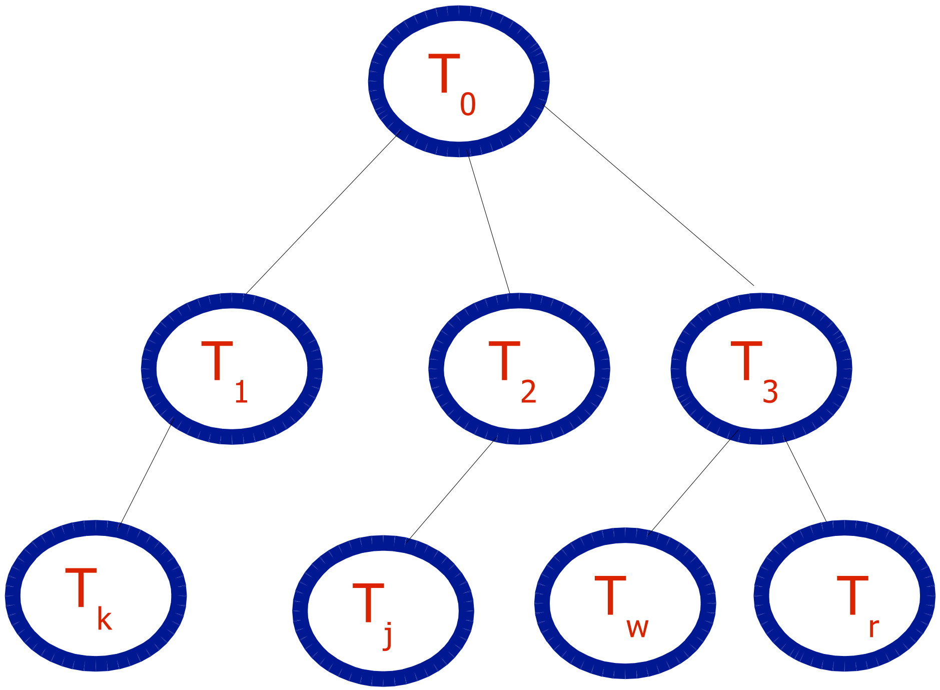

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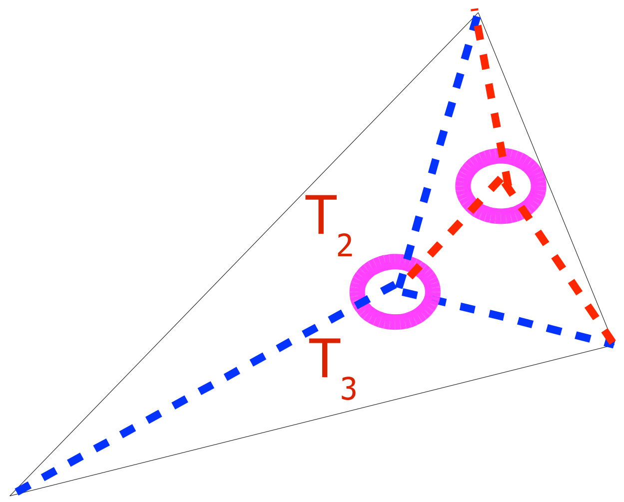

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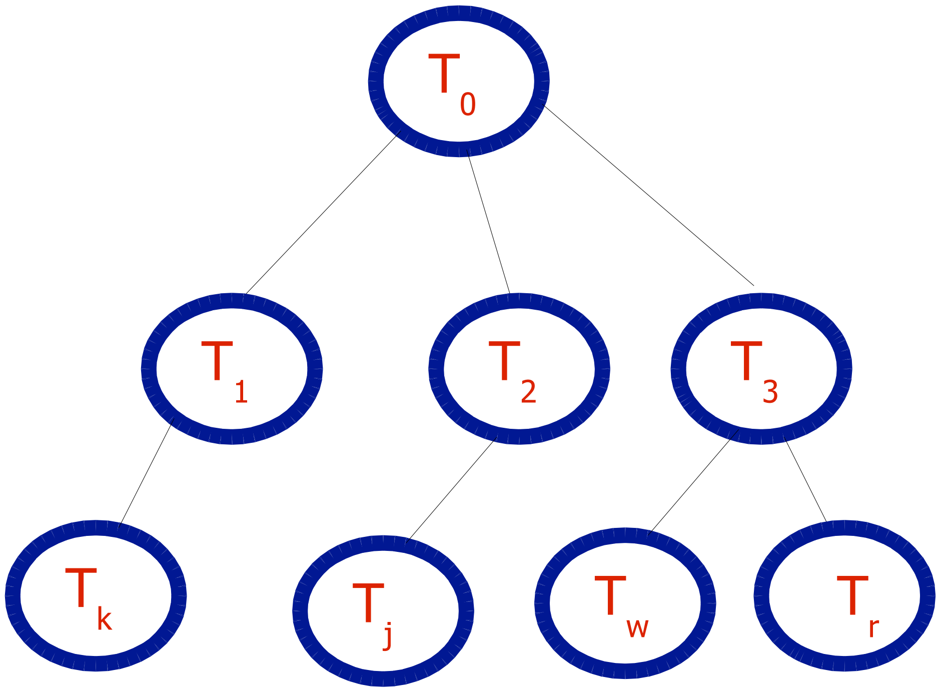

Looks like a: Tree

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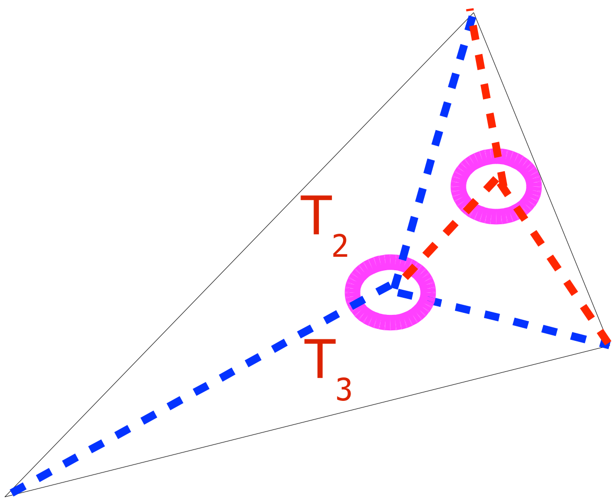

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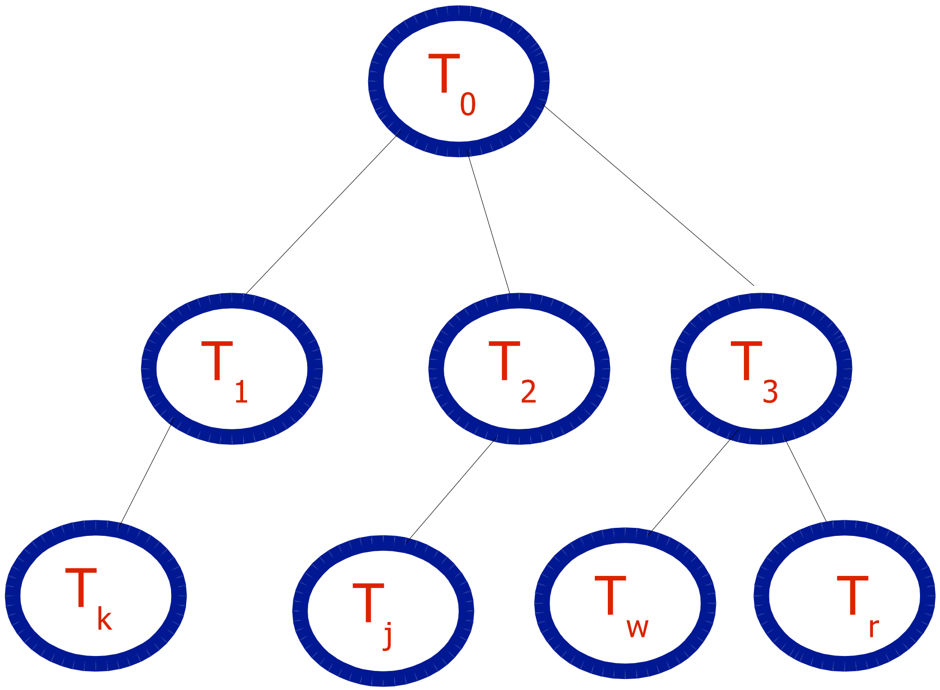

Looks like a: Tree

We name it :

Point Location Tree (PLT)

Characterization of PLT

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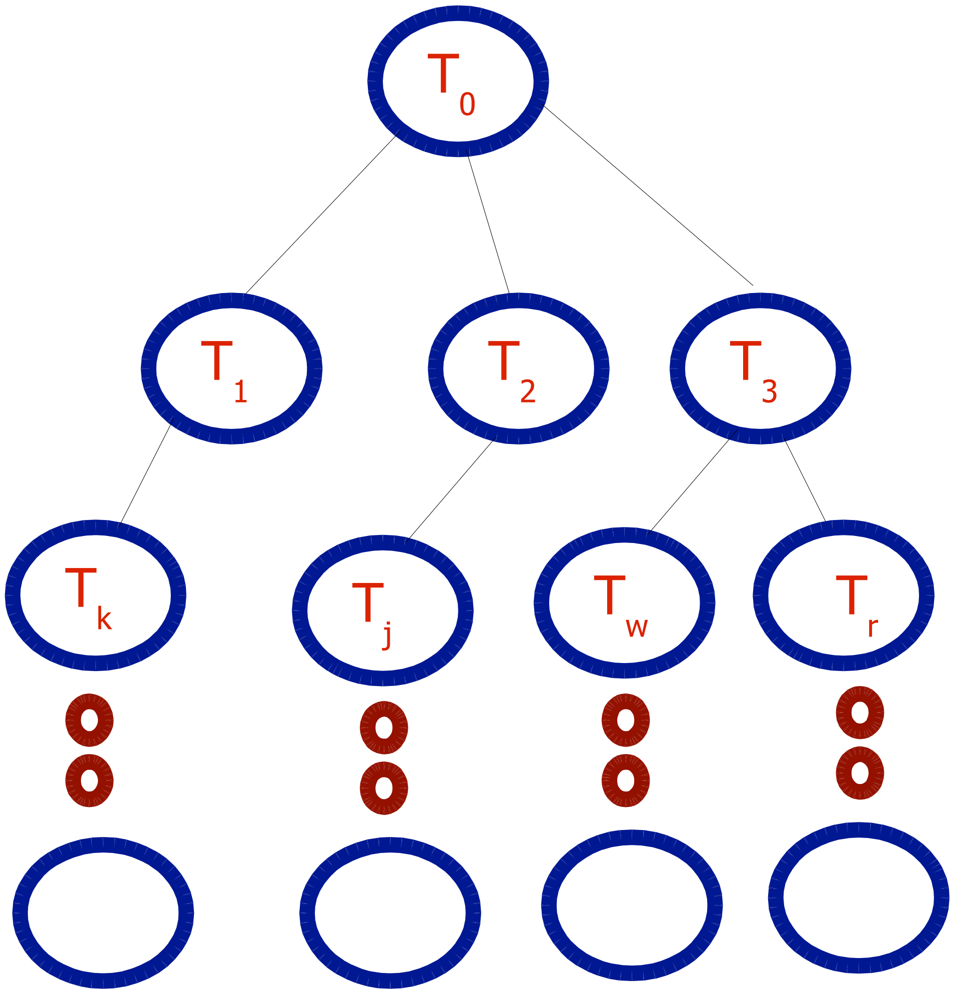

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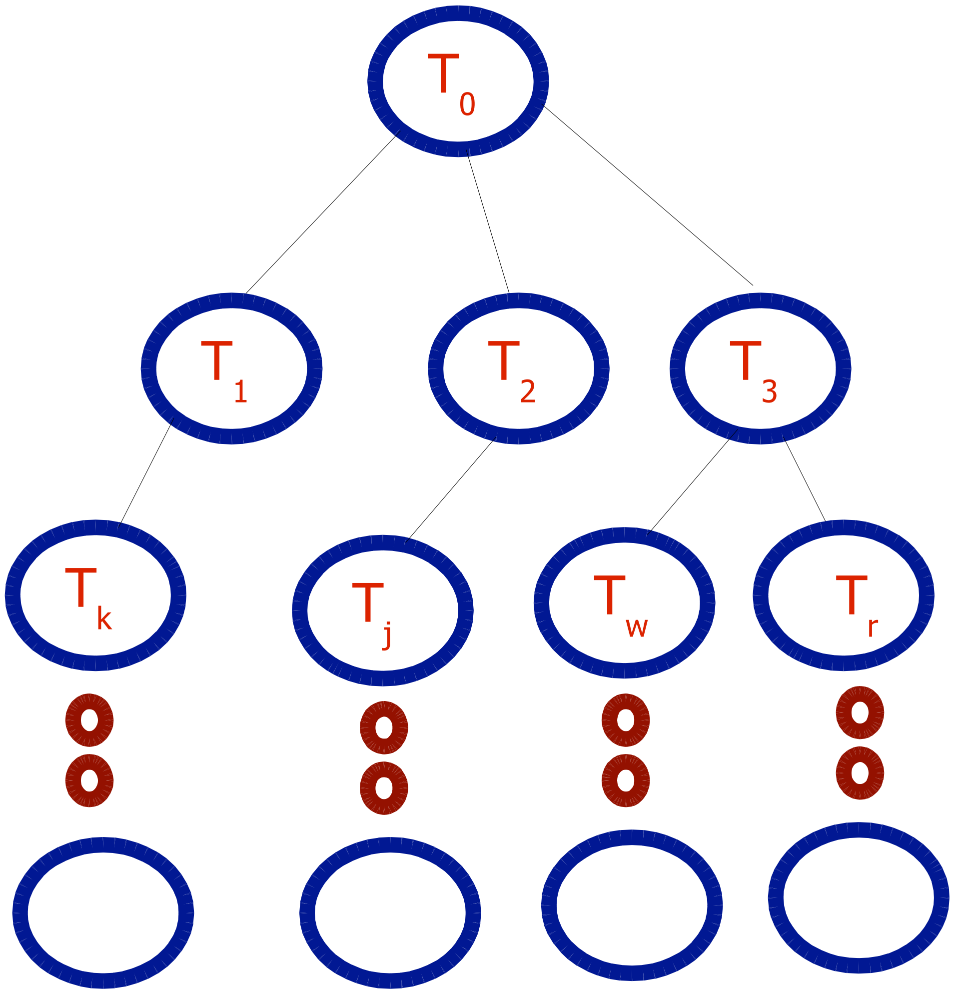

1. Each nodes of PLT is a triangles interacting with O(1) leaf nodes which are also triangles

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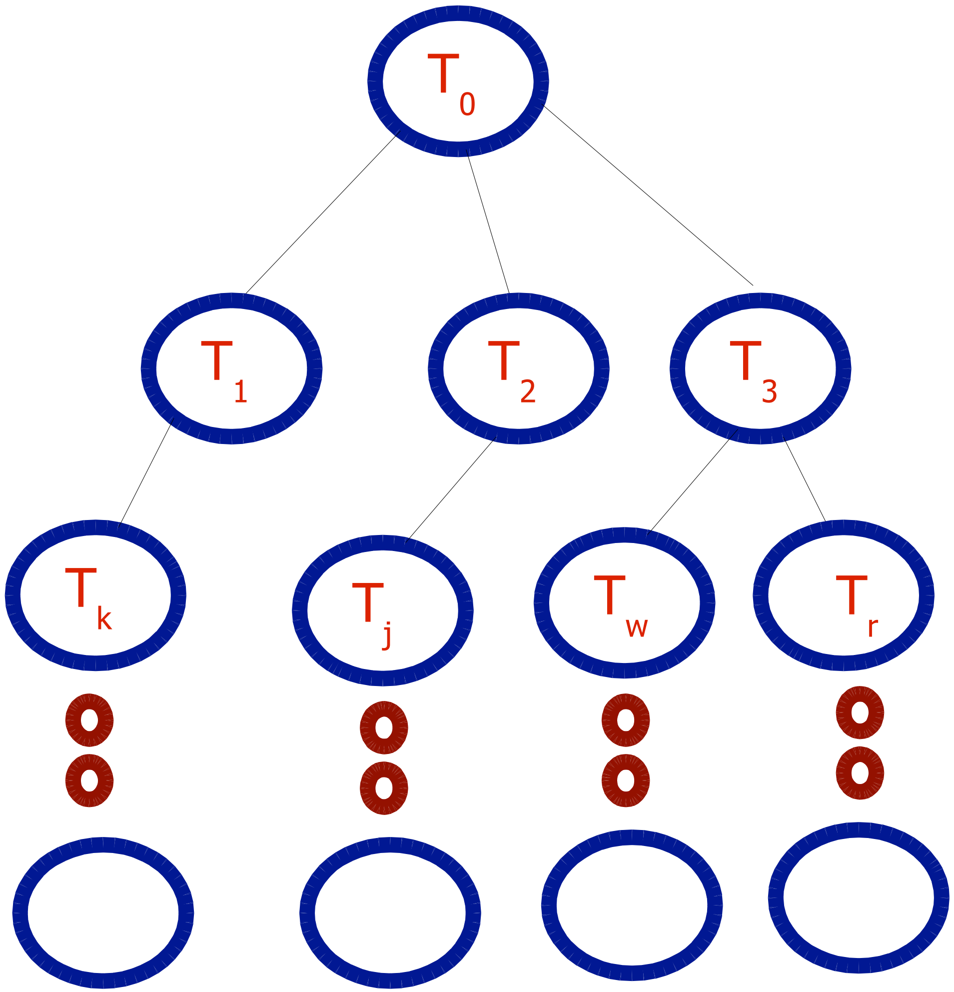

1. Each nodes of PLT is a triangles interacting with O(1) leaf nodes which are also triangles

2. Leaf nodes of the tree are the original triangles of the planar subdivision we started with.

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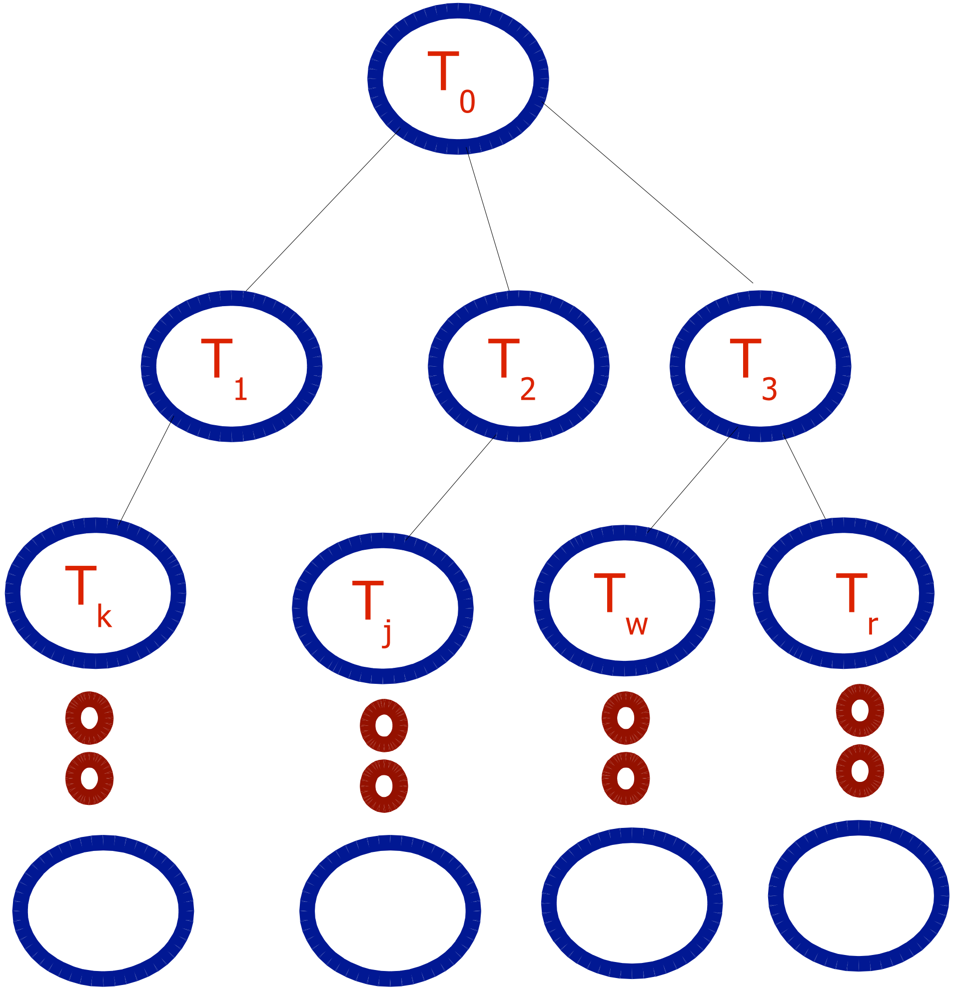

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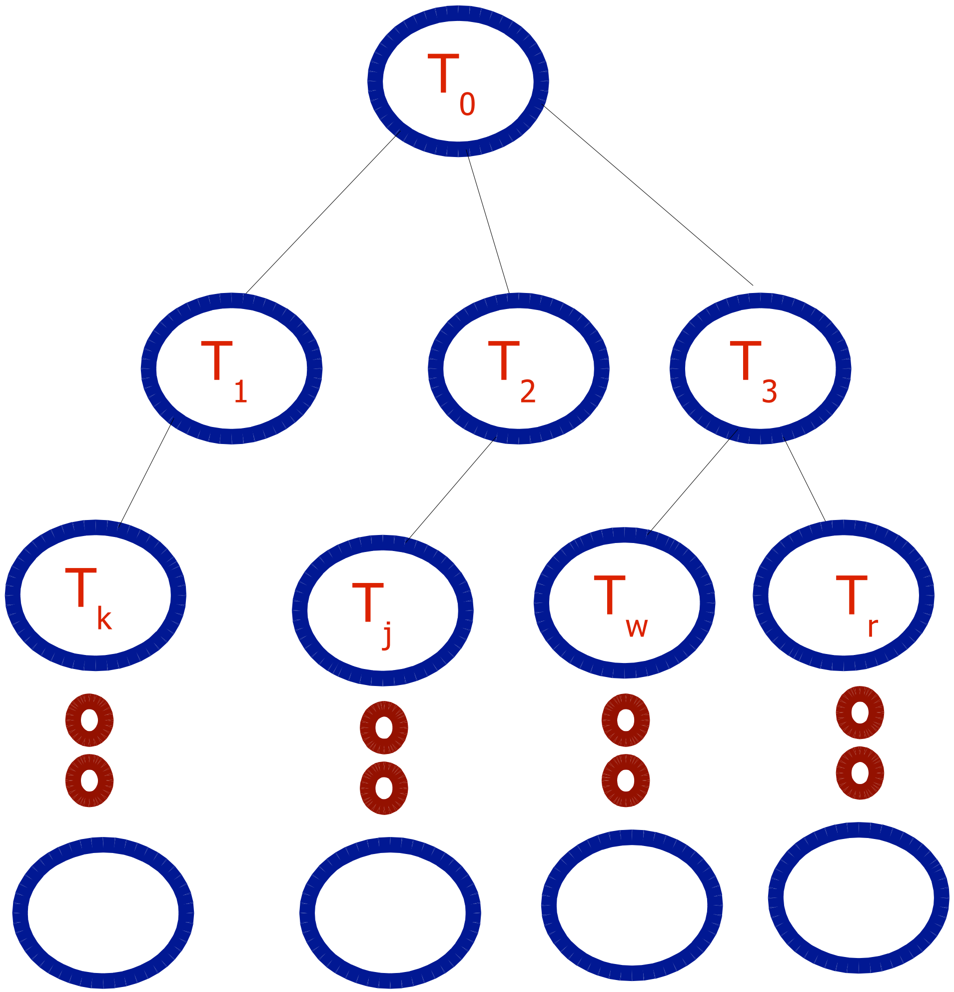

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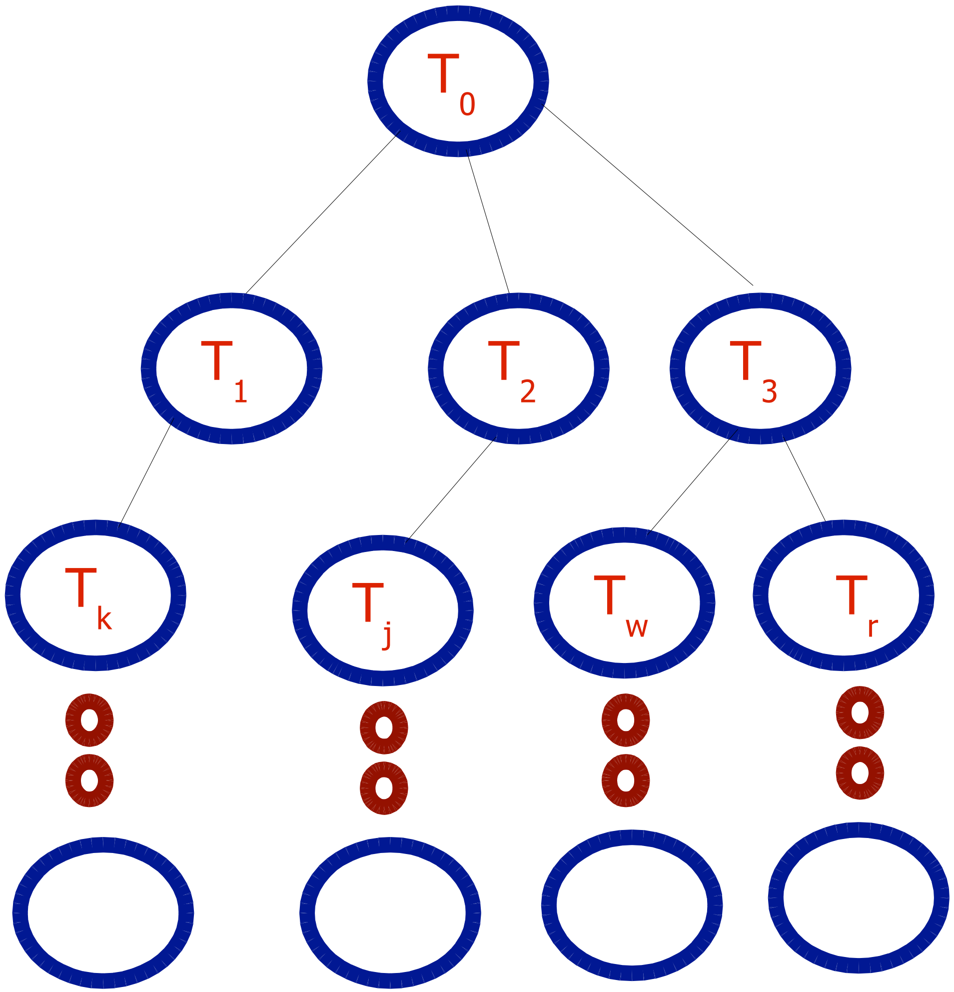

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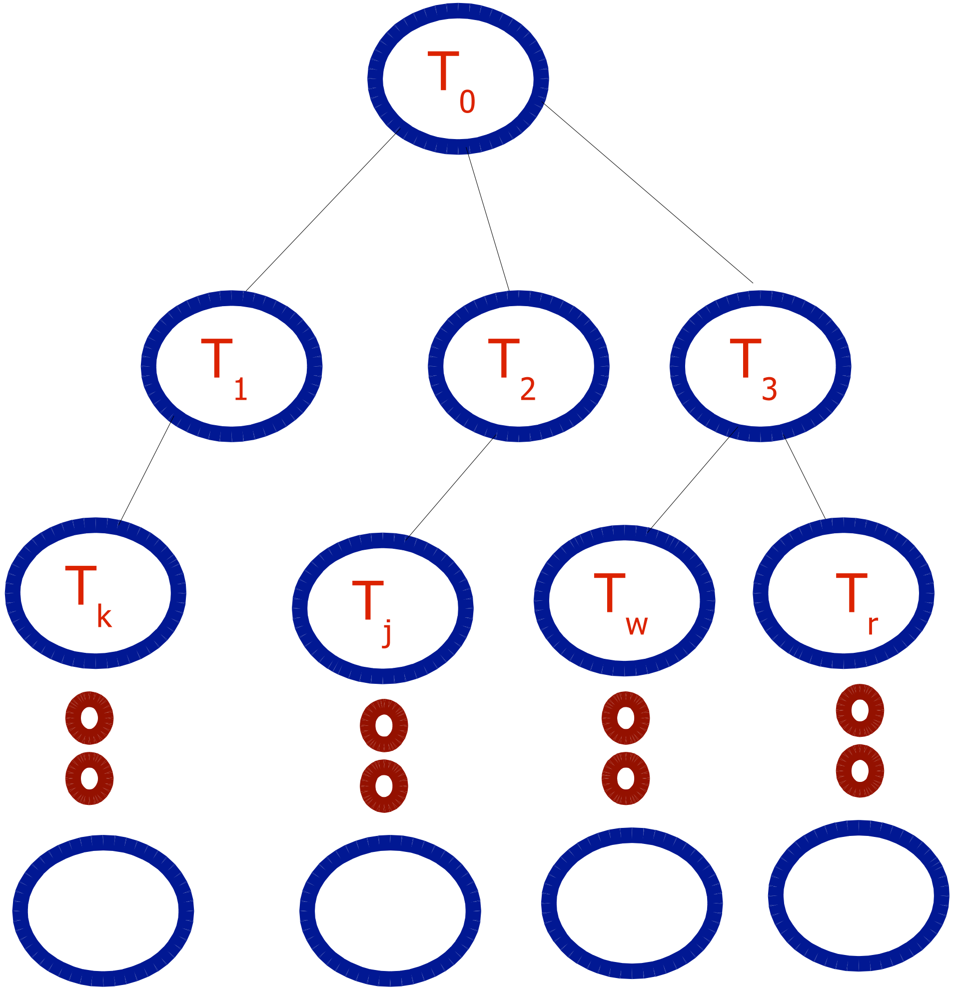

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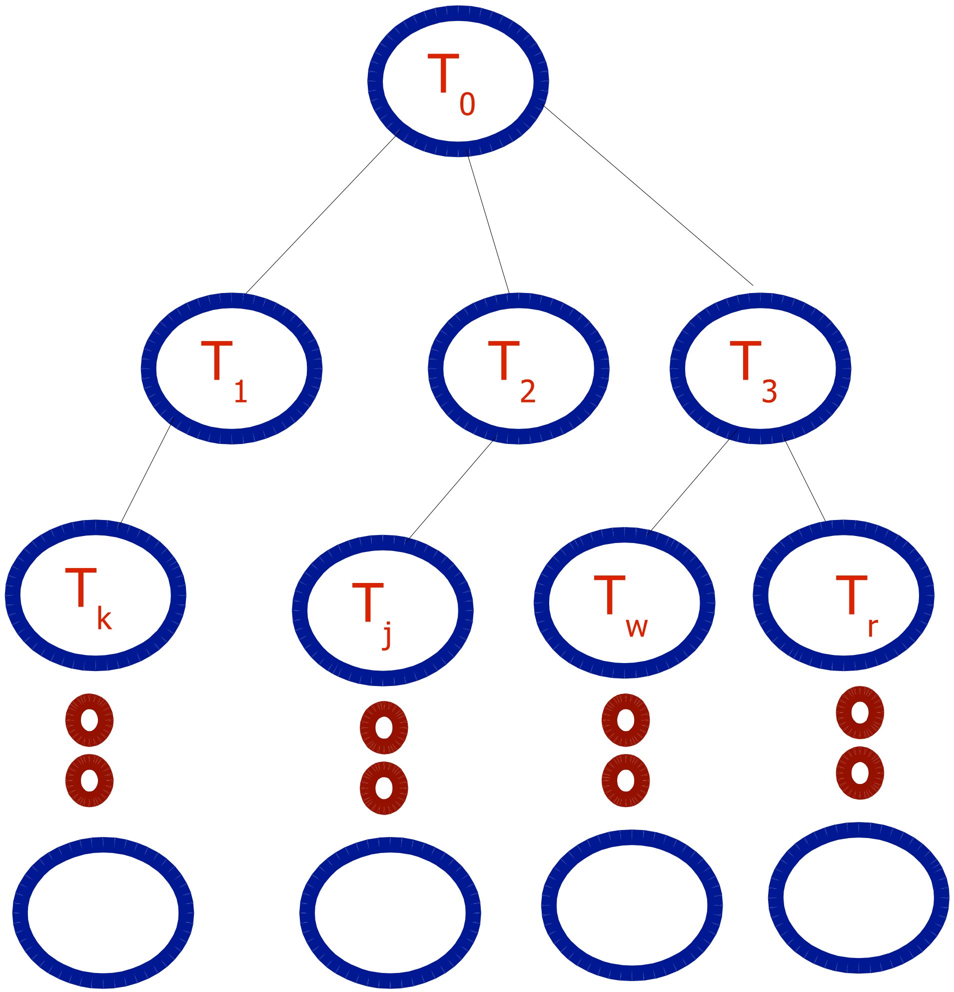

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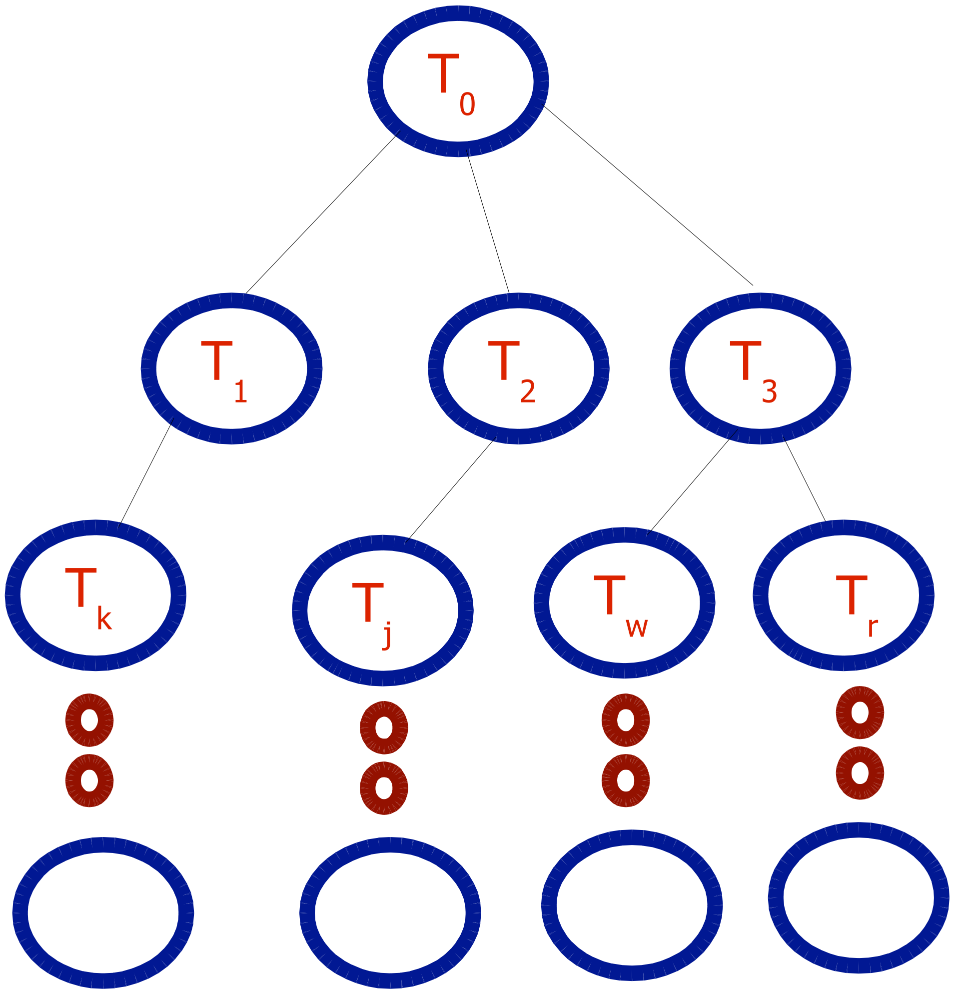
Searching a query point q in PLT



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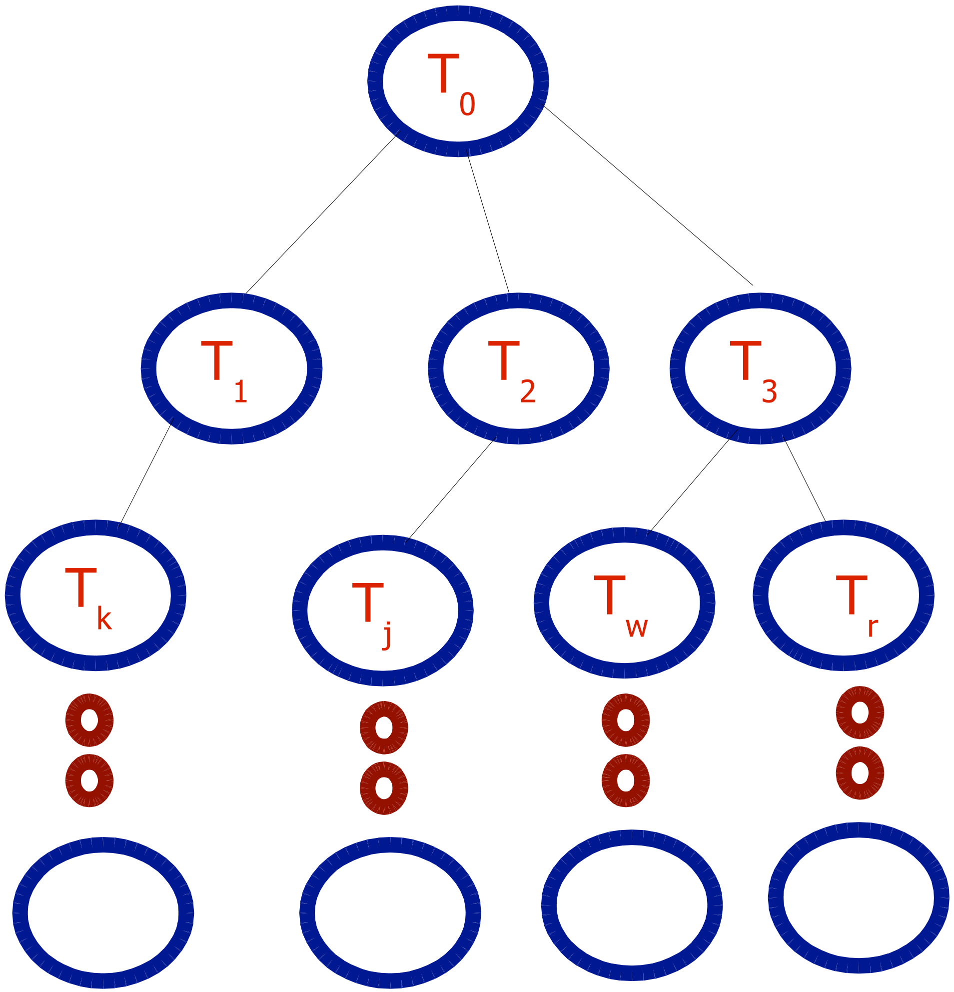

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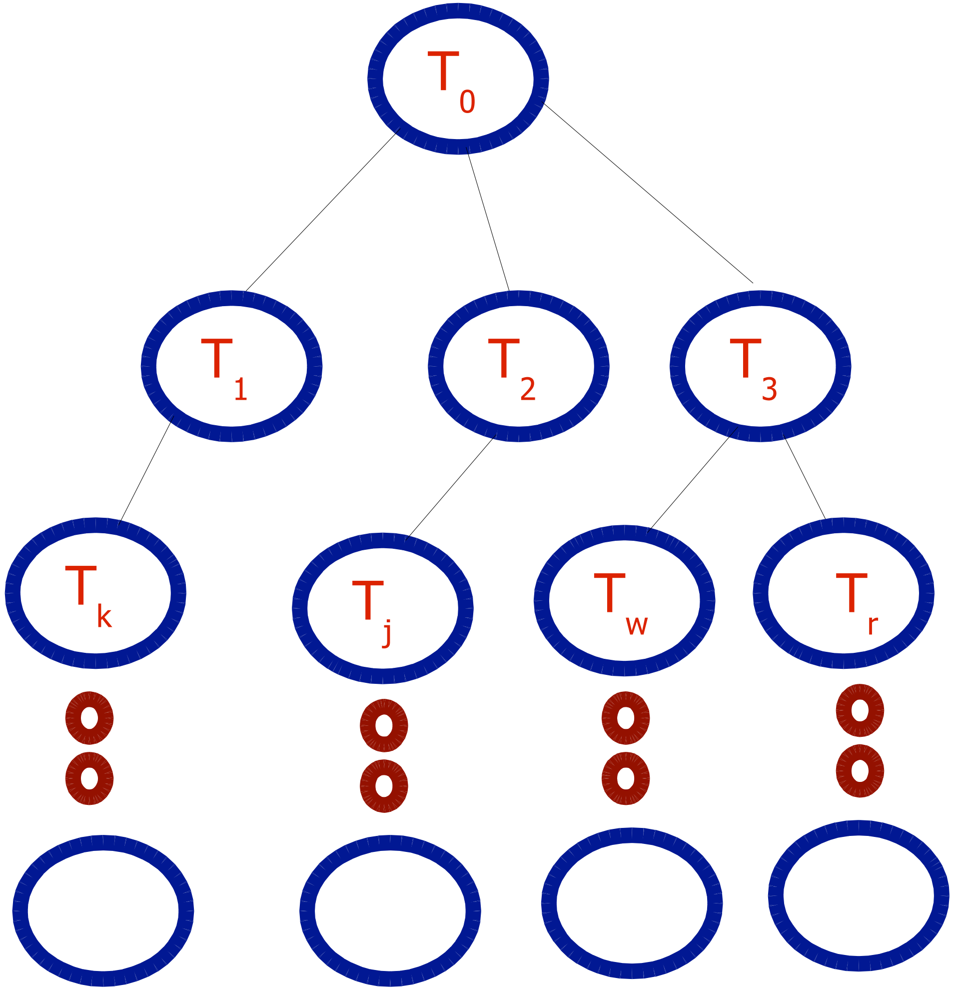

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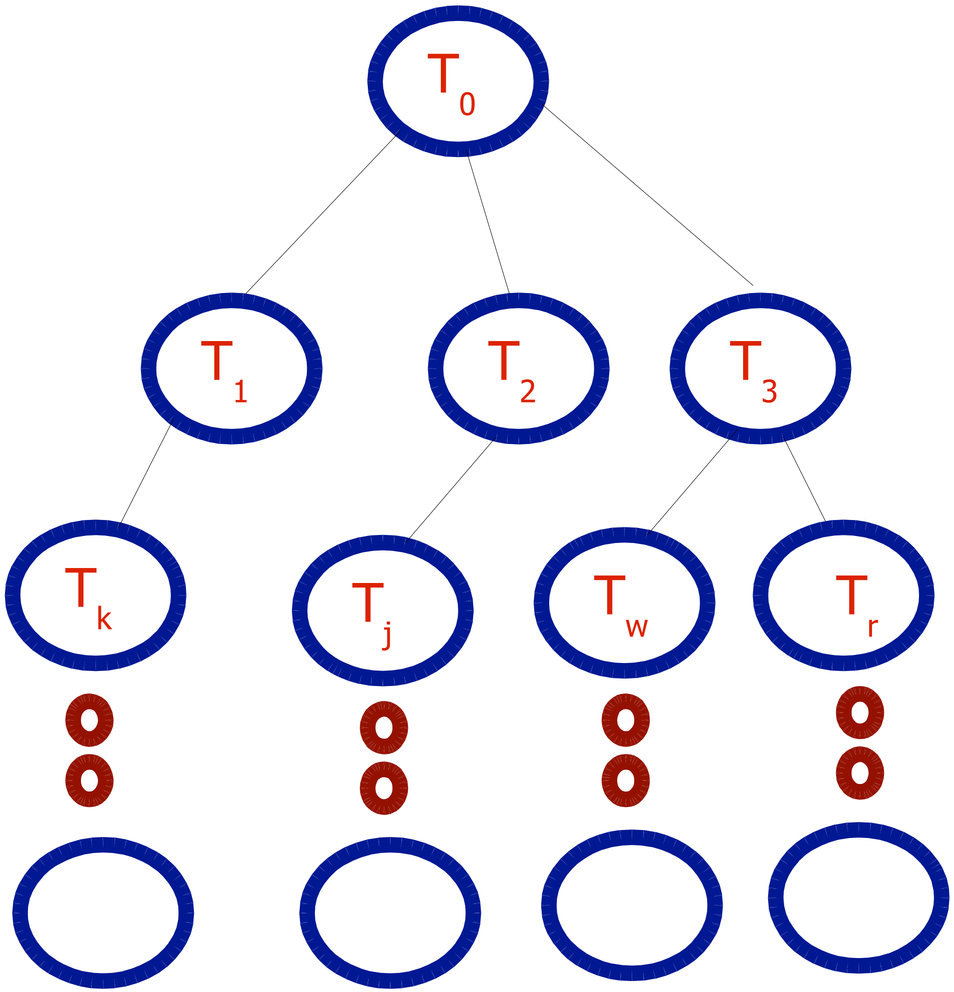

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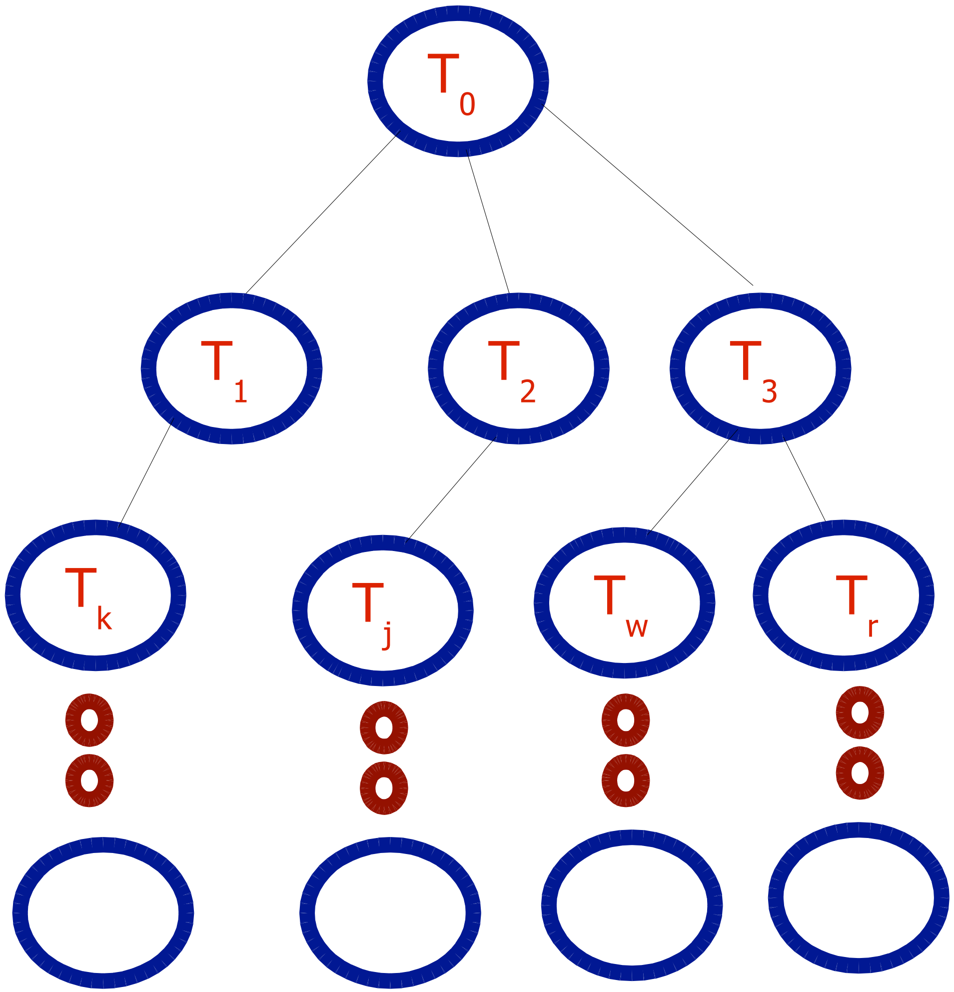
Searching a query point q in PLT



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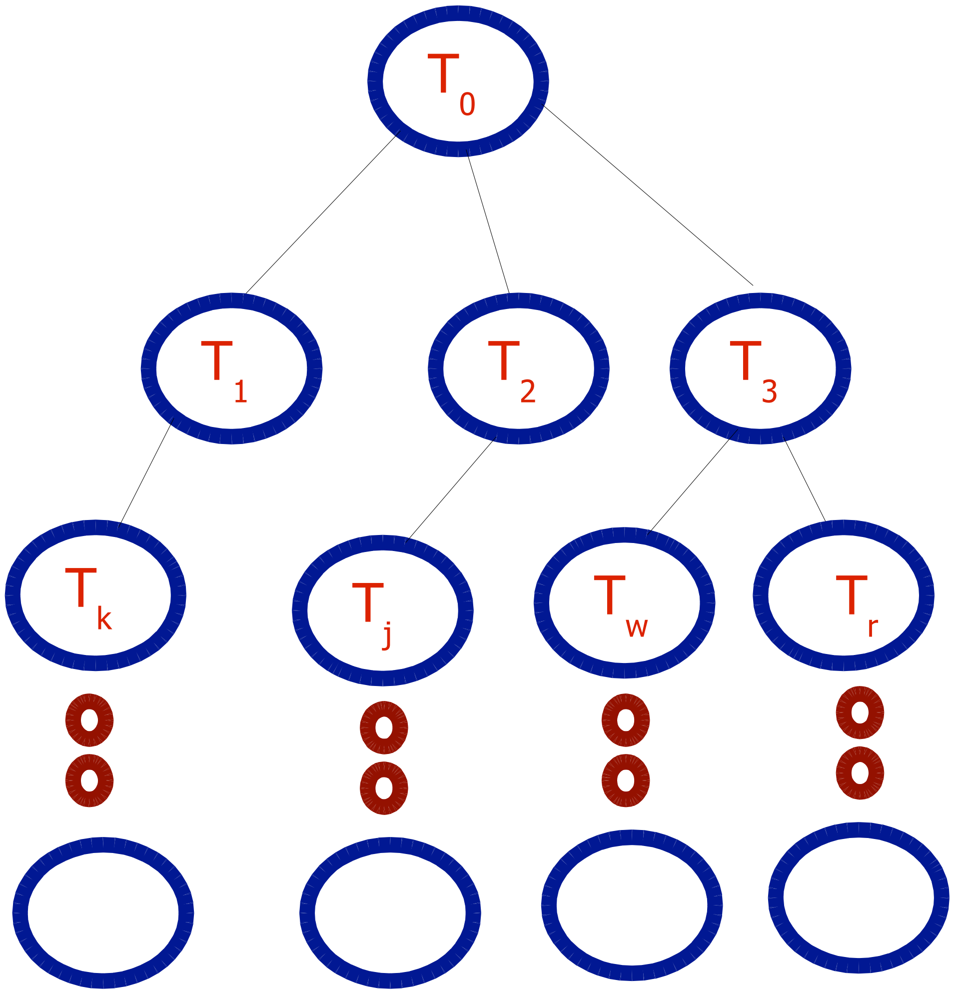

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Query time complexity: $O(\log n)$

Organization of the Talk

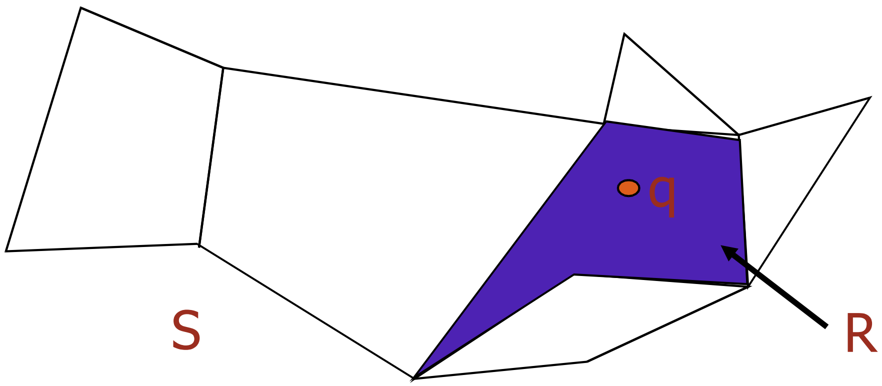
1. Preliminaries, Generic definition and Literature

2. Some technical details of easy versions

3. Conclusion

Planar Point Location

For a given a planar subdivision S



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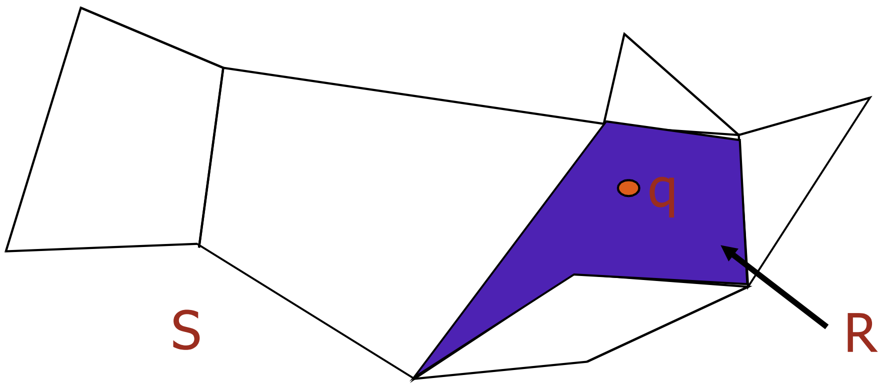
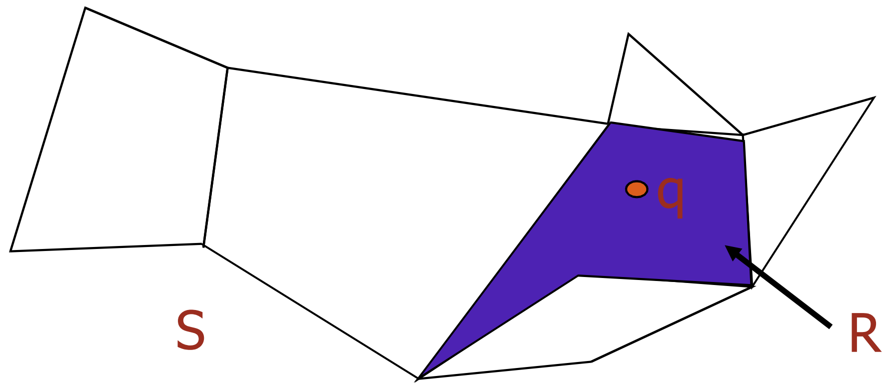


S can be preprocessed such that:

For any query point q,

The region/face R containing q can be reported efficiently.

Results



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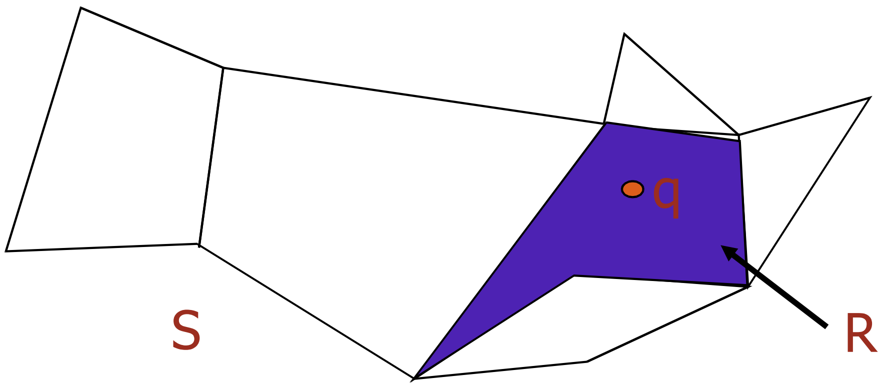
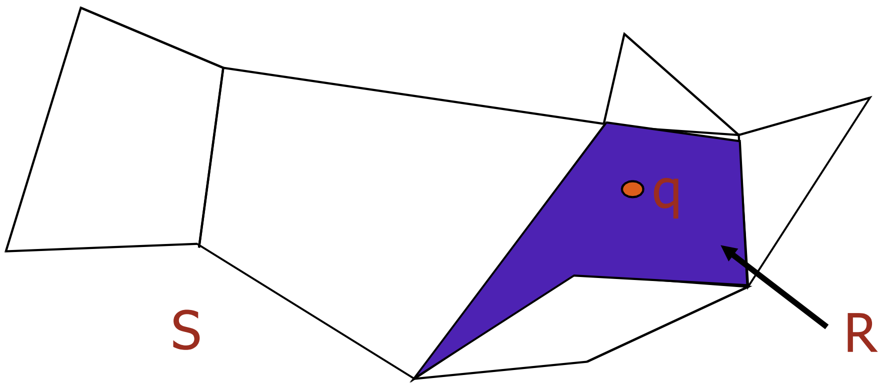


Preprocessing Time: O(n)

Results



Preprocessing Time: $O(n)$

Preprocessing space requirement: $O(n)$

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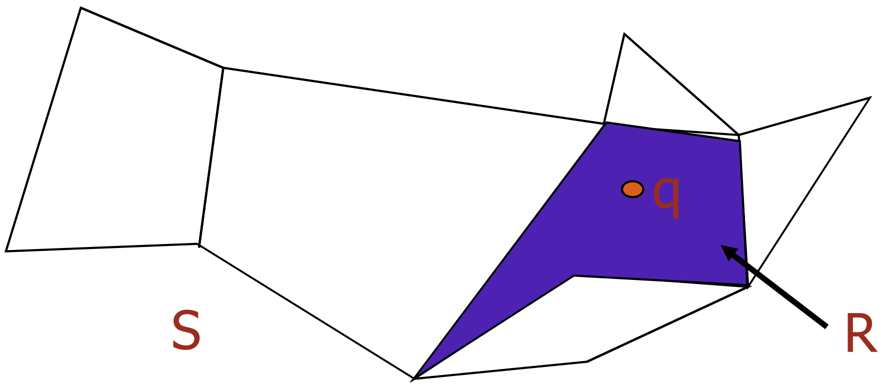

Preprocessing Time: O(n)

Preprocessing space requirement: O(n)

Query Time: O(log n)

Thank You